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Class: X
Subject: Mathematics
Topic: SA1
No. of Questions: 34

Time: 3 Hrs. **M.M. 90**

General Instructions:

1. All questions are compulsory.
2. The questions paper consists of 34 questions divided into four sections, A, B, C and D.
3. Section 'A' comprises 8 questions of 1 mark each, Section 'B' comprises 6 questions of 2 marks each, Section 'C' comprises 10 questions of 3 marks each and Section 'D' comprises 10 questions of 4 marks each.
4. There is no overall choice. However, internal choice has been provided 1 question of two marks, 3 questions of three marks each and 2 questions of four mark each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is not permitted.
6. An additional 15 minutes has been allotted to read this question paper only.

SECTION - A

Q1. The decimal expansion of the rational number $\frac{2^3}{2^2.5}$ $will\ terminate$ after

(a) One decimal place
(b) Two decimal places
(c) Three decimal places
(d) More three decimal

Sol. (a)

If the denominator of a rational number is of the form 2^n5^m, then it will terminate after n places if n > m or m places if m > n.

$Now, \frac{2^3}{2^25} = \frac{2}{5} = \frac{2}{2^05}$

Thus, the decimal expansion of $\frac{2^3}{2^25} = \frac{2}{5} = \frac{2}{2^05}$ will terminate after 1 decimal place.

Q2. For some integer 'm' every odd integer is of form:

(a) m
(b) m+1
(c) 2m
(d) 2m+1

Sol. (d)

For an integer 'm' every odd integer is of form 2m + 1.

Q3. If one of the zeroes of the quadratic polynomial $(k-1)x^2+1$ is -3, then the value of k is

(a) -8/9
(b) 8/9
(c) 4/9
(d) -4/9

Sol. (b)

Since – 3 is the root of quadratic polynomial, we have

$\Rightarrow 9(k-1) = -1 \quad \Rightarrow k-1 = \frac{-1}{9}$

Q4. The lines representing the linear equations 2x-y = 3 and 4x-y = 5:

(a) Intersect at a point
(b) Are parallel
(c) Are coincident
(d) Intersect at exactly two points

Sol. (a)

$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = 1$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Longrightarrow$ intersect at point.

Q5. The mean of 6 numbers is 16. With the removal of a number the mean of remaining numbers is 17. The number removed is:

(a) 2
(b) 22
(c) 11
(d) 6

Sol. We know:

Mean = $\frac{Sum\ of\ observations}{Number\ of\ observation}$

Mean of 6 numbers = 16

Sum of the 6 observations $= 16.6 = 96$

Mean of 5 observations = 17

Thus, number which is removed = 96 – 85 = 11

Q6. If x = $3\sec^2\theta - 1, \quad y = \tan^2\theta - 2\ then\ x - 3y\ \ is\ equal\ to$

(a) 3
(b) 4
(c) 8
(d) 5

Sol. (c)

$x = 3\sec^2\theta - 1, y = \tan^2\theta - 2$

$x - 3y = 3\sec^2\theta - 1 - 3\tan^2 + 6$

$= 3\ (\sec^2\theta - \tan^2\theta\ ) + 5$

$= 3 + 5$
$= 8$

Q7. If $cos\theta + \cos^2\theta = 1, the\ value\ of\ (\sin^2\theta + \sin^4\theta)\ is$

(a) 0
(b) 1
(c) -1
(d) 2

Sol (b)
$\cos\theta + \cos^2\theta = 1 \implies 1 - \cos^2\theta = \cos\theta$
$\sin^2\theta + \sin^4\theta\ \ (1 - \cos^2\theta) +\ (1 - \cos^2\theta)^2$
$= \cos\theta + \cos^2\theta$
$= 1$

Q8. If $\Delta ABC \sim \Delta RQP,\ \angle A = 80°, \angle B = 60°, the\ value\ of\ \angle P\ \ is$

(a) 60°

(b) 50°

(c) 40°

(d) 30°

Sol. (c)

$\Delta ABC \sim \Delta RQP$

$\angle A = \angle R = 80°$

$\angle B = \angle Q = 60°$

Therefore, using angle sum property, we have:

$\angle p = 180°$ - $(80° + 60°) = 40°$

SECTION - B

Q9. Find the largest number which divides 318 and 739 leaving remainder 3 and 4 respectively, is

Sol. We have,

3	735
5	245
7	49
7	7
	1

3	315
3	105
5	35
7	7
	1

$318 - 3 = 315\ ;\ \ 739 - 4 = 735$

$315 = 3 \times 3 \times 5 \times 7$

$= 3^2 \times 5 \times 7$

And

$735\ = 3 \times 5 \times 7 \times 7$

$= 3 \times 5 \times 7$

H.C.F; (315 , 735) = $3 \times 5 \times 7$

$= 105$

Q10. Solve $37x + 43y = 123,\ 43x + 37y = 117.$

Sol.

