

1. Given : AB is diameter
 $\angle CAB = 30^\circ$

To find $\angle PCA$

construction : Join OC

sol : \therefore In $\triangle AOC$

as $AO = OC$

$$\therefore \angle OAC = \angle OCA = 30^\circ$$

$\angle OCP = 90^\circ$ [Radius make an angle of 90° with tangent at point of contact]

$$\therefore \angle PCA + \angle OCA = 90^\circ$$

$$\therefore \angle PCA + 30^\circ = 90^\circ$$

$$\therefore \angle PCA = 60^\circ$$

2. $k + 9, 2k - 1$ and $2k + 7$ are in A.P.

$$\therefore a_2 - a_1 = a_3 - a_2 \quad [\text{where } a_1, a_2 \text{ and } a_3 \text{ are the } 1^{\text{st}}, 2^{\text{nd}} \text{ and } 3^{\text{rd}} \text{ term of the A.P.}]$$

$$2k - 1 - k - 9 = 2k + 7 - 2k + 1$$

$$k - 10 = 8$$

$$k = 18$$

3. In $\triangle ABC$

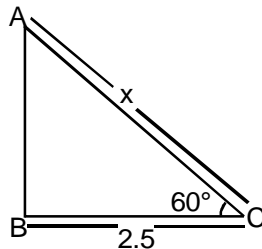
$$\cos 60^\circ = \frac{BC}{AC}$$

$$\frac{1}{2} = \frac{2.5}{AC}$$

$$AC = 2.5 \times 2$$

$$AC = 5 \text{ m}$$

\therefore length of the ladder is 5 m



4. We have to draw a card from 52 playing cards so the total event of drawing a card is = 52 and the event of getting red card and queen is = $26 + 2 = 28$

Acc to question

The probability of getting

$$\text{neither red card nor a queen} = P(\bar{A}) = 1 - P(A)$$

$$= P(\bar{A}) = 1 - \frac{28}{52} = \frac{6}{13}$$

5. Let $-5, \alpha$ be the roots of $2x^2 + px - 15 = 0$

$$\text{so sum of roots} = -5 + \alpha = -\frac{P}{2}$$

$$\text{and product of roots} = -5 \times \alpha = \frac{-15}{2}$$

$$\therefore \alpha = \frac{3}{2}$$

If $\alpha = 3/2$ then

$$P = 7$$

and $P(x^2 + x) + k = 0$ have equal roots

so $D = 0$

$$\Rightarrow P^2 - 4Pk = 0$$

$$\Rightarrow P(P - 4k) = 0$$

$$P = 0 \text{ \& } P - 4k = 0$$

so $4k = p$

$$k = \frac{P}{4} = \frac{7}{4}$$

6.
$$x_1 = \frac{(2x-7)+(1 \times 2)}{2+1}$$

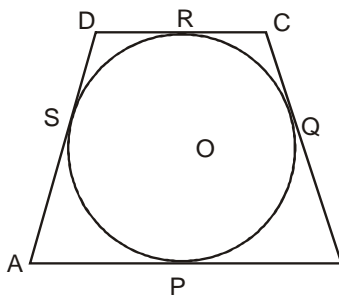
$$x_1 = \frac{-14+2}{3} = \frac{-12}{3} = -4$$

$$y_1 = \frac{(2 \times 4)+(1 \times -2)}{2+1} = \frac{8-2}{3} = \frac{6}{3} = 2$$
 $(x,y) = (-4, 2)$ coordinates of Q.

coordinates of point P(x₂,y₂)
 \Rightarrow mid of AQ is P

So $x_2 = \frac{2+(-4)}{2} = \frac{-2}{2} = -1$

$y_2 = \frac{-2+2}{2} = 0, y = 0$



As we know that tangent from same external points are equal

$\therefore SD = DR$... (1)

$CQ = CR$... (2)

$QB = BP$... (3)

$AS = AP$... (4)

Adding equation (1), (2), (3) & (4)

$SD + CQ + QB + AS = DR + CR + BP + AP$

$AD + BC = AB + DC$ Hence proved

8. To prove : $\triangle ABC$ is a triangle isosceles triangle

Proof : $AB = \sqrt{(3-6)^2 + (0+4)^2}$ (By using distance formula)

$AB = \sqrt{9+16} = \sqrt{25} = 5$

$\therefore AB = 5$

$AC = \sqrt{(3+1)^2 + (0-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

$\therefore AC = 5$

$BC = \sqrt{(6+1)^2 + (4-3)^2} = \sqrt{49+1} = \sqrt{50}$

$\therefore BC = 5\sqrt{2}$

Now as $AB = AC$

$\therefore \triangle ABC$ is isosceles and $(AB)^2 + (AC)^2 = (BC)^2$

\therefore By converse of pythagoras theorem $\triangle ABC$ is a right angle isosceles triangle.

