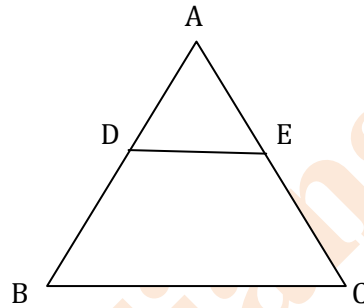


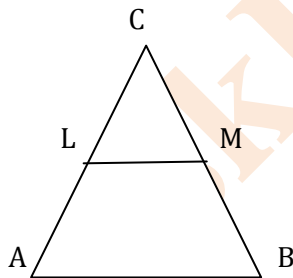
Class: X
Subject: Maths
Topic: Triangles
No. of Questions: 20

Q.1 In a given $\triangle ABC$, $DE \parallel BC$ and $AD/DB = 3/5$. If $AC = 5.6$, find AE .

Solution 1: In $\triangle ABC$,
 $DE \parallel BC$
 $AD/DB = AE/EC$
 $AD/DB = AE/(AC - AE)$
 $3/5 = AE/(5.6 - AE)$
 $16.8 - 3AE = 5AE$
 $8AE = 16.8$
 $AE = 16.8/8 = 2.1$ cm

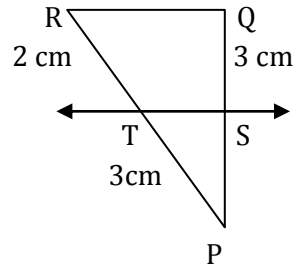


Q.2 In the given figure, $LM \parallel AB$. If $AL = x - 3$, $AC = 2x$, $BM = x - 2$ and $BC = 2x + 3$, find the value of x .



Solution 2: In $\triangle ABC$,
 $LM \parallel AB$
 $AL/LC = BM/MC$
 $AL/(AC - AL) = BM/(BC - BM)$
 $(x - 3)/[2x - (x - 3)] = (x - 2)/[(2x + 3) - (x - 2)]$
 $(x - 3)/(x + 3) = (x - 2)/(x + 5)$
 $(x - 3)(x + 5) = (x - 2)(x + 3)$
 $x^2 + 2x - 15 = x^2 + x - 6$
 $x = 9$

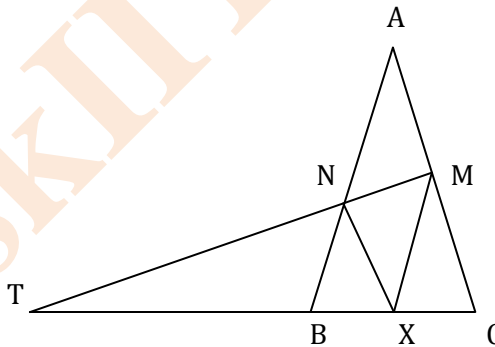
Q.3 In figure, if $ST \parallel QR$. Find PS.



Solution 3: In ΔPRQ ,
 $ST \parallel QR$
 $PS/QS = PT/RT$
 $PS/3 = 3/2$
 $PS = 9/2 \text{ cm}$

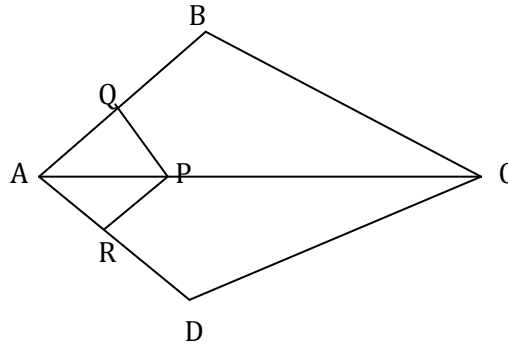
Q.4 Let X be any point on the side BC of a triangle ABC. If XM, XN are drawn parallel to BA and CA meeting CA, BA in M, N respectively; MN meets BC produced in T, Prove that $TX^2 = TB \times TC$.