$37x + 43y = 123$

$43x + 37y = 117$

$(+)\ \ (+)\ \ (+)$

$80X + 80y = 240$

$37x + 436 = 123$

$43x + 37y = 117$

$(-)\ \ (-)\ \ (-)$

$-6X + 6y = 6$

$\Rightarrow x + y = 3$

$-x + y = 1$

On solving the above two equations, we get, x= 1 y=2

OR

$x + \frac{6}{y} = 6$, $3x - \frac{8}{y} = 5.$

Sol. We have

$\left(x + \frac{6}{y} = 6\right) \times 3$

$\Rightarrow 3x + \frac{18}{y} = 18$ ....(1)

Subtracting equation (1) from 3x – y = 5, we get

$-\frac{26}{y} = -13 \Rightarrow y = 2$

Now, from equation (1), we have:

$x = 3$

Q11. α, ß are the roots of the quadratic polynomial p(x) = $x^2 - (k + 6)x + 2$

$\alpha + ß = \frac{1}{2} \times \alpha ß$, $(2x - 1)$. Find the value of K, if

Sol. $\alpha ß$ are root $x^2 - (k + 6)x + 2(2k - 1)$

$\alpha + ß = k + 6, \alpha ß = 2(2k - 1)$

Now $\alpha + ß = \frac{1}{2}\alpha ß \Rightarrow k + 6 = \frac{1}{2} \times 2\ (2k - 1)$

$\Rightarrow k + \ + = 2k - 1$

$\Rightarrow k = 7$

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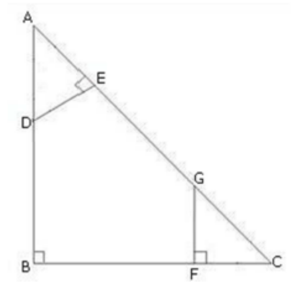
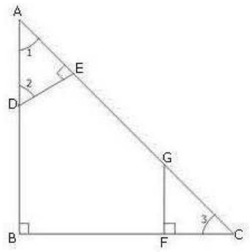
Q12. In fig. AB $\perp BC, GF \perp BC, DE \perp BC. Prove\ that\ \Delta ADE \sim \Delta GCF$.



Sol.



In $\Delta ABC, \angle 1 + \angle 3 = 90°$

In $\Delta ADE, \angle 1 + \angle 2 = 90°$

$\angle 1 + \angle 3 = \angle 1 + \angle 2 \implies \angle 3 = \angle 2$

In $\Delta ADE\ and\ \Delta GCF$

$\angle E = \angle F = 90°$

$\angle 2 = \angle 3$

$\therefore \Delta ADE \sim \Delta GCF$ (By AA similarity criterion)

Q13. If $\cot\theta = \frac{7}{8}, find\ the\ value\ of\ \frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\theta)}$

Sol. $\cot\theta = \frac{7}{8}$ (given)

$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$

$= \frac{49}{64}$

Q14. Find the median class and model class for the following distribution.

C.I	135 – 140	140 – 145	145 – 150	150 – 155	155 – 160	160 – 165
F	4	7	11	6	7	5

Sol.

C.I.	F	$c.f$
135 – 140	4	4
140 – 145	7	11
145 – 150	11	22
150 – 155	6	28
155 – 160	7	35
160 – 165	5	40

Here, $n = 40 \implies \frac{n}{2} = 20$

Median class is 145 -150

Also, since highest frequency is 11, Modal class is 145 – 150.

SECTION – C

Q15. Show that $5 + \sqrt{2}$ is an irrational number.

Sol. To prove $5 + \sqrt{2}$ is irrational, let us assume $5 + \sqrt{2}$ is rational.

$\therefore$ we can find integer a and b were, b are co-prime, $be \neq 0$

Such that,

Now a, b are integers, $\frac{a}{b} - 5$

$\implies \sqrt{2}$ is rational.

Which is a contraction, So 5 $\sqrt{2}$ is irrational.

OR

Prove that $\sqrt{n-1}+\sqrt{n+1}$ is an irrational number.

Sol. Let us assume to the contrary, that $\sqrt{n-1}+\sqrt{n+1}$ is a rational number.