9. Let the first term and common difference of the A.P. be a and d respectively.

Then, $a_n = a + (n-1)d$

$a_4 = a + (4-1)d = 0$

$a_4 = a + 3d = 0$

$a + 3d = 0$

$\therefore a = -3d$ (1)

$$a_{25} = a + (25-1)d$$

$$a_{25} = a + 24d$$

By equation(1)

$$a_{25} = -3d + 24d$$

$$a_{25} = 21d$$

$$a_{11} = a + (11-1)d$$

$$a_{11} = a + 10d$$

By equation(1)

$$a_{11} = -3d + 10d$$

$$\therefore a_{11} = 7d$$

multiply both sides by 3

$$3a_{11} = 21d$$

$$\therefore 3a_{11} = a_{25} \text{ Hence proved}$$

10. In $\triangle OTP$
 $OT = r$, $OP = 2r$ [Given]
 $\angle OTP = 90^\circ$ [radius is perpendicular to tangent at the pair of contact]
 Let $\angle TPO = \theta$

$$\therefore \sin \theta = \frac{OT}{OP} = \frac{r}{2r} = \frac{1}{2}$$

$$\therefore \theta = 30^\circ$$

$$\therefore \text{In } \triangle TOP \angle TOP = 60^\circ \text{ [By angle sum property]}$$

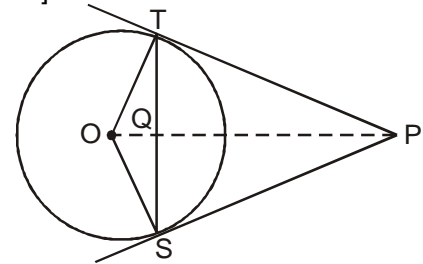
$$\angle TOP = \angle SOP \text{ [As } \triangle \text{'s are congruent]}$$

$$\therefore \angle SOP \text{ is also } 60^\circ$$

$$\therefore \angle TOS = 120^\circ \text{ In } \triangle OTS \text{ as } OT = OS \therefore [\angle OST = \angle OTS]$$

$$\angle OTS + \angle OST + \angle SOT = 180 \Rightarrow 2 \angle OST + 120 = 180^\circ$$

$$\therefore \angle OTS + \angle OST = 30^\circ$$



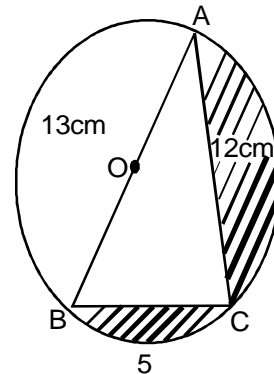
11. $AB^2 = BC^2 + AC^2$
 $\Rightarrow 169 = BC^2 + 144$
 $25 = BC^2$
 $BC = 5$
 Area of shaded region = Area of semicircle
 - Area of $\triangle ABC$

$$= \frac{\pi r^2}{2} - \frac{1}{2} \times BC \times AC$$

$$= \frac{1}{2} [3.14 \times \frac{13}{2} \times \frac{13}{2}] - (5 \times 12)$$

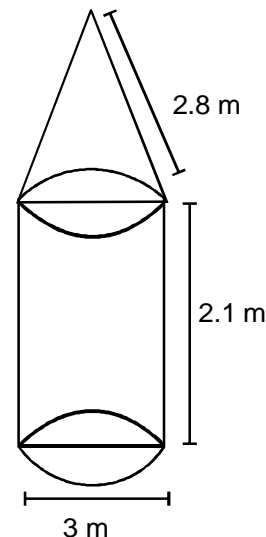
$$= \frac{1}{2} (132.665 - 60)$$

$$= 36.3325 \text{ cm}^2$$

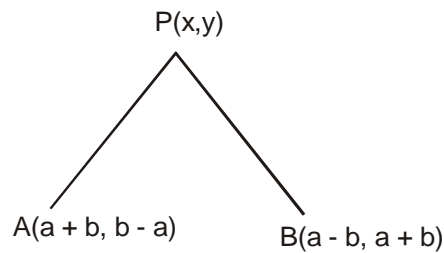


12. Total CSA of tent
 $= 2\pi rh + \pi rl$
 $= \frac{22}{7} [(2 \times \frac{3}{2} \times 2.1) + (\frac{3}{2} \times 1.4)]$
 $\Rightarrow \frac{22}{7} \times 10.5 = 33 \text{ m}^2$

