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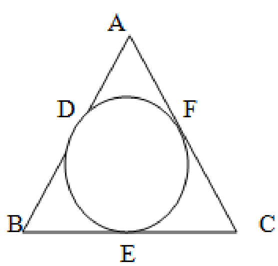
Class: X
Subject: Math's
Topic: Circles
No. of Questions: 20

Q.1 Find the length of the given tangent drawn from a point whose distance from the centre of a circle is 25 cm. Given that the radius of the circle is 7 cm.

Sol: Let P be any given point, O is the centre of the circle and PT be the length of the tangent from P. Then, OP=25 cm and OT=7 cm
Since the tangent is always perpendicular to the radius through the point of contact.
In ΔOTP,
$OP^2=OT^2 + PT^2$
$25^2=7^2 + PT^2$
$PT^2=576$
PT=24 cm

Q.2 In the given figure, if AB=AC, prove that BE=EC. (CBSE-2012)

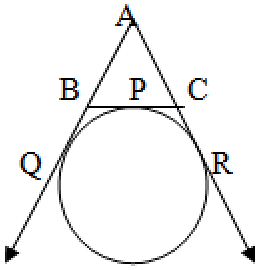
Sol: Since tangents from an exterior point to a circle are equal in length.
AD=AF
BD=BE
CE=CF
Now, AB=AC
AB – AF=AC – AD
AB – AD=AC – AF
BD=CF
BE=C
BE=CE.



Q.3 A circle is touching the side BC of ΔABC at P and touching AB and AC produced at Q and R respectively. Prove that AQ=1/2 (Perimeter of ΔABC).

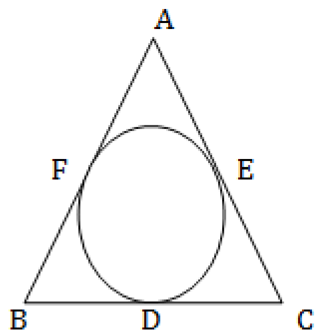
(CBSE-2002)

Sol: Since tangents from the exterior point to a circle are equal in length.
BP=BQ (From B).......(1)
CP=CQ (From C).......(2)
AQ=AR (From A)......(3)
From (3),
AQ=AR
AB + BQ= AC + CR
AB + BP=AC + CP (From (1) and (2))
Perimeter of ΔABC=AB + BC + CA
Perimeter of ΔABC=AB + (BP + PC) + AC
Perimeter of ΔABC=(AB + BP) + (AC + PC)
Perimeter of ΔABC=2(AB + BP)
Perimeter of ΔABC=2(AB + BQ)
Perimeter of ΔABC=2AQ
AQ=1/2(Perimeter of ΔABC)



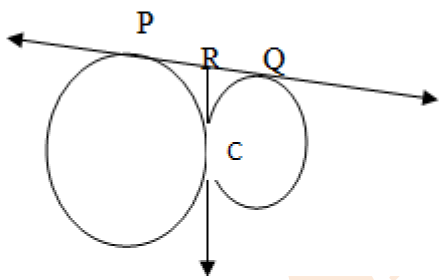
Q.4 In the figure, the incircle of ΔABC touches the sides BC, CA and AB at D,E and F respectively. Show that AF + BD + CE= AE + BF + CD=1/2(Perimeter of ΔABC)

Sol: Since length of tangents from an external point to a circle are equal.
AF=AE (From A)
BD=BF (From B)
CE=CD (From C)
Adding above 3 equations,
AF + BD + CE=AE + BF + CD
Now, Perimeter of ΔABC= AB + BC + CA
Perimeter of ΔABC=(AF + FB) + (BD + CD) + (AE + EC)
Perimeter of ΔABC=2AF + 2BD + 2CE
AF + BD + CE=1/2(Perimeter of ΔABC)
AF + BD + CE=AE + BF + CD=1/2(Perimeter of ΔABC)



Q.5 In the given figure, two circles touch each other at point. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q. (CBSE-2013)

Sol: The tangents drawn from an external point to a circle are equal.
RP=RC and RC=RQ
RP=RQ
R is the mid-point of PQ.



