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Class: 11
Subject: Mathematics
Topic: Circles
No. of Questions: 20
Duration: 60 Min
Maximum Marks: 60

1. The pairs of tangents from (g, f) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ are at right angles if
   a. $2(g^2+f^2) = c$
   b. $g^2+f^2 +c = 0$
   c. $g^2 – f^2+c = 0$
   d. $g^2+f^2 = 2c$

   Sol: B

   Length of the tangent from (g,f) to the given circle
   $= \sqrt{3g^2 + 3f^2 + c}$
   $and..the..radius..of..the..given.circle = \sqrt{g^2 + f^2 - c}$
   $pair..of..\tan gents...are..at.rt..angle..if$
   $\sqrt{3g^2 + 3f^2 - c} = \sqrt{g^2 + f^2 - c}$
   $\rightarrow g^2 + f^2 + c = 0$
   $reqd...condition$

2. Four points (2k, 3k), (1,0), (0,1) and (0,0) are concyclic for
   a. all integral values of k
   b. $0 < k < 1$
   c. $K = 0$ only
   d. $K = 5/13$

   Sol: D

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Equation of the circle be $x^2+y^2+2gx+2fy+c=0$ Passes through(0,0) then c=0
(1,0)and(0,1)
Therefore g= -1/2=f
Equation of the circle $x^2+y^2-x-y=0$
Passes through(2k,3k)
Then $4k^2+9k^2-2k-3k=0$
Or k=0,5/13
When k=0 point is(0,0)

3. The equation of the circle, which has two normals (x – 1)(y - 2) = 0 and a tangent line 3x +4y=6, is
   a. $x^2+y^2-2x-4y+4=0$
   b. $x^2+y^2-2x-4y+5=0$
   c. $x^2+y^2-2x+4y-4=0$
   d. none of these

   Sol: A

   $x^2 + y^2 - 2x - 4y + 4 = 0$
   we can write it as $(x-1)^2+(y-2)^2=1$
   $perpendicular..dis\tan ce..from..the..centre..to...the..\tan gent$
   $$= \frac{3-8-6}{\sqrt{9+16}} = 1 = radius$$
   $but..for(2)and(3)\,prep..dis\tan ce..from..centre..to..the..\tan gent \neq radius$
   $therefore.the..Ans(1)..is..true$

4. The radical axis of the circles $x^2 +y^2 + 4x = 1$ and $4x^2 + 4y^2 =9$ is
   a. 16x + 5 = 0
   b. 16x - 5 = 0
   c. X = 2
   d. X + 2 = 0

   Sol: A

   We can write the equation of the circle $4x^2 + 4y^2 = 9$
   As $x^2 + y^2 = 9/4$
   Radical axis $x^2 + y^2 + 4x - 1 - (x^2 + y^2 - 9/4) = 0$
   Or $4x + 5/4 = 0$
   Or $16x+5=0$

5. If the circles $x^2 + y^2 = 9$ and $x^2 + y^2 + 2\alpha x + 2y + 1 = 0$ touch each other internally, then α =
   a. $\pm\frac{4}{3}$
   b. 1
   c. 4/3
   d. -4/3

   Sol: C

   Centre of $x^2 + y^2 = 9$ (0,0) and radius=3
   $centre..of..the..second..circle(-\alpha,-1)$
   $radius = \sqrt{\alpha^2} = \alpha$
   $distance..between..the..centres$
   $\sqrt{\alpha^2 + 1}$
   $by..the..problem$
   $\sqrt{\alpha^2 + 1} = \alpha - 3$
   $\alpha = 4/3$

6. The power of a point (x_1, y_1) w.r.t. the three circles $a^2(x^2+y^2) = 1$, $b^2(x^2+y^2) = 1$ and $c^2(x^2+y^2) = 1$ are in A.P. Then a^2, b^2, c^2 are in
   a. A.P
   b. G.P
   c. H.P
   d. None of these

   Sol: C

   $$By\ data, x_1^2 + y_1^2 - \frac{1}{a^2}, x_1^2 + y_1^2 - \frac{1}{b^2}, x_1^2 + y_1^2 - \frac{1}{c^2}\ are\ in\ A.P.$$
   $$\Rightarrow -\frac{1}{a^2}, -\frac{1}{b^2}, -\frac{1}{c^2}\ are\ in\ A.P. \Rightarrow a^2, b^2, c^2\ are\ in\ H.P.,$$