Total CSA oftent = 33 m^2
 $1 \text{ m}^2 \text{ cost} \rightarrow \text{Rs. } 500$
 $33 \text{ m}^2 \text{ cost} \rightarrow \text{Rs. } 500 \times 33 = 16500 \text{ Rs}$
 So total cost of canvas needed to make the text is Rs 16500



13. Given : Coordinates of
 $P(x,y)$
 $A(a + b, b - a)$
 $B(a - b, a + b)$
 To prove = $bx = ay$
 According to question
 $PA = PB$



$$(PA)^2 = (PB)^2$$

so according to distance formula

$$[x - (a + b)]^2 + [y - (b - a)]^2 = [(x - (a - b))]^2 + [y - (a + b)]^2$$

$$(a + b)^2 - 2(a + b)x + (b - a)^2 - 2(b - a)y = (a - b)^2 - 2(a - b)x + (a + b)^2 - 2(a + b)y$$

$$2[(a + b)x + (b - a)y] = 2[(a - b)x + (a + b)y]$$

$$(a + b)x + (b - a)y = (a - b)x + (a + b)y$$

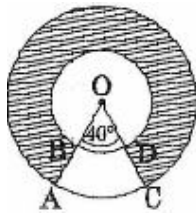
$$(a + b)x - (a - b)x = (a + b)y - (b - a)y$$

$$(a + b - a + b)x = (a + b - b + a)y$$

$$2bx = 2ay$$

$bx = ay$ hence prove

- 14.



Shaded area = Area of larger major sector – area of smaller major sector

$$= \pi(14)^2 \times \frac{40}{360} - \pi(7)^2 \left(\frac{40}{360} \right)$$

$$= \pi \times \frac{40}{360} (14^2 - 7^2)$$

$$= \frac{22}{7} \times \frac{1}{9} (147) = 51.3 \text{ cm}^2$$

- 15.

$$\frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{7n+1}{4n+27}$$

$$\frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{7n+1}{4n+27}$$

$$\frac{a_1 + \frac{(n-1)}{2}d_1}{a_2 + \frac{(n-1)}{2}d_2} = \frac{7n+1}{4n+27} \quad \dots(1)$$

Put $\frac{n-1}{2} = m - 1$

$$n - 1 = 2m - 2$$

$$n = 2m - 2 + 1$$

$$= 2m - 1$$

$$\frac{a_1 + (m-1)d_1}{a_2 + (m-1)d_2} = \frac{7(2m-1)+1}{4(2m-1)+27}$$

$$= \frac{14m-7+1}{8m-4+27} = \frac{14m-6}{8m+23}$$

16. Let $x - 2 = t$

$$\frac{1}{t(t+1)} + \frac{1}{t(t-1)} = \frac{2}{3}$$

$$= \frac{t-1+t+1}{t(t+1)(t-1)} = \frac{2}{3}$$

$$3t = t(t+1)(t-1)$$

$$3t = t(t^2 - 1)$$

$$3t = t^3 - t$$

$$t^3 - 4t = 0$$

$$t(t^2 - 4) = 0$$

$$t = 0 \quad t^2 - 4 = 0$$

$$t = \pm \sqrt{4}$$

$$t = \pm 2$$

$$x - 2 = 0 \quad \& \quad x - 2 = \pm 2$$

$$x = 2 \quad x = 0, 4$$

17. Volume of cone = $\frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 24$$

Volume of cone = volume of cylinder

$$\frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 24 = \frac{22}{7} \times 10 \times 10 \times h$$

