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Class: X
Subject: Mathematics
Topic: SA 1
No. of Questions: 31
Duration: 3hrs
Maximum Marks: 90

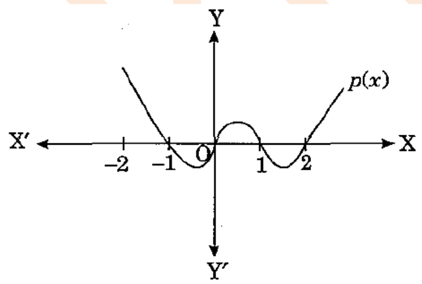
Time: 3 Hours Maximum Marks: 90

General Instruction:

1. All questions are compulsory.
2. The question paper consists of 31 question divided into four sections A, B, C and D.
3. Section 'A' comprises 4 question of 1 mark each, Section 'B' comprises 6 questions of 2 marks each, Section 'C' comprises 10 questions of 3 marks each and Section 'D' comprises 11 question of 4 marks each.
4. Use of calculator is not permitted.

SECTION – A

Q1. In figure, the graph of a polynomial p (x) is shown. The number of zeroes of p(x) is



(a) 4
(b) 3
(c) 2
(d) 1

Sol. (a)

The number of zeroes is 4 as the graph intersects the x-axis in four points, viz., – 1, 0, 1, 2.

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Q2. Find the value of $cosec^2\ 30° \sin^2 45° - \sec^2 60°$

Sol. $cosec^2 30° \sin^2 45° - \sec^2 60°$

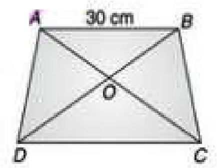
$= (2)^2 \left(\frac{1}{\sqrt{2}}\right)^2 - (2)^2$ $\left[\because cosec\ 30° = 2, \sin 45° = \frac{1}{\sqrt{2}}\ and \sec -60° = 3\right]$

$= \frac{4}{2} - 4$

$= -2$

Q3. ABCD is a trapezium such that $AB \parallel DC\ and\ AB = 3\ cm$. If the diagonals AC and BD intersect at O such that $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$, then calculate DC.

Sol. In the triangle AOB and DOC, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}\ and\ AB = 3\ cm$



∴ $\frac{AO}{OC} = \frac{BO}{OD}$

∴ ∠AOB = ∠DOC

Hence, both triangles are similar

$\frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{DC}$

(In similar triangle corresponding sides are proportional.)

$DC = 2AB$

$DC = 2 \times 3 = 6\ cm$

Q4. If the curves for more than ogive and less than ogive of a grouped data meet at (30, 45), then calculate the median of the data.

Sol. If the curves for more than ogive and less than ogive of a grouped data meet at (30, 45), then median of the data is 30.

SECTION – B

Q5. If the zeroes of the polynomial $x^2 + px + q$ are double in the value of the zeroes of $2x^2 - 5x - 3$, find the value of p and q.

Sol. Let $f(x) = 2^2 - 5x - 3$

Let the zeroes of polynomial are α and ß, then

Sum of zeroes, α + ß $= \frac{5}{2}$

Product of zeroes, $\alpha ß = -\frac{3}{2}$

According to the question, zeroes of $x^2 + px + q$ are 2α and 2ß

Sum of zeroes $= -\frac{Coeff.of\ x}{Coeff.of\ x^2} = \frac{-p}{1}$

$$-p = 2\alpha + 2ß = 2(\alpha + ß)$$

$$-p = 2 \times \frac{5}{2} = 5\ term$$

Product of zeroes $= \frac{contant}{coeff.of\ x^2} = \frac{q}{1}$

$$q = 2\alpha \times 2ß = 4\alpha ß$$

$$q = 4\left(-\frac{3}{2}\right) = -6$$

$$q = -5\ and\ q = \ -6.$$

Q6. If $x = a, y = b$ is the solution of the equations x + y = 50 and $4x + 5y = 225$, then the values of a and b are respectively

(a) 10 and 40
(b) 25 and 25
(c) 23 and 27
(d) 20 and 30

Sol. Choice (b) is correct.

