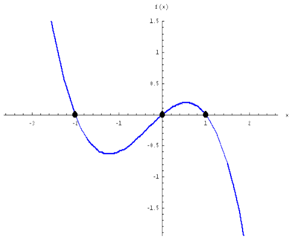
Class: 10
Subject: Math's
Topic: Polynomials
No. of Questions: 20

Q.1 Find the number of zeroes of P(x)



Solution: 3

[Explanation: The graph intersects x-axis at 3 points]

Q.2 Find the zeroes of a polynomial $x^2+7x+10$

Solution: -2 , -5

[Explanation: $x^2+7x+10$
By splitting the middle term
$= x^2+7x+10$
$=x^2+5x+2x+10$
$=x(x+5)+2(x+5)$
$=(x+2)(x+5)$
Now on equating the zeroes to 0
x+5=0 which gives x=-5
and x+2=0 which gives x=-2]

Q.3 Find the zeroes of the polynomial $p(x)=x^2-3$

Solution: $\sqrt{3}$, - $\sqrt{3}$

[Explanation: Using the identity $(a+b)(a-b)= a^2-b^2$
$x^2-3= (x+\sqrt{3})(x-\sqrt{3})$
Equating $(x+\sqrt{3})=0$ we get $x=-\sqrt{3}$
Similarly, $(x-\sqrt{3})=0$, we get $x=\sqrt{3}$]

Q.4 Find the zeroes of the polynomial $p(x)=6x^2-3-7x$ and verify the relationship between them.

Solution: -1/3 and 3/2

[Explanation: $p(x)=6x^2-3-7x = 6x^2-7x-3$
By splitting the middle term,
$p(x)=6x^2-7x-3$
$= 6x^2 -9x+2x-3$
$=3x(2x-3)+1(2x-3)$
$=(2x-3)(3x+1)$
Now on putting zeroes equal to 0, we get x= -1/3 and 3/2
Comparing p(x) with ax^2+bx+c
Sum of zeroes= -1/3 + 3/2= 7/6 = -b/a
Product of zeroes= (-1/3)(3/2)= -1/2= c/a]

Q.5 Find the quadratic polynomial, the sum and product of zeroes are -3 and 2 respectively.

Solution: x^2+3x+2

[Explanation: On comparing p(x) by ax^2+bx+c
Sum of zeroes= -b/a= -3/1
b= 3, a=1
Product of zeroes= c/a=2/1
c=3 , a=1
Now, $ax^2+bx+c= x^2+3x+2$]

Q.6 Find the zeroes of the polynomial $p(x)= 2x^3-5x^2-14x+8$ and find the relationship between them.

Solution: 4, -2, 1/2

[Explanation: $p(x)= 2x^3-5x^2-14x+8$
Factors of 8 are ±1, ±2, ±4
Put x=-2
$P(-2)= 2(-2)^3-5(-2)^2-14(-2)+8$
$= -16-20+36= 0$
(x+2) is a factor of p(x)
Now on dividing p(x) by (x+2), we get
$P(x)= (x+2)(2x^2-9x+4)$
$= (x+2)(2x^2-8x-x+4)$
$= (x+2)[2x(x-4)-1(x-4)]$
$= (x+2)(x-4)(2x-1)$

Zeroes are -2, 4, 1/2
On comparing p(x) by ax^3+bx^2+cx+d

Sum of zeroes= -2+4+1/2= 5/2= -b/a
Product of zeroes= -2 x 4 x 1/2= -4= -d/a
Sum of product of zeroes= 4 x (-2)+ 1/2 x 4 + 1/2 x (-2)= -7= c/a]

Q.7 Verify 3, -1, -1/3 are the zeroes of the cubic polynomial $p(x)= 3x^3-5x^2-11x-3$
Solution: Yes, 3, -1 -1/3 are the zeroes of p(x)

[Explanation: $p(x)= 3x^3-5x^2-11x-3$
$P(3)= 3(3)^3-5(3)^2-11(3)-3$
$= 81-45-33-3= 0$