Q.6 A circle touches all the four sides of a quadrilateral ABCD. Prove that AB + CD=BC + DA.
(CBSE-2012)

Sol: Since tangents drawn from an exterior point to a circle are equal in length.
AP=AS (From A)
BP=BQ(From B)
CR=CQ (From C)
DR=DS (From D)
Add all the above equations,
AP + BP + CR + DR= AS + BQ + CQ + DS

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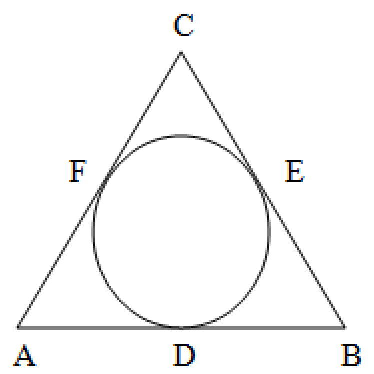
(AP + BP) +(CR + DR)= (AS + DS) +(CQ + BQ)
AB + CD=AD + BC
AB + CD= BC + DA

Q.7 A circle is inscribed in ΔABC having 8 cm, 10 cm and 12 cm as shown in figure. Find AD, BE and CF.

(CBSE-2001)

Sol: Tangents drawn from an external point to a circle are equal in length.
AD=AF=x, say
BD=BE=y, say
CE=CF=z, say
Now, AB=12 cm, BC= 8cm
and CA= 10 cm
x + y=12, y + z=8 and z + x=10
(x + y) + (y + z) + (z + x)=12 + 8 + 10
2(x + y + z)=30
X + y + z=15
Now,
x + y=12 and x + y + z=15
z=3
y + z=8 and x + y + z=15
x=7
z + x=10 and x + y + z=15
y=5
Hence, AD=x=7 cm, BE=y=5 cm and CF=z=3 cm.



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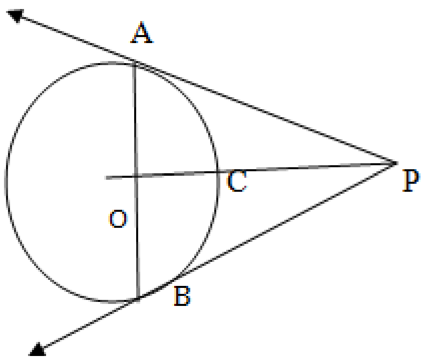
Q.8 Fromm an external point P, two tangents PA and PB are drawn to the circle with centre O. Prove that OP is the perpendicular bisector of AB.

Sol: Suppose OP intersects AB at C.
In triangles PAC and PBC,
PA=PB (tangents from an external point are equal)
∠APC=∠BPC (PA and PB are equally inclined)
And PC=PC
So, by SAS criterion of congruency,
ΔPAC≅ ΔPBC
AC=BC and ∠ACP=∠BCP
∠ACP + ∠BCP=180^0
∠ACP + ∠BCP=90^0
Hence, OP is perpendicular to AB.



Q.9 A circle touches the sides of a quadrilateral ABCD at P, Q, R and S respectively. Show that the angles subtended at the centre by a pair of opposite sides are supplementary.

(CBSE-2012)

Sol: Construction: Join OP, OQ, OR and OS.
Proof: The tangents from an external point to a circle are equal in length.
Let ∠SOA=∠1, ∠SOP=∠2, ∠POB=∠3, ∠BOQ=∠4,
∠QOC=∠5, ∠COR=∠6, ROD=∠7 and ∠DOS=∠8.
∠1=∠2, ∠3=∠4, ∠ 5=∠6 and ∠7=∠8
∠1 + ∠2+ ∠3+ ∠4+ ∠5+∠6 + ∠7+ ∠8=180^0
2(∠2 + ∠3+ ∠6+∠7)=360^0 and 2(∠1+∠8+∠4+∠5)=360^0
(∠2 + ∠3)=180^0 and (∠1 + ∠8) + (∠4 + ∠5)=180^0
∠AOB + ∠COD=180^0
∠AOD + ∠BOC=180^0

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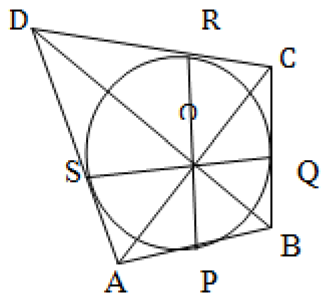
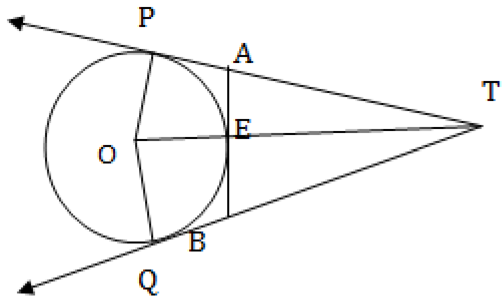


Q.10 O is the centre of a circle of radius 5 cm. T is a point such that OT=13 cm and OT intersects the circle at E. If AB is the tangent to the circle at E, find the length of AB.