$\Longrightarrow\ \left(\sqrt{n-1}+\ \sqrt{n+1}\right)^2$ is rational.

$\Longrightarrow\ (n-1)+(n+1)+2\left(\sqrt{n-1}\times\sqrt{n+1}\right)$

$\Longrightarrow 2n+2\sqrt{n^2-1}$ is rational

But we know that is rational

But we know that $\sqrt{n^2-1}$ is an irrational number

$So\ 2n+2\sqrt{n^2-1}$ is also an irrational number

So our basic assumption that the given number is rational is wrong.

Hence, $\sqrt{n-1}+\sqrt{n+1}$ is an irrational number.

Q16. If α and β are the zeroes of the quadratic polynomial f(x) = x^2-2x+1, find the a quadratic polynomial whose zeroes are $\frac{2\alpha}{ß}\ and \frac{2ß}{\alpha}$.

Sol. $f(x)=x^2-2x+1$

Zeroes of $f(x) are\ \alpha\ and$ ß

Sum of zeroes = α + ß = and

Product of zeroes $=\alpha.ß=1$

Now, $\frac{2\alpha}{ß}+\frac{2ß}{\alpha}=2\ \left(\frac{\alpha}{ß}+\frac{ß}{\alpha}\right)=\left(\frac{\alpha^2+ßB^2}{\alpha ß}\right)$

$$=2\frac{((\alpha+ß)^2-2\alpha ß)}{\alpha ß}=\frac{2\times2}{1}=4$$

$Also,\frac{2\alpha}{ß}\times\frac{2ß}{\alpha}=4$

Required polynomial $=k\ (x^2-4x+4)$, where k is any integer.

Q17. If in a rectangle, the length is increased and breadth is reduced each by 2 meters, the area is reduced by 28 sq. mtrs. If the length is reduced by 1 metre and breadth is increased by 2 metres, the area is increased by 33 sq mtrs. Find the length and breadth of the rectangle.

Sol. Let the length and breadth of the rectangle be x and y respectively.

So the original area of the rectangle = xy

According to question,

$(x+2)(y-2) = xy - 28 \; or \; xy - 2x + 2y - 4 = xy - 28 \; or \; 2x - 2y = 24.$ .....(i)

Next, $(x-1)(y+2) = xy + 33 \; or \; xy + 2x - y - 2 = xy + 33 \; or \; 2x - y = 35$ .....(ii)

Now we need to solve (i) and (ii)

Subtracting (i) from (ii) we get, y = 11

Substituting this value in (ii), we get

$2x = 46$

$x = 23$

So the length and breadth of the rectangle are 23 metres and 11 metres respectively.

OR

A leading library has a fixed change for first three days and an additional change for first three days and an additional charge for each day thereafter. Bhavya paid Rs 27 for a book kept for seven days. While Vrinda paid Rs 21 for a book kept for five days. Find the fixed charge and the charge for each extra day.

Sol. Let the fixed charge = Rs x

And the subsequent charge = Rs y

According to the question, we have:

X + 4y = 24 ......(i)

And x + 2y = 21 .....(ii)

Subtracting (ii) from (i), we have:

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$2y = 6 \; or = 3$

So, from (i)

$x = 27 - 12 = 15$

Thus, the fixed charge is Rs 15 and the charge for each extra day is Rs 3.

Q18. For what values of a and b does the following pairs of linear equations have an infinite number of solutions: $2x + 3y = 7; a(x + y) - b(x - y) = 3a + b - 2$

Sol. The system has infinitely many solutions. Therefore,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \underset{(1)}{\frac{2}{a-b}} = \underset{(2)}{\frac{3}{a+b}} = \underset{(3)}{\frac{7}{3a+b-2}}$

Equating (1) and (2), we get:

$2a + 2b = 3a - 3b$

Or, a = 5b ....(4)

Equating (2) and (3), we get:

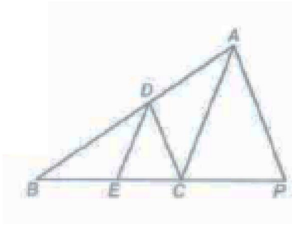
$9a + 3b - 6 = 7a + 7b$

$or, 2a - 4b = 6$ .....(5)

$10b - 4b = 6 \; or \; b = 1$

Thus from (4), we get, a = 5

Q19. In the given figure, DE ∥ AC and $\frac{\text{BE}}{\text{EC}} = \frac{\text{BC}}{\text{CP}}$. Prove that DC ∥ AP.