$$h = 2 \text{ cm}$$

18. The rise in the level of water will be due to the volume of sphere

$$\therefore \frac{4}{3} \pi (6)^3 = \pi x^2 \times 3 \frac{5}{9}$$

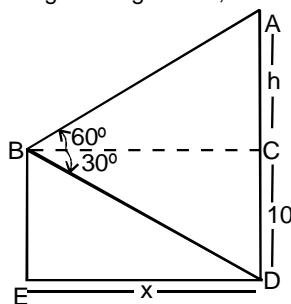
$$\frac{4}{3} \times 6 \times 6 \times 6 = x^2 \times \frac{32}{9}$$

$$x = 9$$

$$\therefore \text{diameter} = 2x = 18 \text{ cm}$$

19. Let x be distance of cliff from man and $h + 10$ be height of hill which is required.

In right triangle ACB,



$$\Rightarrow \tan 60^\circ = \frac{AC}{BC} = \frac{h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}} \quad \dots(i)$$

In right triangle BCD,

$$\tan 30^\circ = \frac{CD}{BC} = \frac{10}{x} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{x}$$

$$\Rightarrow x = 10 \sqrt{3} \quad \dots(ii)$$

From (i) & (ii)

$$\frac{h}{\sqrt{3}} = 10 \sqrt{3}$$

$$\Rightarrow h = 30 \text{ m}$$

\therefore Height of cliff = $h + 10 = 30 + 10 = 40 \text{ m}$.

$$\begin{aligned} \text{Distance of ship from cliff} = x &= 10 \sqrt{3} \text{ m} \\ &= 10 (1.732) = 17.32 \text{ m} \end{aligned}$$

20. Sample space while tossing 3 coins
 $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$
 (i) Favourable cases = $\{HHT, HTH, THH\}$

$$P(\text{exactly 2 heads}) = \frac{\text{Number of favourable outcomes}}{\text{Number of total outcomes}} = \frac{3}{8}$$

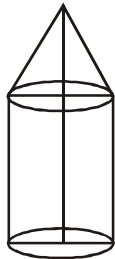
(ii) Favourable cases = $\{HHH, HHT, HTH, THH\}$

$$P(\text{at least 2 heads}) = \frac{4}{8} = \frac{1}{2}$$

(iii) favourable cases = $\{HTT, THT, TTH, TTT\}$

$$P(\text{at least 2 tails}) = \frac{4}{8} = \frac{1}{2}$$

21.



Given $r = 2.8$; $h = 3.5 \text{ m}$
 (ht. of cone) $h_1 = 2.1 \text{ m}$

$$\therefore l = \sqrt{r^2 + (h_1)^2} = 3.5 \text{ m}$$

Area of canvas required per tent
 = [CSA of cone + CSA of cylinder]
 = $\pi r l + 2 \pi r h$
 = $\pi r [3.5 + 7]$

$$= \frac{22}{7} \times \frac{28}{10} \times \frac{105}{10} = \frac{462}{5} \text{ m}^2$$

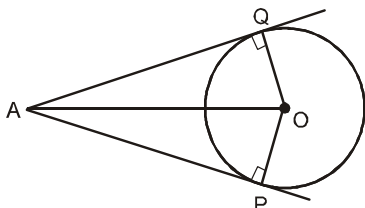
cost of canvas per tent = Rs. $\frac{462}{5} \times 120 = \text{Rs. } 11088$

Total cost of 1500 tents = Rs. 11088×1500

Amount shared by each school;

$$= \text{Rs. } \frac{11088 \times 1500}{50} = \text{Rs. } 332640$$

22.



Given : AP and AQ are two tangents drawn from a point A to a circle C (O, r).

To prove : AP = AQ.

Construction : Join OP, OQ and OA.

Proof : In $\triangle AOQ$ and $\triangle APO$

$$\angle OQA = \angle OPA$$

[Tangent at any point of a circle is perp. to radius through the point of contact]

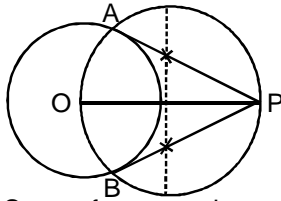
$$AO = AO \quad [\text{Common}]$$

$$OQ = OP \quad [\text{Radius}]$$

So, by R.H.S. criterion of congruency $\triangle AOQ \cong \triangle AOP$

$\therefore AQ = AP$ [By CPCT] **Hence Proved.**

23.