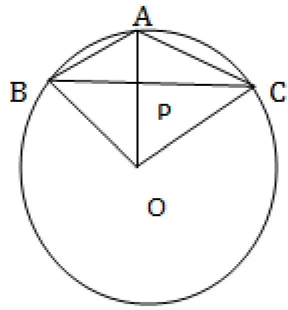
Sol: $\angle OPT=90^0$
Applying Pythagoras theorem,
$OT^2=OP^2 + PT^2$
$13^2= 5^2 + PT^2$
$PT^2= 169 - 25=144$
PT=12 cm
Since the lengths of tangents from an external point to a circle are equal,
AP=AE=x (say)
AT=PT – AP=(12 – x) cm
Since AB is the tangent to the circle E. Therefore, OE is perpendicular to AB.
$\angle OEA=90^0$
$\angle AET=90^0$
$AT^2= AE^2 + ET^2$
$(12 - x)^2= x^2 + (13 - 5)^2$
$144 - 24x + x^2= x^2 + 64$
24x=80
x=10/3 cm
Similarly, BE=10/3 cm
AB=AE + BE=(10/3 + 10/3)=20/3 cm



Q.11 If an isosceles triangle ABC in which AB=AC=6 cm is inscribed in a circle of radius 9 cm, find the area of the triangle.

Sol: Let O be the centre of the circle and let P be the mid-point of BC. Then OP⊥BC.
Since ΔABC is isosceles and P is the mid-point of BC. Therefore, AP⊥BC as median from the vertex in an isosceles triangle is perpendicular to the base.
Let AP=x and PB=CP=y

Applying Pythagoras theorem,
$AB^2= BP^2 + AP^2$ and $OB^2=OP^2 +BP^2$
$36=x^2 + y^2$...(1) and $81=(9 – x)^2 + y^2$ ....(2)
$81 – 36=\{(9 – x)^2 + y^2\} – \{y^2 + x^2\}$
$45=81 – 18x$
$x = 2$ cm
Putting x=2 cm in (1)
$36=y^2 + 4$
$y=4\sqrt{2}$ cm
$BC=2BP=2y=8\sqrt{2}$ cm



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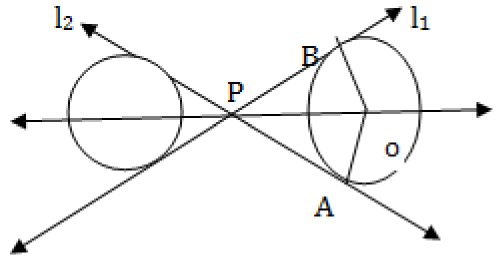
Q.12 Find the locus of centres of circles which touch two intersecting lines.

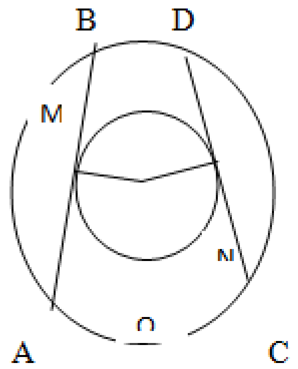
Sol: Let l_1 and l_2 be two intersecting lines which intersect at P.
Let O be the centre of the circle which touches both l_1 and l_2
In ΔOAP and ΔOBP,
OA=OB (radius)
PA=PB (Tangents from external points)
nd OP=OP
So, be SSS congruency criterion, ΔOAP ≅ ΔOBP
∠APO=∠BPO
OP is the bisector of the angle APB.
O lies on the bisector of the angle between l_1 and l_2



Q.13 In concentric circles, prove that all the chords of the outer circle which touch the inner are of equal length.

Sol: Let AB and CD be the two chords of the circle which touch the inner circle at M and N respectively.

Since AB and CD are two tangents to the smaller circle
Om=ON=radius of the smaller circle.
Thus, AB and CD are two chords of the larger circle such that they are equidistant from the centre.
Hence, AB=CD.



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Q.14 Find the locus of the centres of circles which touch a given line at a given point.

Sol: Let APB be the given line, and let a circle with centre O touch APB at P. Then, $\angle OPB=90^0$

Let there be another circle with centre O' which touches the line APB at P. Then, $\angle O'PB=90^0$

This is possible only when O and O' lie on the same line O'OP. Hence, the required locus is a line perpendicular to the given line at the point of contact.

Q.15 In the given figure, O is the centre of the circle. PA and PB are tangent segments. Show that the quadrilateral AOBP is cyclic.

Sol: Since tangents at a point to a circle is perpendicular to the radius through the point.

$OA \perp AP$ and $OB \perp BP$.