Since $x = a\ and\ y = b$ is the solution of the equations, therefore,

$a + b = 50$ .....(1)

And $4a + 5b = 225$ .....(2)

From (1), $a = 50 - b$

Substituting $a = 50 - b$ in $(2), we$ get

$$4(50 - b) + 5b = 225$$

$\Longrightarrow$ $200 - 4b + 5b = 225$

$\Longrightarrow$ $b = 225 - 200$

$\Longrightarrow$ $b = 25$

Substituting $b = 25$ in (1), we get

$$a = 50 - 25 = 25$$

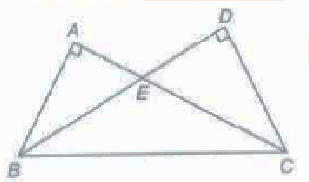
Hence the values of a and b are 25 and 25 respectively.

Q7. Two ΔABC and ΔDBC are on the same base BC and on the same side of BC in which ∠A = ∠D = 90°. If CA and BD meet each other at E, then show that AE. EC = BE. ED.

Sol. **Given** : Two ΔABC and ΔDBC are on the same base BC and on the same side of BC in which ∠A = ∠D = 90° such that CA and BD meet each other at E.

To prove: AE . EC = BE . ED

∠BAC = ∠BDC = 90°



∠AEB = ∠DEC [vertically opposite angles]

∴ $\Delta AEB \sim \Delta DEC$ [by AA similarity]

⇒ $\frac{AE}{ED} = \frac{BE}{EC}$

∴ $AE\,.\,EC = BE.\,ED$

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Q8. If $4\cos\theta = 11\sin\theta$, fine the value of $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta}$.

Sol. Given: $4\cos\theta = 11\sin\theta$

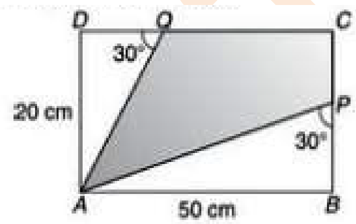
$\implies$ $\cos\theta = \frac{11}{4}\sin\theta$

Now, $\frac{11\cos\theta-7\sin\theta}{11\cos\theta+7\sin\theta} = \frac{11\times\frac{11}{4}\sin\theta-7\sin\theta}{11\times\frac{11}{4}\sin\theta+7\sin\theta}$

$= \frac{\sin\theta\left(\frac{121}{4}-7\right)}{\sin\theta\left(\frac{121}{4}+7\right)}$

$= \frac{121-28}{121+28} = \frac{93}{149}$.

Q9. In the given figure, ABCD is a rectangle in which line segments AP and AQ. Are drawn such that ∠APB = ∠AQD = 30°. Find the length of (AP + AQ).



Sol. In ΔABP, $\sin 30° = \frac{AB}{AP}$

$\frac{1}{2} = \frac{50}{AP} \implies AP = 100\ cm$

In ΔAQD, $sin 30° = \frac{AD}{AQ}$

$\implies$ $\frac{1}{2} = \frac{20}{AQ} \implies AQ = 40\ cm$

Now, the length of $(AP + AQ)$

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Q10. The data regarding the height of 50 girls of class X of a school is given below. Write the above distribution as more than type cumulative frequency distribution.

Hight (in cm)	120 – 130	130 – 140	140 – 150	150 – 160	160 – 170	Total
Number of girls	2	8	12	20	8	50

Sol.

Heights	**No. of girls**
120 and more	50
130 and more	48
140 and more	40
150 and more	28
160 and more	8

SECTION – C

Q11. Show that $n^2 - 1$ is divisible by 8, if n is an odd positive integer.

Sol. We know that any odd positive integer is of the form $4m + 1$ *or* $4m + 3$ for some integer m.
When $n = 4m + 1$, then

$n^2 - 1 = (4m + 1)^2 - 1$
$= (16m^2 + 8m + 1) - 1$
$= 16m^2 + 8m$
$= 8m(2m + 1)$

$\Longrightarrow$ $n^2 - 1$ is divisible by 8 $[\because 8m(2m + 1)$ is divisible by 8]

When $n = 4m + 3$, then

$n^2 - 1 = (4m + 3)^2 - 1$
$= (16m^2 + 24m + 9) - 1$
$= 16m^2 + 24m + 8$
$= 8(2m^2 + 3m + 1)$

$\Longrightarrow$ $n^2 - 1$ is divisible by 8 $[\because 8(2m^2 + 3m + 1)$ is divisible by 8]

Hence, $n^2 - 1$ is divisible by 8.

Q12. An army contingent of 140 members is to march behind an army band of 96 members in parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Sol. Let the number of columns be x.

X is the largest number, which should divide both 104 and 96.

$$104 = 96 \times 1 + 8$$

$$96 = 8 \times 12 + 0$$

∴ HCF of 104 and 96 is 8.'

