

Class: X
Subject: Math's
Topic: Coordinate geometry
No. of Questions: 20

Q.1 Find the distance between the points A(-6, 7) and B(-1, -5).

Sol: $x_1 = -6, y_1 = 7, x_2 = -1, y_2 = -5$
 $AB = \sqrt{\{(-1 + 6)^2 + (-5 - 7)^2\}} = \sqrt{169} = 13$

Q.2 Find a point on x-axis which is equidistant from A(2,-5) and B(-2,9). (CBSE-2009)

Sol: Let P be the point equidistant from A and B.
 $PA = PB$
 $\sqrt{\{(x - 2)^2 + (0 + 5)^2\}} = \sqrt{\{(x + 2)^2 + (0 - 9)^2\}}$
 $(x - 2)^2 + 25 = (x + 2)^2 + 81$
 $-8x = 56$
 $x = -7$

Q.3 The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from Q(2, -5) and R(-3,6), then find the coordinates of P. (CBSE-2010)

Sol: Let the coordinates of P be (x,y)
Also $x = 2y$
 $PQ = PR$
 $\sqrt{\{(x - 2)^2 + (y + 5)^2\}} = \sqrt{\{(x + 3)^2 + (y - 6)^2\}}$
 $5y^2 + 2y + 29 = 5y^2 + 45$
 $2y = 16$
 $y = 8$
Coordinates of P are (16,8)

Q.4 Show that A(6,4), B(5,-2) and C(7,-2) are the vertices of an isosceles triangle. Also, find the length of the median through A. (CBSE-2010)

Sol: Let A(6,4); B(5,-2); C(7,-2)
 $AB = \sqrt{37}$; $AC = \sqrt{37}$
 $AB = AC$
This shows that ΔABC is isosceles.
Let D be the mid-point of BC.
Coordinates of D are (6, -2)
 $AD = \sqrt{\{(6 - 6)^2 + (4 + 2)^2\}}$

$$AD=6$$

- Q.5 Determine the ratio in which the line $3x + y - 9=0$ divides the segment joining the points (1,3) and (2,7). (CBSE-2008)

Sol: Suppose the ratio is $K:1$

$$\text{The coordinates of points are: } \left(\frac{2K+1}{K+1}, \frac{7K+3}{K+1} \right)$$

Also, the point lies on $3x + y - 9=0$

$$3\left(\frac{2K+1}{K+1}\right) + \frac{7K+3}{K+1} - 9=0$$

$$6K + 3 + 7K + 3 - 9K - 9=0$$

$$K = 3/4$$

$$K:1 = 3:4$$

- Q.6 If $A(-2,-1)$, $B(a,0)$, $C(4,b)$ and $d(1,2)$ are the vertices of a parallelogram, find the values of a and b . (CBSE-2006)

Sol: Since the diagonals of a parallelogram bisect each other.
Coordinates of mid-point of AC are same as the coordinates of mid-point of BD.

$$\left(\frac{-2+4}{2}, \frac{-1+b}{2} \right) = \left(\frac{a+1}{2}, \frac{0+2}{2} \right)$$

$$\left(1, \frac{b-1}{2} \right) = \left(\frac{a+1}{2}, 1 \right)$$

$$\frac{a+1}{2} = 1; \frac{b-1}{2} = 1$$

$$a+1=2; b-1=2$$

$$a=1; b=3$$

- Q.7 If $A(5,-1)$, $B(-3,-2)$ and $C(-1,8)$ are the vertices of ΔABC , find the length of median through A and the coordinates of the centroid. (CBSE-2006)

Sol: Let AD be the median through vertex A of ΔABC . Then, D is the mid-point of BC.

$$\text{Coordinates of D} = \left(\frac{-3-1}{2}, \frac{-2+8}{2} \right) \text{ i.e., } (-2,3)$$

$$AD = \sqrt{\{(5+2)^2 + (-1-3)^2\}} = \sqrt{65} \text{ units.}$$

Let G be the centroid.

$$\text{Coordinates of G are } \left(\frac{2x(-2) + 1x5}{2+1}, \frac{2x3 + 1x(-1)}{2+1} \right) = (1/3, 5/3).$$

Q.8 Find the coordinates of the centroid of a triangle whose vertices are (0,6), (8,12) and (8,0).