$P(-1)= 3(-1)^3-5(-1)^2-11(-1)-3$
$= -3-5+11-3= 0$

$P(-1/3)=3(-1/3)^3-5(-1/3)^2-11(-1/3)-3$
$= -1/9 + 5/9 -11/3-3= 0$
Since the value of p(x) is 0 at 3, -1 and -1/3
So they are the zeroes of the polynomial]

Q.8 If the sum of the product of zeroes is -1, sum and product of zeroes are 3 and -3 respectively, find the cubic polynomial.

Solution: x^3-3x^2-x+3

[Explanation: Sum of products of zeroes= -1= c/a
Sum of zeroes= 3= -b/a
Product of zeroes= -3= -d/a
Now, a=1, b=-3, c= -1 and d= 3
On putting the values in ax^3+bx^2+cx+d
$P(x)= x^3-3x^2-x+3$]

Q.9 Find the factors of $4s^2-4s+1$

Solution: (2s-1)(2s-1)

[Explanation: $4s^2-4s+1$
$= 4s^2-2s-2s+1$
$= 2s(2s-1)-1(2s-1)$
$= (2s-1)(2s-1)$]

Q.10 Divide $p(x)= x^4-5x+6$ by $g(x)= 2-x^2$ and find the quotient and remainder.

Solution: Quotient= $-x^2-2$, Remainder= $-5x+10$

[Explanation:

$$\begin{array}{r|l} & -x^2-2 \\ \hline -x^2+2 & x^4 \qquad -5x+6 \\ & x^4-2x^2 \\ & - \quad + \\ \hline & \quad -2x^2-5x+6 \\ & \quad -2x^2 \quad -4 \\ & \quad + \qquad + \\ \hline & \qquad -5x+10 \end{array}$$

]

Q.11 On dividing x^3-3x^2+x+2 by a polynomial g(x), the quotient and remainder were (x-2) and (-2x+4) respectively. Find g(x)

Solution: $g(x)= x^2-x+1$

[Explanation: $p(x)= x^3-3x^2+x+2$

$P(x)= g(x) \times q(x)+ r(x)$

$(x^3-3x^2+x+2)-+2x-4= g(x) \times q(x)$

$(x^3-3x^2+3x-2)/(x-2)= g(x)$

$g(x)= x^2-x+1$]

Q.12 Factorize $p(x)= 2x^3+x^2-2x-1$

Solution: (x-1)(2x+1)(x+1)

[Explanation: $p(x)= 2x^3+x^2-2x-1$

Factors of 1 are ±1

Put x=1

$P(1)= 2(1)^3+(1)^2-2(1)-1$

$= 2+1-2-1=0$

(x-1) is a factor of p(x)

On dividing p(x) by (x-1), we get

$P(x)= (x-1)(2x^2+3x+1)$

$= (x-1)[2x^2+2x+x+1]$

$= (x-1)[2x(x+1)+1(x+1)]$

$= (x-1)(x+1)(2x+1)]$

Q.13 If $x+y+z=0$, then show that $x^3+y^3+z^3=3xyz$

Solution: Refer to explanation

[Explanation: $x+y+z=0$
Using identity $(x+y+z)(x^2 + y^2 + z^2 -xy-yz-zx)= x^3+y^3+z^3-3xyz$
Now $x+y+z=0$
$(0)= x^3+y^3+z^3-3xyz$
$x^3+y^3+z^3=3xyz$]

Q.14 The polynomial $x^4-6x^3+16x^2-25x+10$ is divided by another polynomial x^2-2x+k, the remainder is $x+a$, find k and a

Solution: $k= 5$, $a=-5$

[Explanation: $x^2-4x+(8-k)$
x^2-2x+k | $x^4-6x^3+16x^2-25x+10$
$x^4-2x^3+kx^2$
$- \quad + \quad -$