Hence, 8 columns are required,

Q13. On dividing a polynomial $3x^3 + 4x^2 + 5x - 13$ by a polynomial g(x), the quotient and the remainder were $(3x + 10)$ and $(16x - 43)$ respectively. Find g(x).

Sol. $3x^3 + 4x^2 + 5x - 13 = (3x + 10)g(x) + (16x - 43)$

$\Longrightarrow$ $$\frac{3x^3+4x^2-11x+30}{3x+10} = g(x)$$ (Adding (16x – 43) to the divided)[1/2]

$3x + 10)\overline{3x^3 + 4x^2 - 11x + 30}(x^2 - 2x + 3$

$3x^3 + 10x^2$

$-\quad -$

$-6x^2 - 11x$

$-6x^2 - 20x$

$+\quad +$

$9x - 30$

$9x - 30$

$-\quad -$

0

Hence, $g(x) = x^2 - 2x + 3,$

Q14. Gaura went to a 'sale' to purchase some pants and skirts. When her friends asked her how many of each she had bought, She answered, the number of pants purchased. Also, the number of skirts if four less than four times the number of pants purchased. Help her friend to find.

(a) How many pants and skirts Gaura bought?

(b) Which mathematical concept is used to solve the above question?

(c) Which value(s) are hidden behind conductivity in the question?

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Sol. (a) Let the number of pants be x and the number skirts be y.

According to the question,

$y = 2x - 2$ ... (i)

$y = 4x - 4$ ... (ii)

From Eqs. (i) and (ii). We get

$4x - 4 = 2x - 2$

$\Rightarrow \quad 4x - 2x = -2 + 4$

$\Rightarrow \quad 2x = 2$

$\Rightarrow \quad x = 1$

Putting the value of x in Eq. (i).

$y = 2 \times 1 - 2$

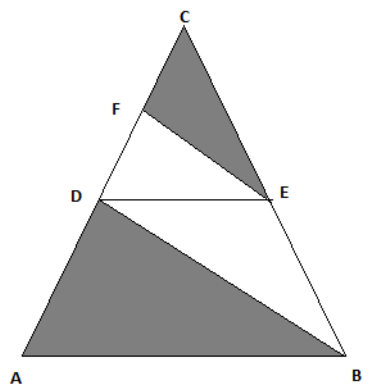
$= 2 - 2 = 0$

Hence, the number of pants, she purchased is 1 and she did not buy any skirt.

(b) Polynomial

(c) Friendly nature and fond of shopping

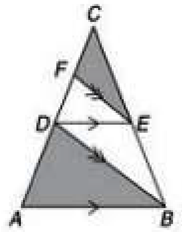
Q15. In the given figure, $DE \parallel AB$ and $EF \parallel DB$. Prove that $DC^2 = CF \times AC$.



Sol. According to the question,

$DE \parallel AB$

$\therefore \quad \frac{CD}{AD} = \frac{CE}{EB}$ (By BPT) .... (i)



Again since FE ∥ DB,

∴ $\frac{CE}{EB} = \frac{CF}{FD}$ (By BPT) ….. (ii)

From equation (i) and (ii)

$$\frac{CD}{AD} = \frac{CF}{FD} \Longrightarrow \frac{AD}{CD} = \frac{FD}{CF}$$

$\Longrightarrow$ $$1 + \frac{DA}{CD} = \frac{FD}{CF} + 1$$

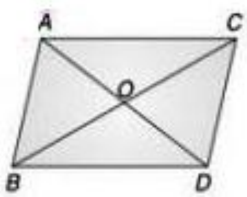
$\Longrightarrow$ $$\frac{CD+DA}{CD} = \frac{FD+CF}{CF}$$

$\Longrightarrow$ $$\frac{AC}{CD} = \frac{CD}{CF}$$

$\Longrightarrow$ $$CD^2 = AC.CF \Longrightarrow DC^2 = CF \times AC$$

Q16. In the given figure, ΔABC and ΔDBC are on the same base BC, AD and intersect at O, Prove that:

$\frac{ar\,(\Delta ABC)}{ar\,(DBC)} = \frac{AO}{DO}$.



Sol. To Prove: $\frac{ar\,(\Delta ABC)}{ar\,(\Delta\, DBC)} = \frac{AO}{DO}$

Construction: Draw$AE \perp BC$ and $DF \perp BC$.