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Sol. In ΔABC,

$DE \parallel AC$ [by basic proportionality theorem]

$\therefore \quad \frac{BD}{DA} = \frac{BE}{EC}$ ....(1)

Now, $\frac{BE}{EC} = \frac{BC}{CP}$ [given].......(ii)

From Eqs.(i) and (ii) we get

$\frac{BD}{DA} = \frac{BC}{CP}$

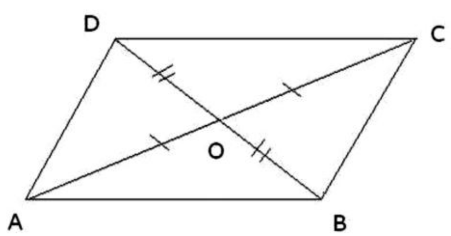
So, in ΔABP,

$\frac{BD}{DA} = \frac{BC}{CP}$ [From above]

$\Rightarrow \quad DC \parallel AP$ [Using converse of basic proportionality theorem]

Q20. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Sol.



Given: ABCD is a rhombus

To prove: $AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$

Proof:

We know, diagonals of a rhombus bisect at right angles.

Therefore, from triangle AOB,

$AB^2 = AO^2 + OB^2$

$= \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2$

$= \frac{1}{4}AC^2 + \frac{1}{4}BD^2$

$\Longrightarrow 4AB^2 = AC^2 + BD^2$

Thus, $AB^2 + BC^2 + CD^2 + DA^2 = 4AB^2 = AC^2 + BD^2$

$[As\ AB = BC = CD = DA]$

Q21. $\frac{\sec 29°}{\text{cosec}61°} + 2\cot 8° \cot 17° \cot 45° \cot 73° \cot 82° - 3(\sin^2 38° + \sin^2 52°)$

Sol. We have,

$\frac{\sec 29°}{\text{cosec}61°} + 2\cot 8° \cot 17° \cot 45° \cot 73° \cot 82° - 3(\sin^2 38° + \sin^2 52°)$

$= \frac{\sec(90°-61°)}{\text{cosec}61°} + 2\cot(90° - 82°)\cot(90° - 73°)\cot 45°.\cot 73°.\cot 82°$

$-3[(\sin^2 38° + \sin^2(90° - 38°))]$

$= \frac{\text{cosec}61°}{\text{cosec}61°} + 2\tan 82°.\tan 73°.\cot 45°.\cot 73°\cot 82° - 3(\sin^2 38° + \cos^2 38°)$

$\left[\because \sec(90° - \theta) = \text{cosec}\theta, \cot(90° - \theta) = \tan\theta \quad \sin(90° - \theta) = \cos\theta\right]$

$= \frac{\text{cosec}\ 61°}{\text{cosec}61°} + 2\tan 82°.\tan 73°.\cot 45°.\frac{1}{\tan 73°}.\frac{1}{\tan 82°} - 3(\sin^2 38° + \cos^2 38°)$

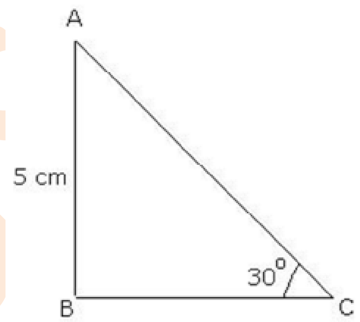
$\left[\because \quad \cot\theta = \frac{1}{\tan\theta}\right]$ and $[\sin^2\theta + \cos^2\theta = 1]$

$= 1 + 2 \times 1 - 3 \times 1$

$= 3 - 3$

$= 0$

Q22. In triangle ABC, right angled at B, AB = 5 cm, $\angle ACB = 30°$. Find the length of BC and AC.



Sol. In ΔBAC, $\angle B = 90°$

We have:

$\frac{AB}{AC} = \sin 30° = \frac{1}{2} \Rightarrow \frac{5}{AC} = \frac{1}{2} \Rightarrow AC = 10\ cm$

And, $\frac{BC}{AC} = cos 30° = \frac{\sqrt{3}}{2} \Rightarrow \frac{BC}{10} = \frac{\sqrt{3}}{2} \Rightarrow BC = 5\sqrt{3}\ cm$

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Q23. If mean of the following data is 86, then what is the value of p?

Wages (in Rs.)	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100	100 – 110
Number of workers	5	3	4	P	2	13

Sol.