$-4x^3+(16-k)x^2-25x$
$-4x^3+8x^2 \quad - 4kx$
$+ \quad - \quad +$

$(8-k)x^2+(-25+4k)x+10$
$(8-k)x^2-(16-2k)x+k(8-k)$
$- \quad + \quad -$

$(-9+2k)x+10-8k+k^2$

Now the remainder is $(-9+2k)x+(10-8k+k^2)$
$(-9+2k)x+(10-8k+k^2)= x+a$
$(-9+2k)x+(k^2-8k+10)= x+a$
$(-9+2k)x= x$ and $(k^2-8k+10)=a$
$2k=10$
$k=5$

Putting $k=5$ in equation $(k^2-8k+10)=a$
$(5)^2-8(5)+10=a$
$a= -5$]

Q.15 Give possible expression for the length and breadth of the rectangle having area $25a^2-35a+12$

Solution: Length= 5a-3 and breadth= 5a-4

[Explanation: Area= $25a^2-35a+12$

On factorizing we will get the required length and breadth

Let $p(x)= 25a^2-35a+12$

$= 25a^2-15a-20a+12$

$= 5a(5a-3)-4(5a-3)$

$=(5a-3)(5a-4)$

The possible value of length is 5a-3 and breadth is 5a-4]

Q.16 Find the value of k in $p(x)=kx^2-\sqrt{2}x+1$ if (x-1) is a factor of p(x)

Solution: $k= \sqrt{2} -1$

[Explanation: $p(x)= kx^2-\sqrt{2}x+1$

Now, p(1)=0

$P(1)= k(1)^2-\sqrt{2}(1)+1$

$0= k-\sqrt{2}+1$

$k= \sqrt{2} -1$]

Q.17 Find all the zeroes of $2x^4-3x^3-3x^2+6x-2$ if you know that two of the zeroes are $\sqrt{2}$ and $-\sqrt{2}$

Solution: 1/2 and 1

[Explanation: Since $\sqrt{2}$ and $-\sqrt{2}$ are the two zeroes then $(x-\sqrt{2})(x+\sqrt{2})= x^2-2$ is a factor of p(x)

On dividing p(x) by (x^2-2) we get $(x^2-2)(2x^2-3x+1)$

On splitting the middle term,

$P(x)= (x^2-2)[x^2-2x-x+1]$

$= (x^2-2)[2x(x-1)-1(x-1)]$

$=(x^2-2)(2x-1)(x-1)$

Zeroes are 1/2 and 1]

Q.18 Find a cubic polynomial with the sum, sum of the product of its zeroes and the product of its zeroes is 2, -7, -14 respectively.

Solution: $x^3-2x^2-7x+14$

[Explanation: Sum of zeroes= 2= -b/a
Sum of product of zeroes= -7= c/a
Product of zeroes= -14= -d/a
a=1, b=-2, c=-7 and d= 14
Now, $a^3+bx^2+cx+d= x^3-2x^2-7x+14$]

Q.19 A polynomial $p(x)= 3x^4-4x^3-3x-1$ is divided by (x-1) and the remainder comes out to be -5, find the quotient.

Solution: $3x^3-x^2-x-4$

[Explanation: $p(x)= 3x^4-4x^3-3x-1$
P(x)= g(x) x q(x) + r(x)
$3x^4-4x^3-3x-1= (x-1)$ x q(x) +(-5)
$(3x^4-4x^3-3x-1)/(x-1)= q(x)$
$q(x)= 3x^3-x^2-x-1$]

Q.20 Using factor theorem determine whether g(x) is a factor of p(x) in $p(x)= x^3-4x^2+x+6$ and g(x)= x-3

Solution: Yes. g(x) is a factor of p(x)

[Explanation: $p(x)= x^3-4x^2+x+6$
Using factor theorem
g(x)= 0
x-3=0
x=3
$p(3)= (3)^3-4(3)^2+3+6$
= 27-36+6=0
Since p(3)=0, g(x) is a factor of p(x)]