Solution 4: In ΔTXM ,
 $XM \parallel BN$
 $TB/TX = TN/TM \dots \dots (2)$
 In ΔTMC ,
 $XN \parallel CM$
 $TX/TC = TN/TM \dots \dots (1)$
 From (1) and (2),
 $TB/TX = TX/TC$
 $TX^2 = TB \times TC$



Q.5 In the given figure, if $PQ \parallel BC$ and $PR \parallel CD$. Prove that i) $AR/AD = AQ/AB$, ii) $QB/AQ = DR/AR$.

(CBSE- 2010)



Solution 5: In $\triangle ABC$,

$PQ \parallel BC$

Therefore, by basic proportionality theorem,

$$AQ/AB = AP/AC \dots \dots \dots (1)$$

In $\triangle ACD$,

$PR \parallel CD$

By basic proportionality theorem,

$$AP/AC = AR/AD \dots \dots \dots (2)$$

From (1) and (2)

$$AQ/AB = AR/AD \text{ or } AR/AD = AQ/AB$$

$$AB/AQ = AD/AR$$

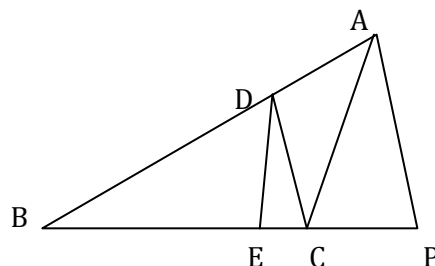
$$(AQ + QB)/AQ = (AR + RD)/AR = 1 + QB/AQ$$

$$= 1 + RD/AR$$

$$QB/AQ = DR/AR$$

Q.6 In the given figure, $DE \parallel AC$ and $DC \parallel AP$. Prove that $BE/EC = BC/CP$

(CBSE-2005)



Solution 6: In $\triangle BPA$,

$DC \parallel AP$

By basic proportionality theorem,

$$BC/CP=BD/DA$$

In $\triangle BCA$,

$$DE \parallel AC \dots \dots \dots (1)$$

By basic proportionality theorem,

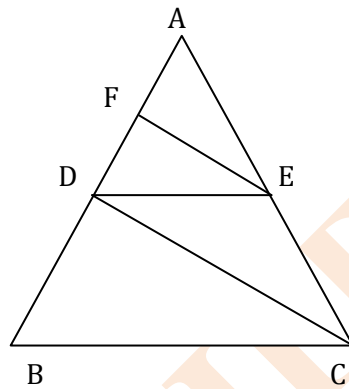
$$BE/EC=BD/DA \dots \dots \dots (2)$$

From (1) and (2),

$$BC/CP=BE/EC \text{ or } BE/EC=BC/CP$$

Q.7 In the given figure, $DE \parallel BC$ and $CD \parallel EF$. Prove that $AD^2 = AB \times AF$

(CBSE-2007)



Solution 7: In $\triangle ABC$,

$$DE \parallel BC$$

$$AB/DE=AC/AE \dots \dots \dots (1)$$

In $\triangle ADC$,

$$FE \parallel DC$$

$$AD/AF=AC/AE \dots \dots \dots (2)$$

From (1) and (2),

$$AB/AD=AD/AF$$

$$AD^2=AB$$

Q.8 ABCD is a quadrilateral; P, Q, R and S are the points of trisection of sides AB, BC, CD and DA respectively and are adjacent to A and C; Prove that PQRS is a parallelogram.

Solution 8: Construction: Join AC.

Proof: Since P, Q, R and S are the points of trisection of AB, BC, CD and DA respectively.

$BP=2PA$, $BQ=2QC$, $DR=2RC$ and $DS=2SA$.

In $\triangle ADC$,

$DS/SA=DR/RC$

S and R divide the sides DA and DC respectively in the same ratio.

$SR \parallel AC$(1)

In $\triangle ABC$,

$BP/PA=2PA/PA=2$ and $BQ/QC=2QC/QC=2$

$BP/PA=BQ/QC$

P and Q divide the sides BA and BC respectively in the same ratio.