CI	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100	100 – 110	Total
f_i	5	3	4	P	2	13	27+p
X_i	55	65	75	85	95	105	
f_iX_i	275	195	300	85p	190	1365	2325+85p

$Mean = \frac{\sum f_i x_i}{\sum f_i}$

Substituting the values, we ge:

$86 = \frac{2325+85p}{27+p}$

$86p + 2322 = 2325 + 85p$

$p = 3$

Q24. Find the modal age of 100 residents of a colony from the following data:

Age in yrs.(more than or equal)	0	10	20	30	40	50	60	70
No. of Persons	100	90	75	50	28	15	5	0

Sol.

Age in yrs.	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
No. of persons (f_i)	10	15	25	22	13	10	5

Since, the maximum frequency is 25 and it lies in the class interval 20 – 30.

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Therefore, modal class = 20 – 30

$l = 20, h = 10, f_0 = 15\ f_1 = 25, f_2 = 22$

Mode $= 1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$= 20 + \left(\frac{25-15}{2(25)-15-22}\right) \times 10$

$= 20 + 7.69 = 27.69$ years (approx.)

SECTION – D

Q25. A number of the form 15^n, where n $\in N$ (the set of natural numbers), can never end with a zero. Justify this statement.

Sol. If the number 15^n, where $n\ \varepsilon\ N$, was to end with a zero, then its prime factorization must have 2 and 5 as its factors.

But 15 = 5 × 3

$15^n = (5 \times 3)^n = 5^n\ '\ 3^2$

So, prime factors of 15^n includes only 5 but not 2.

Also, from the Fundamental theorem of Arithmetic, the prime factorization of a number is unique.

Hence, a number of the form 15^n, where $n \epsilon N$, will never end with a zero.

Q26. Solve the equations $2x - y + 6 = 0\ and\ 4x + 5y - 16 = 0$ graphically. Also determine the coordinate of the vertices of the triangle formed by these lines and the x- axis.

Sol. To solve the equations, make the table corresponding to each equation.

$2x - y + 6 = 0$

$\Rightarrow y = 2x + 6$

X	-1	-2	-3
Y	4	2	0

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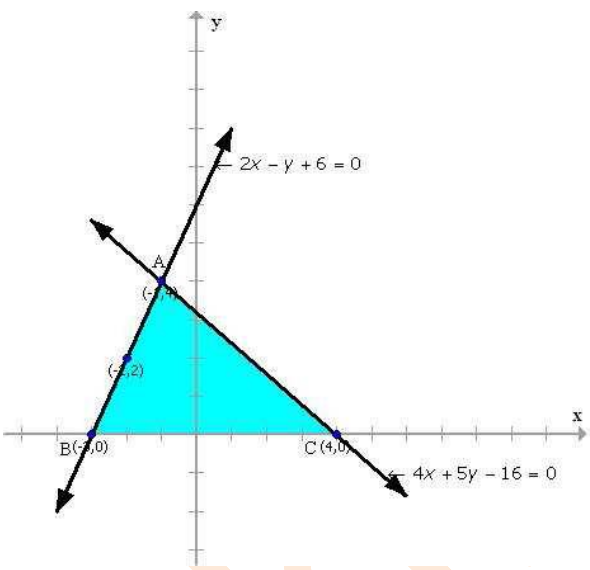
$4x + 5y - 16 = 0$

$\Rightarrow y = \frac{16-4x}{5}$

X	4	-1
Y	0	4

Now plot the point and draw the graph.



Since, the lines intersect at the point (-1, 4), so x = -1 and $y = 4$ be the solution.

Also, by observation vertices of triangle formed by lines and $x - axis$ are A $(-1, 4), B\ (-3, 0)$ and $C(4, 0)$.

Q27. The remainder on dividing $x^3 + 2x^2 + kx + 3\ by\ x - 3\ \ is\ 21$. Sanju was asked to find the quotient. He was a little puzzle and was thinking how to proceed. His classmate qunjan helped him by suggesting that he should first find k and then proceed further.

Sol. Let $p(x) = x^3 + 2x^2 + kx + 3$

Using Remainder theorem, we have:

$p(3) = 3^3 + 2 \times 3^2 + 3k + 3 = 21$

$\Rightarrow k = -9$

Thus, $p(x) = x^3 + 2x^2 - 9x + 3$

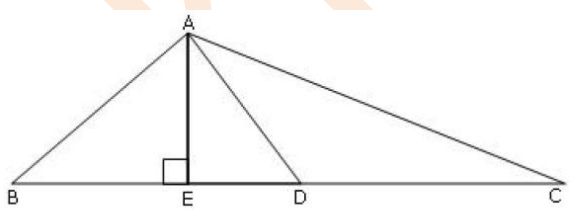
$$\begin{array}{r|l} & x^2 + 5x + 6 \\ \hline x - 3 & x^3 + 2x^2 - 9x + 3 \\ & x^3 - 3x^2 \\ & - \quad + \\ \hline & 5x^2 - 9x + 3 \\ & 5x^2 - 15x \\ & - \quad + \\ \hline & 6x + 3 \\ & 6x - 18 \\ & - \quad + \\ \hline & 21 \end{array}$$

When $p(x)$ is divided by $(x - 2)$, the quotient is $x^2 + 5x + 6$.