Sol: $\{(x_1 + x_2 + x_3)/3, (y_1 + y_2 + y_3)/3\}$
 $\{(0+8+8)/3, (6+ 12+0)/3\} = (16/3, 6)$

Q.9 If $x - 2y + k=0$ is a median of the triangle whose vertices are at points A(-1,3), B(0,4) and C(-5,2), find the value of k.

Sol: $(\frac{-1+0-5}{3}, \frac{3+4+2}{3}) = (-2,3)$
Now, the point lies on the median $x - 2y + k=0$
 $-2 -2(3) + k=0$
 $k=8$

Q.10 Find the area of the triangle whose vertices are $(t, t - 2)$, $(t + 2, t + 2)$ and $(t + 3, t)$

Sol: Area= $1/2 |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$
 $= 1/2 | \{ t(t+2 - t) + (t + 2)(t - t+2) + (t+3)(t - 2 - t - 2) \} |$
 $= 1/2 | \{ 2t + 2t + 4 - 4t - 12 \} | = |-4| = 4 \text{ sq. units}$

Q.11 Prove that the points $(a, b + c)$; $(b, c + a)$ and $(c, a + b)$ are collinear. (CBSE-2010)

Sol: $\Delta = 1/2 | \{ a(c + a) + b(a + b) + c(b + c) \} - \{ b(b + c) + c(c + a) + a(a + b) \} |$
 $= 1/2 | (ac + a^2 + ab + b^2 + bc + c^2) - (b^2 + bc + c^2 + ca + a^2 + ab) |$
 $= 0$
Hence, the given points are collinear.

Q.12 For what value of x will the points $(x, -1)$, $(2,1)$ and $(4,5)$ lie on a line?(CBSE-2013)

Sol: Area =0
 $| \{ x * 1 + 2 * 5 + 4 * (-1) \} - \{ 2 * (-1) + 4 * 1 + x * 5 \} | = 0$
 $(x + 10 - 4) - (-2 + 4 + 5x) = 0$
 $(x + 6) - (5 + 2) = 0$
 $-4x + 4 = 0$
 $x = 1$

Q.13 If P(x,y) is any point on the line joining the points A(a,0) and B(0,b), then find the value of $x/a + y/b$. (CBSE-2009)

Sol: $(x * 0 + a * b + 0 * y) - (a * y + 0 * 0 + x * b) = 0$
 Since A,B and C are collinear points
 $ab - (ay + bx) = 0$
 $ab = ay + bx$
 $ab/ab = ay/ab + bx/ab$
 $1 = y/b + x/a$ or $x/a + y/b = 1$

Q.14 Find k so that the point P(-4,6) lies on the line segment joining A(k,10) and B(3,-8). Also, find the ratio in which P divides AB. (CBSE-2010)

Sol: P, A and B are collinear
 $(-4 * 10 + k * -8 + 3 * 6) - (6k + 30 - 4 * -8) = 0$
 $(-40 - 8k + 18) - (6k + 30 + 32) = 0$
 $-14k - 84 = 0$

Let P divides AB in the ratio X:1

Coordinates are $(\frac{3X - 6}{X + 1}, \frac{-8X + 10}{X + 1})$

$$\frac{3X - 6}{X + 1} = -4 \text{ and } \frac{-8X + 10}{X + 1} = 6$$

$$X = 2$$

Ratio = 2:7

Q.15 Three vertices of a parallelogram ABCD are A(3,-4), B(-1,-3) and C(-6,2). Find the coordinates of vertex D and find the area of the parallelogram ABCD. (CBSE-2013)

