

Class: X
Subject: Math's
Topic: Trigonometry
No. of Questions: 20

Q.1 If $\sin x = \frac{a^2 - b^2}{a^2 + b^2}$, find $\tan x$.

Sol: $\sin x = \frac{a^2 - b^2}{a^2 + b^2} = \frac{P}{H}$
 $H^2 = P^2 + B^2$
 $(a^2 + b^2) = (a^2 - b^2) + B^2$
 $a^4 + b^4 + 2a^2b^2 = a^4 + b^4 - 2a^2b^2 + B^2$
 $B = 2ab$
 $\tan x = \frac{P}{B} = \frac{a^2 - b^2}{2ab}$

Q.2 If $\operatorname{cosec} x = 2$, find the value of $\frac{1}{\tan x} + \frac{\sin x}{1 + \cos x}$

Sol: $\operatorname{cosec} x = \frac{H}{P} = 2$
 $H^2 = B^2 + P^2$
 $4 = B^2 + 1$
 $4 - 1 = B^2$
 $B = \sqrt{3}$
 $\tan x = \frac{P}{B} = \frac{1}{\sqrt{3}}$
 $\sin x = \frac{P}{H} = \frac{1}{2}$
 $\cos x = \frac{B}{H} = \frac{\sqrt{3}}{2}$
 $\frac{1}{\tan x} + \frac{\sin x}{1 + \cos x} = 2$

Q.3 In $\triangle ABC$, right angled at C, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of $\sin A \cos B + \cos A \sin B$.

(CBSE-2008)

Sol: $\tan A = \frac{1}{\sqrt{3}} = \frac{P}{B}$
 $P^2 + B^2 = H^2$
 $1 + (\sqrt{3})^2 = H^2$
 $\sin A = \frac{1}{2}$, $\cos A = \frac{\sqrt{3}}{2}$, $\cos B = \frac{1}{2}$, $\sin B = \frac{\sqrt{3}}{2}$
 $\sin A \cos B + \cos A \sin B = 1$

Q.4 If $\sin B=1/2$, find the value of $3 \cos B - 4 \cos^2 B$

Sol: $\sin B=1/2 = P/H$
 $H^2 = P^2 + B^2$
 $4 = 1 + B^2$
 $B = \sqrt{3}$
 $\cos B = \sqrt{3}/2 = B/H$
 $3 \cos B - 4 \cos^2 B = 0$

Q.5 If $\sin x = 4/5$, find the value of $\frac{4 \tan x - 5 \cos x}{\sec x + 4 \cot x}$

Sol: $\sin x = 4/5$
 $H^2 = P^2 + B^2$
 $25 = 16 + B^2$
 $B = 3$
 $\cos x = B/H = 3/5$, $\tan x = P/B = 4/3$, $\cot x = B/P = 3/4$, $\sec x = H/B = 5/3$
 $\frac{4 \tan x - 5 \cos x}{\sec x + 4 \cot x} = 1/2$

Q.6 Given $16 \cot A = 12$, find the value of $\frac{\sin A + \cos A}{\sin A - \cos A}$

Sol: $\frac{\sin A + \cos A}{\sin A - \cos A}$
Dividing the numerator and denominator by $\sin A$,
 $= \frac{1 + \cot A}{1 - \cot A}$
Now, $16 \cot A = 12$
 $\cot A = 12/16 = 3/4$
 $\frac{\sin A + \cos A}{\sin A - \cos A} = 7$

Q.7 If $\tan x = 12/13$, evaluate $(2 \sin x \cos x) / (\cos^2 x - \sin^2 x)$

Sol: $\tan x = 12/13$
 $(2 \sin x \cos x) / (\cos^2 x - \sin^2 x)$
Dividing the numerator and denominator by $\cos^2 x$
 $= 2 \tan x / (1 - \tan^2 x) = 24/13 * 169/25 = 312/25$

Q.8 Find the value of $2(\cos^2 45^\circ + \tan^2 60^\circ) - 6(\sin^2 45^\circ - \tan^2 30^\circ)$

Sol: $\cos 45^\circ = 1/\sqrt{2}$, $\tan 60^\circ = \sqrt{3}$, $\sin 45^\circ = 1/\sqrt{2}$, $\tan 30^\circ = 1/\sqrt{3}$
 $2(\cos^2 45^\circ + \tan^2 60^\circ) - 6(\sin^2 45^\circ - \tan^2 30^\circ) = 2[(1/\sqrt{2})^2 + (\sqrt{3})^2] - 6[(1/\sqrt{2})^2 - (1/\sqrt{3})^2]$
 $= 6$

Q.9 Find the value of x in $\tan 3x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$

Sol: $\tan 3x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$
 $\tan 3x = 1/\sqrt{2} * 1/\sqrt{2} + 1/2 = 1$
 $\tan 3x = \tan 45^\circ$
 $3x = 45^\circ$
 $x = 15^\circ$

Q.10 Find the acute angle x. when $(\cos x - \sin x)/(\cos x + \sin x) = (1 - \sqrt{3})/(1 + \sqrt{3})$

Sol: $(\cos x - \sin x)/(\cos x + \sin x) = (1 - \sqrt{3})/(1 + \sqrt{3})$
Dividing LHS by $\cos x$
 $(1 - \tan x)/(1 + \tan x) = (1 - \sqrt{3})/(1 + \sqrt{3})$
 $\tan x = \sqrt{3}$
 $x = 60^\circ$

Q.11 If $\sin(A + B) = 1$ and $\cos(A - B) = \sqrt{3}/2$, $0^\circ < A + B \leq 90^\circ$, $A > B$ then find A and B.