Q28. In triangle ABC, D is the mid – point of BC and AE $\perp BC. If\ AC > AB$. Show that

$AB^2 = AD^2 - BC \times DE + \frac{BC^2}{4}$

Sol. AD is the median of triangle ABC, since D is mid-point of BC.



$\Rightarrow BD = DC = \frac{BC}{2}$ …..(i)

In right triangle AEB,

$AB^2 = AE^2 + BE^2$ (Pythagoras theorem)

$AB^2 = (AD^2 - DE^2) + (BD - DE)^2$

(By using Pythagoras theorem for right triangle AED and BE = BD - DE)

$AB^2 = AD^2 - DE^2 + \left(\frac{BC}{2} - DE\right)^2$

$AB^2 = AD^2 - DE^2 + \frac{BC^2}{4} + DE^2 - 2$

$\Longrightarrow AB^2 = AD^2 - BC\ x\ DE + \frac{BC^2}{4}$

Hence proved

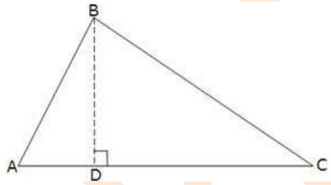
OR

In a right angled triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides.

Sol. Given: A right triangle ABC right angled at B.

To prove: $AC^2 = AB^2 + BC^2$

Construction: Draw BD $\perp AC$



Proof:

We know that:If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

$\Delta ADB \sim \Delta\ ABC$

So, $\frac{AD}{AB} = \frac{AB}{AC}$ (Sides are proportional)

$Or, AD.AC = AB^2$ …….(1)

Also, $\Delta BDC \sim \Delta ABC$

So, $\frac{CD}{BC} = \frac{BC}{AC}$

$Or, CD.AC = BC^2$ …….(2)

Adding (1) and (2),

$AD.AC + CD.AC = AB^2 + BC^2$

$AC\ (AD + CD) = AB^2 + BC^2$

$AC. = AB^2 + BC^2$

Hence Proved.

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Q29. Formulate the following problems as a pair of equations and hence find their solutions.
2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 women alone to finish the work and also that taken by 1 man alone.

Sol. Let 1 women finish the work in x day and let 1 man finish the work in y days.

Work of 1 woman in 1 day = $\frac{1}{x}$

Work of 1 man in 1 day = $\frac{1}{y}$

Work of 2 women and 5 men in one day

$$= \frac{2}{x} + \frac{5}{y} = \frac{5x-2y}{xy}$$

The number of days required for complete work

$$= \frac{2}{x} + \frac{5}{y} = \frac{5x+2y}{xy}$$

The number of days required for complete work

$$= \frac{xy}{5x+2y}$$

Work are given that $\frac{xy}{5x+2y} = 4$

Similarly, in second case

$$\frac{xy}{6x+3y} = 3$$

Then, $\frac{5x+2y}{xy} = \frac{1}{4}$ and $\frac{6x+3y}{xy} = \frac{1}{3}$

$\Rightarrow \frac{20}{y} + \frac{8}{x} = 1$ and $\frac{18}{y} + \frac{9}{x} = 1$

On putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$

$\therefore \quad 20v + 8u = 1 \quad$ ... (i)

and $\quad 18v + 9u = 1 \quad$ ... (ii)

On multiplying Eq.(i) by 9 Eq. (ii) by 8, then subtracting later from first, we get

$180v - 144v = 9 - 8$

$\Rightarrow \quad 36v = 1 \Rightarrow v = \frac{1}{36}$

On substituting v = $\frac{1}{36}$ in Eq. (ii), we get

$$18 \times \frac{1}{36} + 9u = 1$$

$\Rightarrow \ 9u = 1 - \frac{1}{2} \Rightarrow u = \frac{1}{18}$

Now, $\ u = \frac{1}{18}$ and $v = \frac{1}{36}$

$\Rightarrow \ \frac{1}{x} = \frac{1}{18} \quad$ and $\frac{1}{y} = \frac{1}{36}$

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Hence, $u = \frac{1}{18}$ and $v = \frac{1}{36}$

$\Rightarrow \frac{1}{x} = \frac{1}{18}$ and $\frac{1}{y} = \frac{1}{36}$

$\Rightarrow x = 18$ and $y = 36$

Hence, single women finish the work in 18 days and single man finishes the work in 36 days.