Proof:

In ΔAOE and ΔDOF, ∠AOE = ∠ DOF (Vertically oppsite angles)

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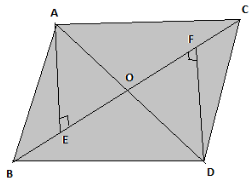
$\angle AEO = \angle DFO = 90°$ (Construction)

$\Rightarrow$ $\Delta AOE \sim \Delta DOF$ (By AA similarity)

$\therefore$ $\frac{AO}{DO} = \frac{AE}{DF}$



Now, $\frac{ar\,(\Delta\,ABC)}{ar\,(\Delta\,DBC)} = \frac{\frac{1}{2}\times BC\times AE}{\frac{1}{2}\times BC\times DF} = \frac{AE}{DF}$

$= \frac{AO}{DO}$ [From equation (i)]

Q17. Prove that: $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A$.

Sol. L.H.S. = $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A}$

$= \frac{\cos A}{1-\left(\frac{\sin A}{\cos A}\right)} + \frac{\sin A}{1-\left(\frac{\cos A}{\sin A}\right)}$

$= \frac{\cos^2 A}{coa\ A-\sin A} + \frac{\sin^2 A}{\sin A-\cos A}$

$= \frac{\cos^2 A}{\cos A-\sin A} - \frac{\sin^2 A}{\cos A-\sin A}$

$= \frac{\cos^2 A-\sin^2 A}{\cos A-\sin A}$

$= \frac{(\cos A-\sin A)\,(\cos A+\sin A)}{(\cos A-\sin A)}$

$= \cos A + \sin A$

$= R.H.S.$ Hence Proved

Q18. In ΔABC, ∠B = 90°, BC = 5 cm, AC – AB = 1, evaluate $\frac{1+\sin C}{1+\cos C}$.

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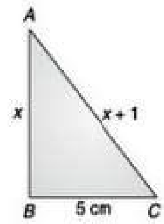

Sol. Let $AB = x$

$\because$ $AC - AB = 1$

$\Longrightarrow$ $AC = x + 1$

$\because$ $AC^2 = AB^2 + BC^2$

$\therefore$ $(x+1)^2 = x^2 + (5)^2$

$\Longrightarrow$ $x^2 + 2x + 1 = x^2 + 25$

$\Longrightarrow$ $2x = 24 \Longrightarrow x = \frac{24}{2} = 12\ cm'$

Hence, $AB = 12\ cm, AC, = 13\ cm$

$\sin C = \frac{AB}{AC} = \frac{12}{13}$

$\text{Cos}\, C = \frac{AB}{AC} = \frac{5}{13}$

Now $\frac{1+\sin C}{1+\cos C} = \frac{1+\frac{12}{13}}{1+\frac{5}{13}} = \frac{\frac{25}{13}}{\frac{18}{13}} = \frac{25}{18}$

Q19. The following is the age distribution of patients admitted during a month in a hospital. Find the model age of a patient.

Age in years	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	Total
No. of Patients	5	10	20	25	12	18	10	100

Sol. $\text{Mode} = 1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

Here, Modal class $= 30 - 40$,

$$l = 30, f_1 = 25,\ f_0 = 20, f_2 = 12, h = 10$$

$$\text{Mode} = 30 + \frac{25-20}{50-20-12}$$

$$= 30 + \frac{5\times10}{18}$$

$$= 30 + 2.77 = 32.77\ (Modal\ age)$$

Q20. The mean of the following distribution is 53. Find the missing frequency p.

Class Interval	0 – 20	20 – 40	40 – 60	60 – 80	80 – 1000
Frequency	12	15	32	P	13

Sol.

Sol. x_i	f_i	$f_i x_i$
10	12	120
30	15	450
50	32	1600
70	P	70p
90	13	1170
Total	$\sum f_i = 72 + p$	$\sum f_i x_i = 3340 + 70p$

$\therefore$ Mean, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow$ $53 = \frac{3340+70p}{72+p}$

$\Rightarrow$ $3340 + 70p = 53(72 + p)$

$\Rightarrow$ $3340 + 70p = 3816 + 53p$

$\Rightarrow$ $70p + 53p = 3816 - 3340$

$\Rightarrow$ $17p = 476$

$$p = \frac{476}{17} = 28$$

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SECTION – D

Q21. Using Euclid's division algorithm, find the HCF of 56, 96 and 404

Sol. Given, integers are 56, 96 and 404.

First, we will find the HCF of 56 and 96.