$PQ \parallel AC$(2)

From equations (1) and (2),

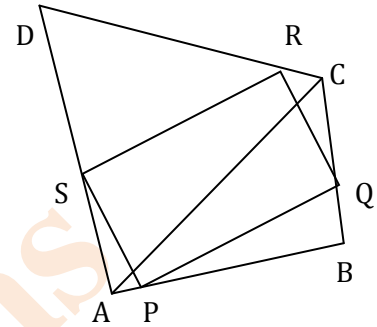
$SR \parallel AC$ and $PQ \parallel AC$

$SR \parallel PQ$

Similarly, by joining BD, we can prove that

$QR \parallel PS$

Hence, PQRS is a parallelogram.



Q.9 The bisector of interior angle A of triangle ABC meets BC in D, and the bisector of exterior angle A meets BC produced in E. prove that $BD/BE=CD/CE$.

Solution 9: Since AD is the bisector of $\angle A$ meeting BC at D.

$AB/AC=BD/DC$(1)

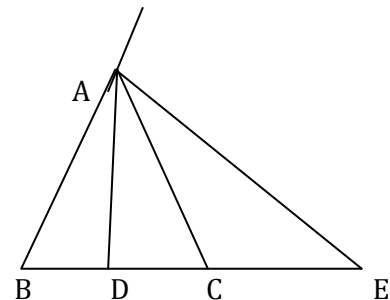
Since AE is the external bisector of $\angle A$ meeting BC produced in E.

$AB/AC=BE/CE$(2)

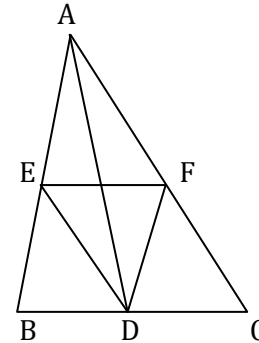
From (1) and (2),

$BD/DC=BE/CE$

$BD/BE=CD/CE$



Q.10 AD is a median of ΔABC . The bisector of $\angle ADB$ and $\angle ADC$ meet AB and AC in E and F respectively. Prove that $EF \parallel BC$.



In ΔADB , DE is the bisector of $\angle ADB$.

$$AD/DB = AE/EB \dots\dots(1)$$

In ΔADC , DF is the bisector of $\angle ADC$.

$$AD/DC = AF/FC$$

$$AD/DB = AF/FC \dots\dots(2)$$

From (1) and (2),

$$AE/EB = AF/FC$$

Thus, in ΔABC , line segment EF divides the sides AB and AC in the same ratio.

Hence, EF is parallel to BC.

Q.11 If the bisector of an angle of a triangle bisect the opposite side, prove that the triangle is isosceles. (CBSE- 2002)

Solution 11: In ΔABC , AD is the angle bisector of $\angle A$.

$$AB/AC = BD/DC$$

$$AB/AC = 1 \text{ (D is the mid-point of BC)}$$

$$AB = AC$$

Hence, the triangle ABC is isosceles.

Q.12 The bisectors of the angles B and C of a triangle ABC, meet the opposite sides in D and E respectively. If $DE \parallel BC$, prove that the triangle is isosceles.

Solution 12: Construction: Join DE

In ΔABC , BD is the bisector of $\angle B$.

$$AB/BC = AD/DC \dots\dots(1)$$

In ΔABC , CE is the bisector of $\angle C$.

$$AC/BC = AE/BE \dots\dots(2)$$

Now, $DE \parallel BC$

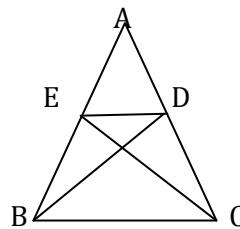
$$AE/BE = AD/DC \dots\dots(3)$$

From (3), we find the RHS of (1) and (2) are equal. Therefore, their LHS are also equal, i.e.,

$$AB/BC = AC/BC$$

$$AB = AC$$

Hence, ΔABC is isosceles.