Q30. Prove that: $\frac{cosA - sinA + 1}{cosA + sinA - 1} = cosec\ A + cotA$

Sol. Prove that: $\frac{cosA - sinA + 1}{cosA + sinA - 1} = cosec\ A + cotA$

We will make use of the identity: $cosec^2 A = 1 + \cot^2 A$

$$L.H.S = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$$

$$= \frac{\cot A - 1 + cosec\ A}{\cot A + 1 - cosec\ A}$$

$$= \frac{\{(\cot A) - (1 - cosec\ A)\ (\cot A) - (cosec\ A)\}}{\{(\cot A) + (1 - cosec\ A)\}\{(\cot A) - (1 - cosec\ A)\}}$$

$$= \frac{(\cot A - 1 + cosec\ A)^2}{(\cot A)^2 - (1 - cosec\ A)^2}$$

$$= \frac{\cot^2 A + 1 + cosec^2 A - 2\cot A - 2\ cosec\ A + \cot A\ cosec\ A}{\cot^2 A - (1 + cosec^2 A - 2\ cosec\ A)}$$

$$= \frac{2\ cosec^2 A + 2\cot A\ cosec\ A - 2\cot A - 2\ cosec\ A}{\cot^2\ A - 1\ cosec^2\ A + 2\ cosec\ A}$$

$$= \frac{2\ cosec\ A\ (cosec\ A + \cot A) - 2\ (\cot A + cosec\ A)}{\cot^2\ A - cosec^2\ A - 1 + 2\ cosec\ A}$$

$$= \frac{(cosec\ A + \cot A)\ (2\ cosec\ A - 2)}{-1 - 1 + 2\ cosec\ A}$$

$$= \frac{(cosec\ A + \cot A)\ (2\ cosec\ A - 2)}{(2\ cosec\ A - 2)}$$

$$= cosec\ A + \cot A$$

$$= R.H.S$$

OR

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Prove that : $\sqrt{\frac{1+sinA}{1-SinA}} = secA + tanA$

Sol. $LHS = \sqrt{\frac{1+\sin A}{1-\sin A}} = \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$

$= \sqrt{\frac{(1+\sin A)^2}{(1-\sin A)\,(1+\sin A)}}$

$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$

$= \frac{1+\sin A}{\sqrt{\cos^2 A}}$

$= \frac{1+\sin A}{\cos A}$

$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$

$= \sec A + \tan A = RHS$

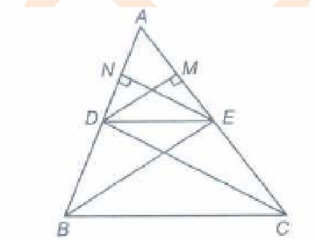
Q31. Prove that, if a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, the other two sides are divided in the same ratio.

Sol. **Given**: A ΔABC in which a line parallel to side BC intersect other two sides AB and AC at D and E respectively.

To prove: $\frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$

Construction: Join BE and CD then draw DM$\perp$ AC and EN $\perp$ AB.

Proof: In ΔADE and ΔBDE



Area of ΔADE = $\frac{1}{2} \times \text{AD} \times \text{EN}$ ..(i)

Area of ΔADE = $\frac{1}{2} \times \text{DB} \times \text{EN}$ …(ii)

On dividing Eq. (i) by Eq. (ii), we get

$\frac{\text{Area of } \Delta\text{ADE}}{\text{Area of } \Delta\text{BDE}} = \frac{\frac{1}{2}\times\text{AD}\times\text{EN}}{\frac{1}{2}\times\text{DB}\times\text{EN}} = \frac{\text{AD}}{\text{DB}}$ ...(iii)

Similarly, Area of ΔADE = $\frac{1}{2} \times \text{AE} \times \text{DM}$ ...(iv)

Area of $\Delta\text{DEC} = \frac{1}{2} \times \text{EC} \times \text{DM}$ ...(v)

On dividing Eq. (iv) by Eq. (v), we get

$\frac{\text{Area of } \Delta\text{ADE}}{\text{Area of } \Delta\text{DEC}} = \frac{\frac{1}{2}\times\text{AE}\times\text{DM}}{\frac{1}{2}\times\text{EC}\times\text{DM}} = \frac{\text{AE}}{\text{EC}}$ (vi)