On applying Euclid's division algorithm, we get

$$96 = 56 \times 1 + 40$$

Since, remainder, 40≠0, so we apply Euclid's division algorithm to 56 and 40.

$$56 = 40 \times 1 + 16$$

$\because$ Remainder, $16 \neq 0$, so we apply Euclid's division algorithm to 40 and 16.

$$40 = 16 \times 2 + 8$$

$\because$ Remainder $8 \neq 0$, so again apply Euclid's division algorithm, we get $16 = 8 \times 2 + 0$

Clearly, HCF of 56 and 96 is 8.

Now, we find the HCF of 8 and third number 404.

On applying Euclid's division, we get

$$404 = 50 \times 8 + 4$$

Since, remainder, 4≠ 0, so we apply Euclid's division algorithm to 8 and 4.

$$8 = 4 \times 2 + 0$$

Since, remainder is 0

i.e., HCF of 56, 96 and 404 is 4.

Q22. If a and β are the zeroes of polynomial $p(x) = 3x^2 + 2x + 1$, find the polynomial whose zeroes are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$.

Sol. Since, α and ß are the zeroes of polynomial $3x^2 + 2x + 1$.

Hence, $\alpha + ß = -\frac{2}{3}$

And $\alpha ß = \frac{1}{3}$,

Now for the new polynomial,

Sum of the zeroes $= \frac{1-\alpha}{1+\alpha} + \frac{1-ß}{1+ß}$

$$= \frac{(1-\alpha+ß-\alpha ß)+(1+\alpha-ß-\alpha ß)}{(1+\alpha)\,(1+ß)}$$

$$= \frac{2-2\alpha ß}{1+\alpha+ß+\alpha ß} = \frac{2-\frac{2}{3}}{1-\frac{2}{3}+\frac{1}{3}}$$

$$\text{Sum of the zeroes} = \frac{\frac{4}{3}}{\frac{2}{3}} = 2$$

$$\text{Product of zeroes} = \left[\frac{1-\alpha}{1+\alpha}\right]\left[\frac{1-ß}{1+ß}\right] = \frac{(1-\alpha)(1-ß)}{(1+ß)(1+ß)}$$

$$= \frac{1-\alpha-ß+\alpha ß}{1+\alpha+ß+\alpha ß} = \frac{1-(\alpha+ß)+\alpha ß}{1+(\alpha+ß)+\alpha ß}$$

$$\text{Product of zeroes} = \frac{1+\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}} = \frac{\frac{6}{3}}{\frac{2}{3}} = 3$$

Hence, required polynomial $x^2 - \ (sum\ of\ zeroes)\ x + Product\ of\ zeroes$

$$= x^2 - 2x + 3.$$

Q23. A and B are two points 150 km apart on a highway. Two cars stars from A and B at the same time. If they move in the same direction, they meet in 15 hours. But if they move in the opposite direction, they meet in 1 hour. Find their speed.

Sol. Let the speed of the car I from A $= x$ km/hr.

Speed of the car II from B $= y$ km/hr.

Same direction:

Distance covered by car I = 150 + (distance covered by car II)

$\Longrightarrow \quad 15x = 150 + 15y$

$\Longrightarrow \quad 15x - 15y = 150$

$\Longrightarrow \quad x - y = 10$ .....(i)

Opposite direction:

Distance covered by car I + distance covered by car II

$$= 150\ km$$

$x + y = 150$ ......(ii)

Adding eq. (i) and (ii), we get $2x = 160$

$\Rightarrow \qquad x = 80$

Putting $x = 80$ in eq. (i), $\qquad y = 70$

$\therefore$ speed of the car I = 80 km/hr

And speed of the car II = 70 km/hr

Or

A and B are fiends and their ages differ by 2 years. A's father D is twice as old as A and B is twice as old as his sister C. The age of D and C differ by 40 years. Find the ages of A and B

Sol. Let the ages of A and B be x and y years respectively, then ....(1)

$x - y = \pm 2$

D's age = Twice the age of A = $2x$ years

$\Rightarrow$ C's age = Half the age of B = $\frac{y}{2}$ years

Then, $2x - \frac{y}{2} = 40$ ....(2)

When $x - y = 2..(1a)$ and $2x - \frac{y}{2} = 140$

Multiplying (2) by 2, we get

$4x - y = 80$

Subtracting $(1a) from$ (3), we get

$(4x - y) - (x - y) = 80 - 2$

$\Rightarrow \quad 3x = 78$

$\Rightarrow \quad x = 26$

Substituting $x = 26$ in $(1a)$, we get

$26 - y = 2 \Rightarrow y = 26 - 2 = 24$

When $x - y = -2 \ldots \ldots (1b) and\ 2x - \frac{y}{2} = 40.$

Multiplying (2) by 2, we get ....(3)