Sol: Mid-points of diagonals AC and BD are same as they bisect each other.

$$\left(\frac{x-1}{2}, \frac{y-3}{2}\right) = \left(\frac{3-6}{2}, \frac{-4+2}{2}\right)$$

$$\frac{x-1}{2} = \frac{3}{2} \text{ and } \frac{y-3}{2} = -1$$

$$x = -2, y = 1$$

coordinates of D are (-2,1)

Area of parallelogram ABCD = 2(Area of ΔABC)

$$\begin{aligned} \text{Area of } \Delta ABC &= \frac{1}{2} | \{3 * (-3) + (-1) * 2 + (-6) * (-4)\} - \{3 * 2 + (-6) * (-3) + (-1) * (-4)\} | \\ &= \frac{15}{2} \text{ sq. units} \end{aligned}$$

$$\text{Area of parallelogram ABCD} = 2 * \frac{15}{2} = 15 \text{ sq. units}$$

Q.16 If the area of ΔABC formed by $A(x,y)$, $B(1,2)$ and $C(2,1)$ is 6 square units, then prove that $x+y=15$ or $x+y+9=0$. (CBSE-2010)

Sol: Area of $\Delta ABC=6$
 $\frac{1}{2} |(2x + 1 + 2y) - (x + 4 + y)|=6$
 $|x + y - 3|=12$
 $x + y - 3=\pm 12$
 $x + y - 15=0$ or $x + y + 9=0$
 $x + y=15$ or $x + y + 9=0$

Q.17 Find the area of the quadrilateral, the coordinates of whose vertices are $(-3,2)$, $(5,4)$, $(7,-6)$ and $(-5,-4)$.

Sol: Let $A(-3,2)$, $B(5,4)$, $C(7,-6)$ and $D(-5,-4)$
 Area of $\Delta ABC = \frac{1}{2} |(-12 - 30 + 14) - (10 + 28 + 18)| = 42$ sq. units
 Area of $\Delta ACD = \frac{1}{2} |(18 - 28 - 10) - (14 + 30 + 12)| = 38$ sq. units
 Area of quadrilateral ABCD = $(42 + 38) = 80$ sq. units

Q.18 If the points $A(1,-2)$, $B(2,3)$, $C(-3,2)$ and $D(4,-3)$ are the vertices of a parallelogram ABCD, then taking AB as the base, find the height of the parallelogram. (CBSE-2013)

Sol: : Let the height be h , BD is the diagonal
 Area of $\Delta ABD = \frac{1}{2} * b * h$
 $H = 2(\text{Area of } \Delta ABD)/AB$
 $b = \sqrt{(2 - 1)^2 + (3 + 2)^2} = \sqrt{26}$
 Area of $\Delta ABD = \frac{1}{2} | \{1 * 3 + 2 * (-3) + (-4) * (-2)\} - \{1 * (-3) + (-4) * 3 + 2 * (-2)\} |$
 $= \frac{1}{2} (5 + 19) = 12$ sq. units
 $h = 2 * 12 / \sqrt{26} = 24 / \sqrt{26}$ units

Q.19 If the points (p,q) , (m,n) and $(p - m, q - n)$ are collinear, show that $pn=qm$. (CBSE-2010)

Sol: Points are collinear
 $\{p * n + m(q - n) + (p - m)q\} - \{m * q + (p - m)n + p(q - n)\} = 0$
 $(pn + qm - mn + pq - mq) - (mq + pn - mn + pq - pn) = 0$
 $(pn + pq - mn) - (mq - mn + pq) = 0$
 $pn - mq = 0$
 $pn = mq$

Q.20 Find the area of a triangle whose vertices are A(3,2), B(11,8) and C(8,12)

Sol: Area of $\Delta ABC = \frac{1}{2} | \{ 3(8 - 12) + 11(12 - 2) + 8(2 - 8) \} | = \frac{1}{2} | \{-12 + 110 - 48\} | = 25$ sq. units

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