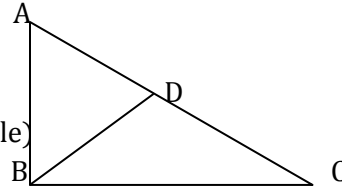
Q.13 In ΔABC , $\angle B = 2\angle C$ and the bisector of $\angle B$ intersects AC at D . Prove that $BD/DA = BC/BA$.

Solution 13: In ΔABC , bisector of $\angle B$ meets AC at D .

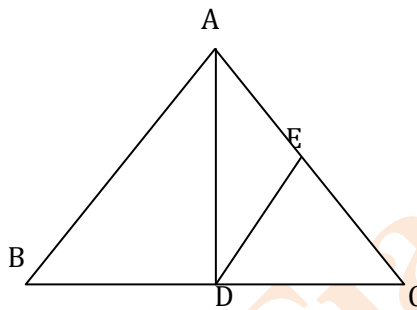
$$CD/AD = BC/BA$$

$$BD/AD = BC/BA \quad (\text{CD} = BD \text{ as } \Delta BDC \text{ will be isosceles triangle})$$

$$BD/DA = BC/BA$$



Q.14 In the figure, $\angle BAC = 90^\circ$, AD is the bisector. If DE is perpendicular to AC , prove that $DE \times (AB + AC) = AB \times AC$.



Solution 14: It is given that AD is the bisector of $\angle A$ of ΔABC ,

$$AB/AC = BD/DC$$

$$AB/AC + 1 = BD/DC + 1$$

$$(AB + AC)/AC = (BD + DC)/DC$$

$$(AB + AC)/AC = BC/DC \dots \dots \dots (1)$$

In ΔCDE and ΔCBA ,

$$\angle DCE = \angle BCA = \angle C \quad (\text{Common})$$

$$\angle BAC = \angle DEC = 90^\circ$$

So, by AA criterion,

$$\Delta CDE \sim \Delta CBA$$

$$CD/CB = DE/BA$$

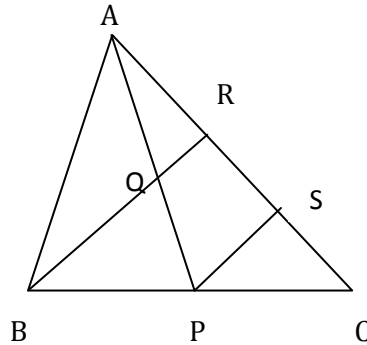
$$AB/DE = BC/DA \dots \dots \dots (2)$$

From (1) and (2),

$$(AB + AC)/AC = AB/DE$$

$$DE \times (AB + AC) = AB \times AC$$

Q.15 In the figure, P is the mid-point of BC and Q is the mid-point of AP. If BQ when produced meets AC at R, prove that $RA = \frac{1}{3} CA$.



Solution 15: Construction: Draw $PS \parallel BR$, meeting AC at S.

Proof: In $\triangle BCR$, P is the mid-point of BC and $PS \parallel BR$.

S is the mid-point of CR.

$$CS = SR \dots \dots (1)$$

In $\triangle APS$, Q is the mid-point of AP and $QR \parallel PS$.

R is the mid-point of AS.

$$AR = RS \dots \dots (2)$$

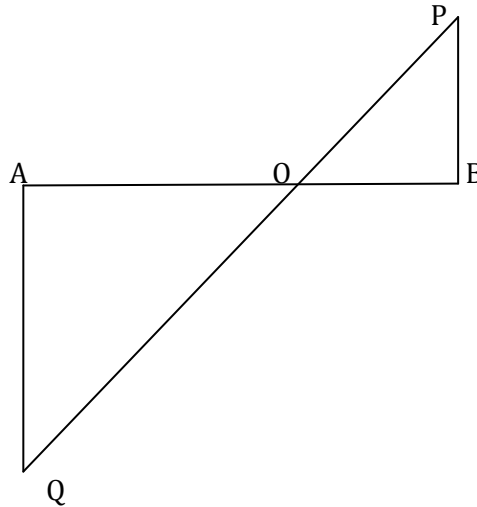
From (1) and (2),

$$AR = RS = SC$$

$$AC = AR + RS + SC = 3AR$$

$$AR = \frac{1}{3} AC = \frac{1}{3} CA$$

Q.16 In the given figure, QA and PB are perpendiculars to AB. If AO =10 cm, BO=6 cm and PB=9 cm. Find AQ.