$4x - y = 80$

Subtracting (1b) from (3), we get

$(4x - y) - (x - y) = 80 + 2$

$\Rightarrow \quad 3x = 82$

$\Rightarrow \quad x = \frac{82}{3} = 27\frac{1}{3}$

Substituting $x = 27\frac{1}{3}\ in\ (1b)$, we get

$\frac{82}{3} - y = -2$

$\Rightarrow \quad y = \frac{82}{3} + 2$

$\Rightarrow \quad y = \frac{82+6}{3} = \frac{88}{3} = 29\frac{1}{3}$

Hence, A's age = 26 years and B's age = 24 years

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or

A's age $= 27\frac{1}{3}$ years and B's age $-29\frac{1}{3}$ years.

Q24. A two-digit number is obtained either by multiplying the sum of digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and adding 3. Find the number.

Sol. Let the two digits number be $10x + y$

According to the question, $8(x + y) - 5 = 10x + y$

$\Longrightarrow \quad 2x - 7y + 5 = 0$ ......(i)

And $\quad 16(x - y) + 3 = 10x + y$

$6x - 17y + 3 = 0$ ......(ii)

Solving equation (i) and (ii) by cross-multiplication method, we get

$$\frac{x}{(-7)(3)-(-17)(5)} = \frac{y}{(5)(6)-(2)(3)} = \frac{1}{(2)(-17)-(6)(-7)}$$

$\Longrightarrow \quad \frac{x}{-12+85} = \frac{y}{30-6} = \frac{1}{-34+42}$

$\Longrightarrow \quad \frac{x}{64} = \frac{y}{24} = \frac{1}{8}$

$\Longrightarrow \quad \frac{x}{8} = \frac{y}{3} = 1$

Hence, $\quad x = 8, y = 3$

So, the required number $= 10 \times 8 + 3 = 83$.

Q25. In figure, ΔABC is right angled at C. DE ⊥ AB. If BC = 12 cm. AD = 3 cm and DC = 2 cm, then prove that ΔABC ~ ΔADE and hence find the lengths of AE and DE.

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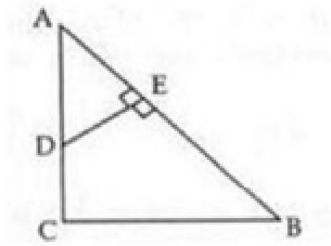
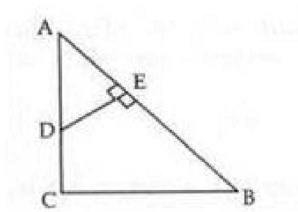


Sol.



In triangles ABC and ADE, we have:

$\angle A = \angle A$

$\angle ABC = \angle AED$

Therefore, $\angle ABC \sim ADE$ (by AA similarity criterion)

$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$ …… (1)

Now, in right $\angle ABC$,

$AB^2 = BC^2 + AC^2$ (by Pythagoras theorem)

$\Longrightarrow AB^2 = 12^2 + 5^2 = 144 + 25 = 169$

$\Longrightarrow AB = 13\ cm$

Substituting the lengths of the sides in (1), we get:

$DE = \frac{36}{13}$ and $AE = \frac{15}{13}$

Q24. Prove that the area of semi- circle drawn on the hypotenuse of a right angled triangle is equal to the sum of the area of the semi – circles drawn on the other two sides of the triangle.

Sol. Let ABC be a right angled triangle at B.
Let AB = y, BC = x

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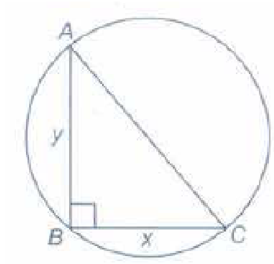

A semi-circle is drawn all three sides of a triangle.