Steps of constructions are as follows

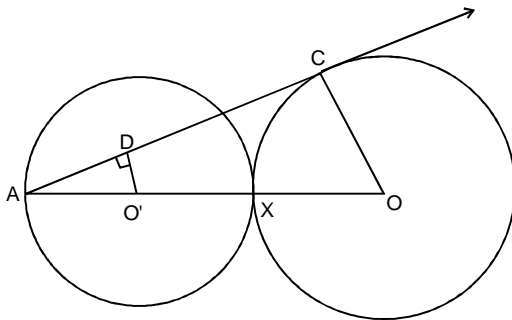
(1) Draw a circle of radius 4 cm

(2) Let O be its centre and P be any external point such that $OP = 8$ cm

(3) Join OP and then taking OP as diameter draw a circle intersecting the given circle at two points A and B. Join AP and BP.

(4) Hence, AP and BP are the required tangents

24.



Let $AO' = OX' = XO = r$

\therefore Radius is always perpendicular to tangent, $\therefore \angle ACO = 90^\circ$

In $\triangle ADO'$ and $\triangle ACO$

$$\angle DAO' = \angle CAO \quad [\text{Common}]$$

$$\angle ADO' = \angle ACO \quad [\text{each } 90^\circ]$$

\therefore By AA similarity criteria

$\triangle ADO' \sim \triangle ACO$

$$\Rightarrow \frac{DO'}{CO} = \frac{AO'}{AO} = \frac{r}{3r} = \frac{1}{3}$$

25.

We have

$$\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$$

$$\Rightarrow \frac{x+2+2(x+1)}{x^2+3x+2} = \frac{4}{x+4}$$

$$\Rightarrow (3x+4)(x+4) = 4(x^2+3x+2)$$

$$\Rightarrow 3x^2 + 16x + 16 = 4x^2 + 12x + 8$$

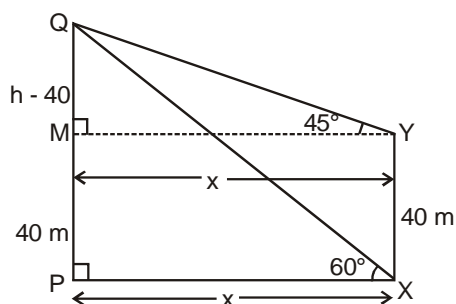
$$\Rightarrow x^2 - 4x - 8 = 0$$

Using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 + 32}}{2} = \frac{4 \pm 4\sqrt{3}}{2}$$

$$x = 2 + 2\sqrt{3} \text{ or } 2 - 2\sqrt{3}$$

26.



Let PQ be the tower

Let PQ = h

Clearly

XY = PM = 40 m

QM = (h - 40)

Let PX = MY = x

$$\text{In } \triangle MQY, \tan 45^\circ = \frac{QM}{MY} \Rightarrow 1 = \frac{h - 40}{x}$$

$$\Rightarrow x = h - 40 \quad \dots(i)$$

$$\text{In } \triangle QPX, \tan 60^\circ = \frac{QP}{PX} \Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow h = x\sqrt{3} \quad \dots(ii)$$

From (i) and (ii) we get

$$x = x\sqrt{3} - 40$$

$$(\sqrt{3} - 1)x = 40$$

$$PX = x = \frac{40}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)} = 20(\sqrt{3} + 1)$$

$$PQ = h = 20(\sqrt{3} + 1)\sqrt{3} = 20\sqrt{3}(\sqrt{3} + 1)$$

27.

$$\underbrace{1, 2, 3, \dots, x-1, x, x+1, \dots, 49}_S$$

$$S = 1 + 2 + 3 + \dots + (x - 1)$$

$$= \left(\frac{x-1}{2}\right) [1 + x - 1]$$

$$= \left(\frac{x-1}{2}\right) (x)$$

$$S = (x + 1) + (x + 2) + \dots + 49$$

$$= \left(\frac{49-x}{2}\right) (x + 1 + 49)$$

$$= \frac{49-x}{2} (x + 50)$$

$$S = S'$$

$$\left(\frac{x-1}{2}\right) x = \left(\frac{49-x}{2}\right) (x + 50)$$

$$x^2 - x = 49x + 49 \times 50 - x^2 - 50x$$

$$2x^2 = 49 \times 50$$

$$x^2 = 49 \times 25$$

$$x = 35$$

28. Coordinates of D = $\frac{2(4)+(1)}{2+1}$
 $= \frac{8+1}{3} = \frac{9}{3} = 3 = \frac{2(6)+(1)(5)}{2+1}$

Coordinates of D = $(3, \frac{17}{3})$

Coordinates of E = $\frac{2(4)+(1)(7)}{2+1} = \frac{8+7}{3} = \frac{15}{3} = 5$

$\frac{2(6)+1(2)}{2+1} = \frac{12+2}{3} = \frac{14}{3}$

area $\Delta(ADE) = \frac{1}{2} [4 \frac{17}{3} - \frac{14}{3} + 3(\frac{14}{3} - 6) + 5(6 - \frac{17}{3})]$