Solution 16: Triangle AOQ ~ BOP (Using AA criterion)

$$AO/BO = AQ/BP$$

Using this, find AQ.

Q. 17 The perimeters of two similar triangles are 30 cm and 20 cm respectively. If one side of the first triangle is 12 cm, determine the corresponding side of the second triangle.

Let ΔABC and ΔDEF be the two similar triangles of perimeter P_1 and P_2 respectively. Also, let $AB=12$ cm, $P_1=30$ cm and $P_2=20$ cm

$$AB/DE = BC/EF = AC/DF = P_1/P_2$$

$$AB/DE = P_1/P_2$$

$$12/DE = 30/20$$

$$E = 12 \times 20/30 \text{ cm} = 8 \text{ cm}$$

Q.18 D is a point on the side BC of triangle ABC such that $\angle ADC = \angle BAC$. Prove that $CA/CD = CB/CA$ or $CA^2 = CB \times CD$. (CBSE-2004)

Solution 18: In ΔABC and ΔDAC ,

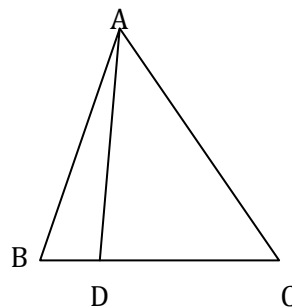
$$\angle ADC = \angle BAC \text{ and } \angle B = \angle C$$

Therefore, by AA~ criterion,

$$\Delta ABC \sim \Delta DAC$$

$$AB/DA = BC/AC = AC/DC$$

$$CB/CA = CA/CD$$



Q.19 Two poles of height a metres and b metres are p metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $ab/(a + b)$ metres.

Solution 19: Let AB and CD be the two poles of heights a and b metres respectively such that the poles are p metres apart, i.e., $AC=p$ metres. Suppose the lines AD and BC meet at O such that $OL=h$ metres.

Let $CL=x$ and $LA=y$. Then, $x + y=p$

In $\triangle ABC$ and $\triangle LOC$,

$\angle CAB = \angle CLO = 90^\circ$

$\angle C = \angle C$ (Common)

$\triangle CAB \sim \triangle CLO$

$CA/CL = AB/LO$

$x = ph/a$(1)

In $\triangle ALO$ and $\triangle ACD$,

$\angle ALO = \angle ACD$

$\angle A = \angle A$

$\triangle ALO \sim \triangle ACD$

$AL/AC = OL/DC$

$y/p = h/b$

$y = ph/b$(2)

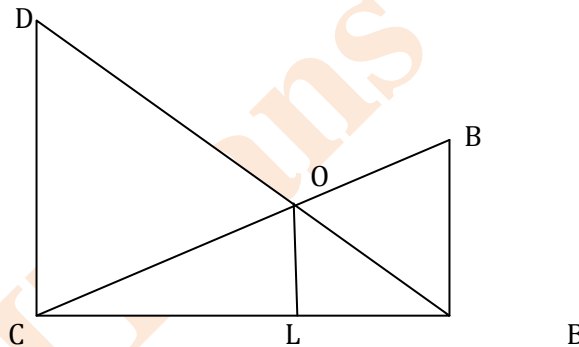
From (1) and (2),

$x + y = ph/a + ph/b$

$p = ph(1/a + 1/b)$

$1 = h(1/a + 1/b)$

$h = ab/(a + b)$ metres



Q.20 E is a point on side AD produced of a parallelogram $ABCD$ and BE intersects CD at F . prove that $\triangle ABE \sim \triangle CFB$. (CBSE-2008)

Solution 20: In $\triangle ABE$ and $\triangle CFB$,

$\angle AEB = \angle CBF$ (alternate angles)

$\angle A = \angle C$ (Opposite angles of ||gm)

Thus, by AA ~ criterion,

$\triangle ABE \sim \triangle CFB$