In ΔABC, by Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= y^2 + x^2$

$\Rightarrow \quad AC = \sqrt{y^2 + x^2}$

$\therefore$ Area of semi – circle drawn on AC, is

$A_1 = \frac{\pi r_1^2}{2}$

$= \frac{\pi}{2}\left(\frac{\sqrt{y^2+x^2}}{2}\right)^2$

$= \pi\left(\frac{y^2+x^2}{8}\right)\left(\because r = \frac{AC}{2}\right) \ldots .(i)$

Now, are of semi-circle drawn on BC, is

$A_2 = \frac{\pi r_2^2}{2} = \frac{\pi}{2}\left(\frac{\pi}{2}\right)^2$ ....(ii)

Also, area of semi-circle drawn on AB, is

$A_3 = \frac{\pi r_3^3}{2} = \frac{\pi}{2}\left(\frac{y}{2}\right)^2$

$= \frac{\pi y^2}{8}$ ....(iii)

$\therefore$ From Eq. (i), we get

$A_1 = \pi\left(\frac{y^2}{8} + \frac{x^2}{8}\right)$

$= \frac{\pi y^2}{8} + \frac{\pi x^2}{8} = A_3 + A_2$ [From Eqs. (ii) and (iii)]

Q.27. If $x = a\sec\theta + \tan\theta$ and $y = a\tan\theta + b\sec\theta$, prove that $x^2 - y^2 = a^2 - b^2$

Sol. We have

$x = a\sec\theta + b\tan\theta$ ....(1)

And $y = a\tan\theta + b\sec\theta$ ....(2)

Squaring (1) and (2), we get

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$x^2 = a^2\theta + b^2 \tan^2 + 2ab \sec\theta \, tan\,\theta$ ....(3)

And $y^2 = a^2 \tan^2\theta + b^2 \sec^2\theta + 2ab \sec\theta \tan\theta$ ....(4)

Subtracting (4) from (3), we get

$x^2 - y^2 = (a^2 \sec^2\theta + b^2 \tan^2\theta + 2ab \sec\theta \tan\theta) - (a^2 \tan^2\theta + b^2 \sec^2\theta +)$
$2ab \sec\theta \tan\theta$

$\Rightarrow$ $x^2 - y^2 = a^2(\sec^2\theta - \tan^2\theta) + b^2(\tan^2\theta - sec^2\theta)$

$\Rightarrow$ $x^2 - y^2 = a^2(\sec^2\theta - \tan^2\theta - \tan^2\theta) + b^2(\tan^2\theta - 1 - \tan^2\theta)$

$\Rightarrow$ $x^2 - y^2 = a^2(1) + b^2(-1)$

$\Rightarrow$ $x^2 - y^2 = a^2 - b^2$

Q28. Prove that:

$$\sqrt{\frac{cosec\,A-1}{cosec\,A+1}} + \sqrt{\frac{cosec\,A+1}{cosec\,A-1}} = 2\sec A$$

Sol. We have

$$L.H.S. = \sqrt{\frac{cosec\ A-1}{cosec\ A+1}} + \sqrt{\frac{cosec\ A+1}{cosec\ A-1}}$$

$$= \frac{(\sqrt{cosec\ A-1})\,(\sqrt{cosec\ A-1}) + (\sqrt{cosec\ A+1})\,(\sqrt{cosec\ A+1})}{\sqrt{cosec\ A+1}\,\sqrt{cosec\ A-1}}$$

$$= \frac{(cosec\ A-1)+(cosec\ A+1)}{\sqrt{cosec^2\ A-1}} \qquad [\because \sqrt{a+b} \times \sqrt{a-b} = a-b]$$

$$= \frac{2\ cosec\ A}{\sqrt{1+cosec\ A}}$$

$$= \frac{2\ cosec\ A}{\sqrt{\cot^2 A}}$$

$$= \frac{2\ cosec\ A}{\cot A}$$

$$= \frac{2}{\sin A} \times \frac{\sin A}{\cos A}$$

$$= \frac{2}{\cos A}$$

$$= 2\sec A$$

Q29. $\sqrt{\frac{sec\theta-1}{sec\theta+1}} + \sqrt{\frac{sec\theta+1}{sec\theta-1}} = 2\ cosec\ \theta$

Sol. $LHS = \sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}}$

$$= \frac{(\sqrt{\sec\theta+1})^2 + (\sqrt{\sec\theta+1})^2}{(\sqrt{\sec\theta+1})(\sqrt{\sec\theta-1})}$$

$$= \frac{\sec\theta-2+\sec\theta+2}{\sqrt{\sec^2\theta-1}}$$

$$= \frac{2\sec\theta}{\sqrt{\tan^2\theta}}$$

$$= \frac{2\sec\theta}{\tan\theta} = 2 \times \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} = 2\, cosec\,\theta = \boldsymbol{RHS}$$

Hence, LHS = RHS.