Sol: $\sin(A + B) = 1$
 $\sin(A + B) = \sin 90^\circ$
 $\cos(A - B) = \sqrt{3}/2$
 $\cos(A - B) = \cos 30^\circ$
 $A + B = 90^\circ$
 $A - B = 30^\circ$
Add both the above equations
 $2A = 120^\circ$
 $A = 60^\circ$
 $60^\circ + B = 90^\circ$
 $B = 30^\circ$

Q.12 If x is an acute angle and $\sin x = \cos x$, find the value of $2\tan^2x + \sin^2x - 1$

Sol: $2\tan^2x + \sin^2x - 1$
 $\sin x = \cos x$ (given)
 $\sin x / \cos x = \cos x / \cos x$
 $2\tan^2x + \sin^2x - 1 = 2 + 1/2 - 1 = 3/2$

Q.13 Given that $\sin(A+B) = \sin A \cos B + \cos A \sin B$, find the value of $\sin 75^\circ$.

Sol: $75^\circ = 45^\circ + 30^\circ$
 $\sin(30^\circ + 45^\circ) = \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$
 $\sin 75^\circ = 1/2 * 1/\sqrt{2} + \sqrt{3}/2 * 1/\sqrt{2} = (\sqrt{3} + 1) / 2\sqrt{2}$

Q.14 If each of α , β and γ is a positive acute angle such that $\sin(\alpha + \beta + \gamma) = 1/2$, $\cos(\beta + \gamma - \alpha) = 1/2$ and $\tan(\gamma + \alpha - \beta) = 1$, find the values of α , β and γ .

Sol: $\sin(\alpha + \beta + \gamma) = 1/2$
 $\sin(\alpha + \beta + \gamma) = \sin 30^\circ$
 $\alpha + \beta + \gamma = 30^\circ$
 $\cos(\beta + \gamma - \alpha) = 1/2$
 $\cos(\beta + \gamma - \alpha) = \cos 60^\circ$
 $\beta + \gamma - \alpha = 60^\circ$
 $\tan(\gamma + \alpha - \beta) = 1$
 $\tan(\gamma + \alpha - \beta) = \tan 45^\circ$
 $\gamma + \alpha - \beta = 45^\circ$
add the 3 equations
 $2\beta = 90^\circ$ and $2\alpha = 75^\circ$
 $\beta = 45^\circ$ and $\alpha = 37.5^\circ$
 $\gamma = 52.5^\circ$

Q.15 Evaluate $\cos(40^\circ - x) - \sin(50^\circ + x) + (\cos^2 40^\circ + \cos^2 50^\circ) / (\sin^2 40^\circ + \sin^2 50^\circ)$
(CBSE-2002)

Sol: $\cos(40^\circ - x) - \sin(50^\circ + x) + (\cos^2 40^\circ + \cos^2 50^\circ) / (\sin^2 40^\circ + \sin^2 50^\circ)$
 $\cos(40^\circ - x) - \sin(90^\circ - (40^\circ - x)) + [\cos^2 40^\circ + \cos^2(90^\circ - 40^\circ)] / [\sin^2 40^\circ + \sin^2(90^\circ - 40^\circ)]$
 $\cos(40^\circ - x) - \cos(40^\circ - x) + (\cos^2 40^\circ + \sin^2 40^\circ) / (\sin^2 40^\circ + \cos^2 40^\circ)$
 $= 0 + 1 = 1$

Q.16 Evaluate $\cot 12^\circ \cot 38^\circ \cot 52^\circ \cot 60^\circ \cot 78^\circ$

(CBSE-2001)

Sol: $\cot 12^\circ \cot 38^\circ \cot 52^\circ \cot 60^\circ \cot 78^\circ$
 $= (\cot 12^\circ \cot 78^\circ) (\cot 38^\circ \cot 52^\circ) (\cot 60^\circ)$
 $= (\cot 12^\circ \tan 12^\circ) (\cot 38^\circ \tan 38^\circ) (\cot 60^\circ)$
 $= 1 * 1/\sqrt{3} = 1/\sqrt{3}$

Q.17 Evaluate $\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$.

Sol: $\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$
 $= \tan(90^\circ - 89^\circ) \tan(90^\circ - 88^\circ) \dots \tan 88^\circ \tan 89^\circ$
 $= \cot 89^\circ \cot 88^\circ \dots \tan 88^\circ \tan 89^\circ$
 $= (\cot 89^\circ \tan 89^\circ) \dots (\cot 44^\circ \tan 44^\circ) \tan 45^\circ$
 $= 1 * 1 * 1 * 1 \dots * 1 = 1$

Q.18 If $\sec 4A = \operatorname{cosec}(A - 20^\circ)$, where $4A$ is an acute angle, find the value of A .

(CBSE-2008)

Sol: $\sec 4A = \operatorname{cosec}(A - 20^\circ)$
 $= \sec[90^\circ - (A - 20^\circ)]$
 $4A = 110^\circ - A$
 $5A = 110^\circ$
 $A = 22^\circ$

Q.19 Evaluate $\cot^2 x - 1/\sin^2 x$

Sol: $\cot^2 x - 1/\sin^2 x = \cot^2 x - \operatorname{cosec}^2 x = -1$

Q.20 $(1 + \tan^2 x)(1 + \sin x)(1 - \sin x)$, find the value.

Sol: $(1 + \tan^2 x)(1 + \sin x)(1 - \sin x)$
 $= (1 + \tan^2 x)(1 - \sin^2 x) = \sec^2 x * \cos^2 x = 1$