$\angle OAP=90^0$ and $\angle OBP=90^0$

$\angle OAP + \angle OBP=180^0$

In quadrilateral OABP,

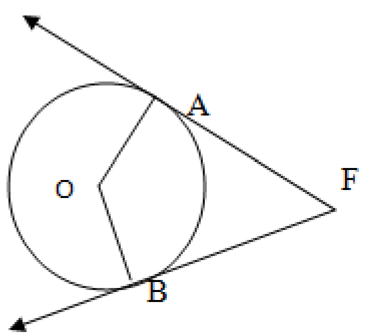
$\angle OAP + \angle APB + \angle AOB + \angle OBP=360^0$

$(\angle APB + \angle AOB) + (\angle OAP + \angle OBP)=360^0$

$\angle APB + \angle AOB+ 180^0=180^0$

$\angle APB + \angle AOB=180^0$

Hence, we can say that AOBP is a quadrilateral.



Q.16 In two concentric circles, a chord of length 24 cm of larger circle becomes a tangent to the smaller circle whose radius is 5 cm. Find the radius of the larger circle.

Sol: Let O be the centre of the concentric circles and APB be the chord of length 24 cm, of the

larger circle touching the smaller circle at P. Then, $OP \perp AB$ and P is the mid-point of AB.

AP=PB=12 cm

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In ΔOPA,
$OA^2= OP^2 + AP^2$
$OA^2=5^2 + 12^2= 169$
OA=13 cm
Hence, the radius of the smaller circle is 13 cm.

Q.17 In two concentric circles, prove that a chord of larger circle which is tangent to smaller circle is bisected at the point of contact.
(CBSE-2012)

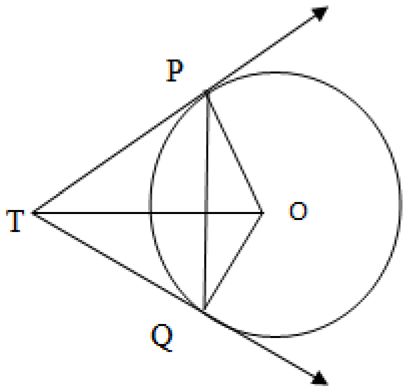
Sol: Let O be the common centre of two concentric circles, and let AB be the chord of the larger circle touching the smaller circle at P.

Join OP.
Since OP is the radius of the smaller circle and AB is a tangent to this circle at a point P.
OP ⊥AB.
We know that the perpendicular drawn from the centre of a circle to any chord of the circle, bisects the chord. So,
OP⊥AB
AP=BP
Hence, AB is bisected at P.

Q.18 Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ=2∠OPQ.
(CBSE-2009)

Sol: We know that lengths of tangents drawn from an external point to a circle are equal in length.
TP=TQ
ΔTPQ is an isosceles triangle.
∠TPQ=∠TQP
In ΔTPQ,
$\angle TPQ + \angle TQP + \angle PTQ=180^0$
$2\angle TPQ = 180^0 - \angle PTQ$
$\angle TPQ=180^0 – 1/2\ \angle PTQ$
$1/2\angle PTQ= 90^0 – \angle TPQ$
OP⊥TP
$\angle OPT=90^0$
$\angle OPQ + \angle TPQ=90^0$

∠OPQ=90^0 – ∠TPQ
1/2∠ PTQ=∠OPQ
∠PTQ=2∠OPQ



Q.19 If all the sides of a parallelogram touch the circle, show that the parallelogram is a rhombus.

(CBSE-2013)

Sol: Let ABCD be a parallelogram such that its sides touch the circle with centre O.
AP=AS
BP=BQ
CR=CQ
DR=DS
Add all the above equations,
AP + BP + CR + DR= AS + BQ + CQ + DS
(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)
AB + CD=AD + BC
2AB= 2BC
AB=BC
AB=BC=CD=DA
Hence, ABCD is a rhombus.

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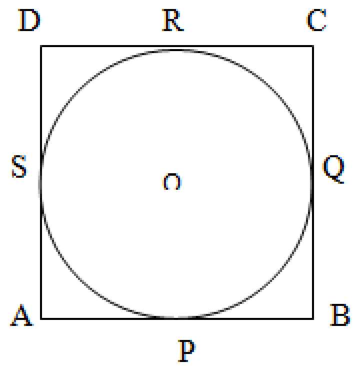

Q.20 In the given figure, the sides AB, BC and CA of triangle ABC touch the circle with centre O and radius P, Q and R respectively. Prove that Area(ΔABC)=1/2(perimeter of ΔABC) x r

Sol: Area(ΔABC)=Area(ΔOBC) + Area(ΔOAB) + Area(ΔOAC)
=1/2(BC x OQ) + 1/2(AB x OP) + 1/2(AC x OR)
=1/2(BC x r) + 1/2(AB x r) + 1/2(AC x r)
=1/2(BC + AB + AC) x r
=1/2(Perimeter of ΔABC) x r.