$= \frac{1}{2} [4 \times 1 + 3 \frac{(-4)}{3} + 5 \times \frac{1}{3}]$

$= \frac{1}{2} \times \frac{5}{3} = \frac{5}{6}$

area ΔABC

$= \frac{1}{2} [4(5-2) + 1(2-6) + 7(6-5)]$

$= \frac{1}{2} [4 \times 3 + 1 \times -4 + 7 \times 1]$

$= \frac{1}{2} [12 - 4 + 7] = \frac{15}{2}$

$\Rightarrow \frac{\text{area } \Delta ABC}{\text{area } \Delta ADE} = \frac{\frac{15}{2}}{\frac{5}{6}} = 9$

$\therefore \text{area } \Delta ABC = 9 \text{ area } (\Delta ADE)$

29. Total no. of events 16

$\{1 \times 1, 1 \times 4, 4 \times 9, 1 \times 16$

$2 \times 1, 2 \times 4, 2 \times 9, 2 \times 16$

$3 \times 1, 3 \times 4, 3 \times 9, 3 \times 16$

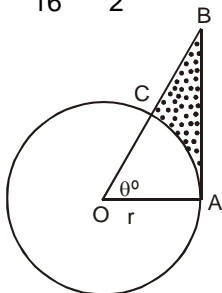
$4 \times 1, 4 \times 4, 4 \times 9, 4 \times 16\}$

Events when product is less than 16 = 8

$\{1 \times 1, 1 \times 4, 1 \times 9, 2 \times 1, 2 \times 4, 3 \times 1, 3 \times 4, 4 \times 1\}$

$\therefore \text{Probability that product of } x \text{ \& } y \text{ is less than } 16 = \frac{\text{events when product is less than } 16}{\text{Total no. of events}}$

$= \frac{8}{16} = \frac{1}{2}$



30.

(i) length of sector $\widehat{CA} = \pi r \frac{\theta}{180}$

In $\triangle OAB$

$$\tan \theta = \frac{AB}{OA}$$

$$AB = r \tan \theta$$

Now, $\sec \theta = \frac{BO}{r}$ so, $BO = r \sec \theta$

length of $CO = r$

$$\begin{aligned} \text{So length of } BC &= OB - OC \\ &= r \sec \theta - r \end{aligned}$$

So perimeter = $\widehat{AC} + AB + BC$

$$= \pi r \frac{\theta}{180} + r \tan \theta + r \sec \theta - r$$

$$= r \left[\tan \theta + \sec \theta + \frac{\pi \theta}{180} - 1 \right]$$

- 31.** Speed of boat in still water = 24 km/hr
Let the speed of stream be 'x'
Upstream = Speed of boat = 24 - x

$$\text{Upstream} = \frac{\text{Distance}}{\text{speed}} = \frac{32}{24 - x}$$

Downstream

Speed of boat = 24 + x

$$T_{\text{downstream}} = \frac{\text{distance}}{\text{speed}} = \frac{32}{24 + x}$$

ATP

$$T_{\text{upstream}} - T_{\text{downstream}} = 1$$

$$\frac{32}{24 - x} - \frac{32}{24 + x} = 1$$

$$32 \left[\frac{24 + x - (24 - x)}{(24 - x)(24 + x)} \right] = 1$$

$$32 [24 + x - 24 + x] = (24 - x)(24 + x)$$

$$64x = (24)^2 - x^2$$

$$x^2 + 64x - 576 = 0$$

$$x^2 + 72x - 8x - 576 = 0$$

$$x(x + 72) - 8(x + 72) = 0$$

$$(x - 8)(x + 72) = 0$$

$$x = 8, -72$$

\therefore speed of stream = 8 km/hr