On dividing Eq. (iv) by Eq. (v), we get

Now, ΔBDE and ΔDEC are on the same base DE and between the same parallel line BC and DE. (vii)

From Eq. (vi),

$$\frac{\text{Area of } \Delta\text{ADE}}{\text{Area of } \Delta\text{BDE}} = \frac{\text{AE}}{\text{EC}} \quad \text{[From Eq. (vii) ... (viii)]}$$

From Eqs. (iii) and (viii), we get

$$\frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$$

Q32. If θ is an acute angle and cosec $\theta = \sqrt{5}$ then,

(a) Evaluate: $\cot\theta - cosec\ \theta$

(b) Verify the identity: $\sin^2\theta + \cos^2\theta = 1$.

Sol. Given: $cosec\ \theta = \sqrt{5}$

$\Rightarrow$ $\sin\theta = \frac{1}{\sqrt{5}}$

So, $\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{1}{5} = \frac{4}{5}$

$\therefore$ $\cos\theta = \frac{2}{\sqrt{5}}$

(i) $cos\theta - cosec\ \theta = \frac{\cos\theta}{\sin\theta} - cosec\ \theta$

$$= \frac{2/\sqrt{5}}{1/\sqrt{5}} - \sqrt{5} = 2 - \sqrt{5}$$

(ii) $\sin^2\theta + \cos^2\theta = \left(\frac{1}{\sqrt{5}}\right)^2 + \left(\frac{2}{\sqrt{5}}\right)^2$

$$= \frac{1}{5} + \frac{4}{5} = 1$$

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Q33. For the data given below draw less than ogive curve.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of students	7	10	23	51	6	3

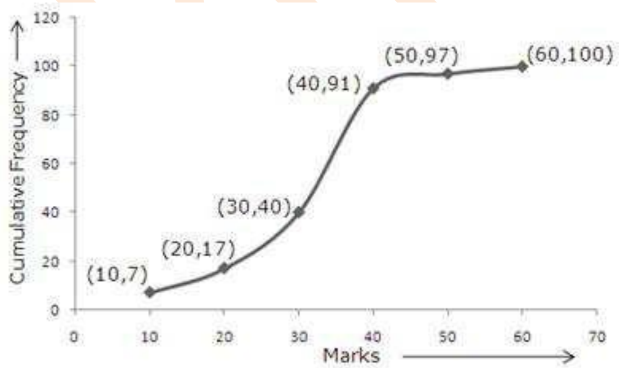
Sol. We first prepare the cumulative frequency distribution table as given blelow:

Marks	No. of students	Marks less than	Cumulative frequency
0 – 10	7	10	7
10 – 20	10	20	17
20 – 30	23	30	40
30 – 40	51	40	91
40 – 50	6	50	97
50 – 60	3	60	100

Now, we mark the upper class limits along x-axis bt taking a suitable scale and the cumulative frequencies along y-axis by taking a suitable scale.

Thus, we plot the points (10, 7), (20, 17), (30, 40), (40, 91), (50, 97) and (60, 100).

Join the plotted points by a free hand to obtain the required ogive.



Q34. The following table gives the distributions of the life of 400 neon lamps. Find mean and median life time.

Life time (in hours)	Number of lamps

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1500- 2000	14
2000- 2500	56
2500 – 3000	60
3000 – 3500	86
3500 - 4000	74
4000 – 4500	62
4500 - 5000	48

Sol.

Life time (in hours)	Number of lamps(f)	Cumulative frequency (cf)
1500- 2000	14	14 = 14
2000- 2500	56	(14 + 56) = 70
2500 – 3000	60	(70+60) = 130 = cf
(Median class) 3000 – 3500	86 = f	(130+86) = 216
3500 - 4000	74	(216 + 74) = 290
4000 – 4500	62	(290 + 62) = 352
4500 - 5000	48	(352+48) = 400
Total	N = 400	

$\because \qquad \frac{N}{2} = \frac{400}{2} = 200$

Since, cumulative frequency 20 lies in the interval 3000 – 3500.

Here (Lower median) l = 3000, f = 86, cf = 130

(class width)h = 500, (Total observations) N = 400

$$\text{Median} = l + \left\{\frac{\frac{N}{2} - cf}{f}\right\} \times h$$

$$= 3000 + \frac{\{200 - 130\}}{86} \times 500$$

$$= 3000 + \frac{70 \times 500}{86}$$

$$= 3000 + \frac{35000}{86} = 3000 + 406.98$$

$$= 3406.98$$

Hence, median life time of a lamp is 3406.98 h.