Q30. In the distribution given below 51% of the observations is more than 14.4. Find the values of x and y. if the total frequency is 20.

Class Interval	0 – 6	6 – 12	12 – 18	80 – 24	24 – 30
Frequency	4	X	5	Y	1

Sol.

Class Interval	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
Frequency	4	X	5	Y	1
Cumulative frequency	4	4+x	9+x	9+x+y	10+x+y

It is given that total frequency N is 20

So, 10+x+y = 20 i.e. x +y = 10 ......(i)

Given 50% of the observation are greater than 14.4.

So median = 14.4, which lies in the class interval 12 – 18.

$I = 12, cf = 4 + x, h = 6, f = 5, N = 20$

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Median $\ell + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$

$14.4 = 12 + \left(\frac{10-(4+x)5}{5}\right) \times 6$

$14.4 - 12 = \frac{(6-x)}{5} \times 6$

$\frac{2.4 \times 5}{6} = 6 - x$

$x = 4$

Now using equation, $10 + x + y = 20$, we get y = 6.

Hence $x = 4$ and $y = 6$.

Or

100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows

Number of letters	1 – 4	4 – 7	7 – 10	10 – 13	13 – 16	16 – 19
Number of surnames	6	30	40	16	4	4

Determine the median number of letter in the surnames. Also, find the modal size of the surnames.

Sol.

Number of surnames	Number of surnames	Cumulative frequency
1 – 4	6	6 = 6
4 – 7	30	6+30 = 36 = cf
(Median class) 07 - 10	40 = f	36+40 = 76
10 – 13	16	76 +16 = 92
13 – 16	4	92 + 4 = 96
16 – 19	4	96 + 4 = 100
Total	N = 100	

(a)

$\because \frac{N}{2} = \frac{100}{2} = 50$

Since, cumulative frequency 50 lies in the interval 7 – 10.

Here, (Lower median class)$l = 7, f = 40, cf = 36$.

$\text{(Class width)h} = 3. \text{(Total observations)N} = 100$

$$\text{Median} = l + \left\{\frac{\frac{N}{2} - cf}{f}\right\} \times h$$

$$= 7 + \left\{\frac{50-36}{40}\right\} \times 3$$

$$= 7 + \frac{14 \times 3}{40}$$

$$= 7 + \frac{21}{20} = 7 + 1.05 = 8.05$$

Hence, the median number of letters in the surnames is 8.05.

(b) Modal class is (7 - 10) (because it has maximum frequency f_m = 40) .

$\text{(Lower modal clss)}l = 7, f_m = 40, f_1 = 30, f_2 = 16, \text{(class width)h} = 3.$

$$\text{Mode} = l + \left\{\frac{f_m - f_1}{2f_m - f_1 - f_2}\right\} \times h = 7 + \left\{\frac{40-30}{80-30-16}\right\} \times 3$$

$$= 7 + \frac{30}{34} = 7 + 0.88 = 7.88$$

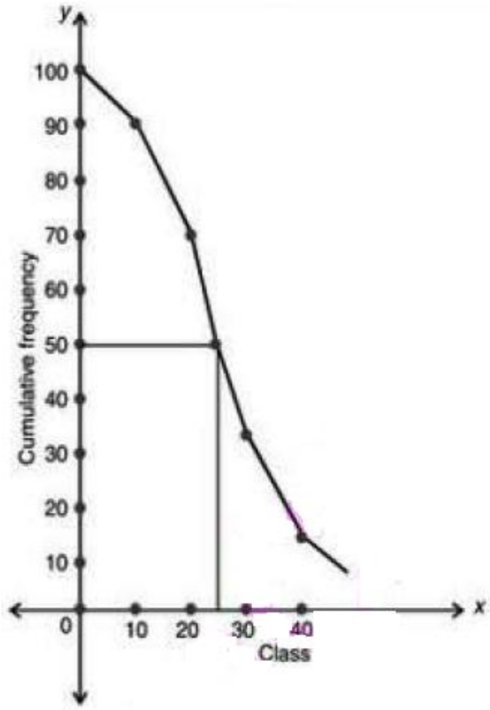
Hence, the modal size of the surname is 7.88

Q31. Draw more than ogive for the following distribution. Find the median from the curve.

Class Interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	10	18	40	20	12

Sol.

More than	$c.f.$
0	100
10	90
20	72
30	32
40	12



From graph, $\frac{N}{2} = \frac{100}{2} = 50$

Hence, Median = 25