7. The circle $x^2 + y^2 - 9x + 9y + 9 = 0$ touches
   a. The x-axis but not the y – axis
   b. The y- axis but not the x-axis
   c. Both the axes
   d. Neither of the coordinate axes

Sol: D

Putting x =0
We get $y^2+9y+9=0$
Here y has 2 real values
The circle cuts y axis in 2 points
It crosses y axis but not touch y axis
We get the similar result putting y=0
This circle touches neither of the coordinate axes

8. The greatest distance of the point (1,2) from the circle $x^2 + y^2+4x+4y-1 = 0$ is
   a. 2
   b. 5
   c. 8
   d. 10

   Sol: C

   Greatest distance = CP + r [where C = (-2-2); P = (1,2) ; r = 3]
   = 5+ 3 = 8

9. If the circles $3x^2 + 3y^2 – 20x – 72y + 137 = 0$ and $x^2 + y^2 +4x – 5y – k = 0$ cut at right angles, then k =
   a. -1
   b. $-\frac{267}{3}$
   c. -3
   d. None of these

   Sol: A

   Circle: $3x^2 + 3y^2 - 20x - 72y + 137 = 0$, centre(10/3,12)
   Circle: $x^2 + y^2 + 4x - 5y - k = 0$ centre (-2,5/2)
   If they cut at right angles then
   $2g_1g_2+2f_1f_2=c_1+c_2$
   Putting the values we get k=-1

10. If the circles $x^2+y^2 +ax =0$ and $x^2 + y^2= b^2$, ab> 0, touch each other then
   a. a+b = 0
   b. a-b = 0
   c. 2a= b
   d. a= 2b

Sol: B

the circle $x^2 + y^2 + ax = 0$ centre(-a/2,0) radius=a/2
and $x^2 + y^2 = b^2$centre(0,0) radius =b
they touch internally
then b - a/2 = a/2
or b = a
or a-b =0

11. The coordinates of the point at which the circles $x^2 + y^2 - 4x-2y+4=0$ and $x^2+y^2 - 12x-8y+36 = 0$ touch each other are
    a. $\left(\frac{14}{5},\frac{8}{5}\right)$
    b. $\left(\frac{13}{5},\frac{6}{5}\right)$
    c. $\left(-\frac{2}{5},0\right)$
    d. $\left(-\frac{2}{5},\frac{8}{5}\right)$

Sol: A

the circles $x^2 + y^2 - 4x - 2y + 4 = 0$
centre(2,1)
$its..dis\tan ce..from..po\text{int}\left(\frac{14}{5},\frac{8}{5}\right) = 1 = its..radius$
circle $x^2 + y^2 - 12x - 8y + 36 = 0$ centre(6,4)
$its..dis\tan ce..from..po\text{int}\left(\frac{14}{5},\frac{8}{5}\right) = 4 = its..radius$
Reqd point

12. The extremities of a diameter of a circle are (-4,3) and (12,-1). Then, the length of the intercept made by the circle on the y- axis is
    a. $4\sqrt{13}$
    b. $6\sqrt{6}$
    c. $3\sqrt{6}$
    d. $2\sqrt{13}$

Sol: A

The extremities of a diameter of a circle are (-4, 3) and (12, -1)Now the centre of the circle is the mid point of these 2 points−(4,1)

$radius = \sqrt{(-4-4)^2 + (3-1)^2} = \sqrt{68}$

$equation..of...the..circle$

$(x-4)^2 + (y-1)^2 = 68$

$cut..y\,axis..therefore..putting..x = 0$

$we..have...(0,1+\sqrt{52}),(0,1-\sqrt{52})$

$length = \sqrt{0+(2\sqrt{52})^2} = 4\sqrt{13}$

13. Which of the following is ⊥ to the line of centres of the circles $x^2 + y^2 + 2x + y + 1 = 0$ and $x^2 + y^2 - x + 2y + 3 = 0$?
    a. $3x - y = 0$
    b. $x + 3y + 4 = 0$
    c. $3x + y + 2 = 0$
    d. None of these

    Sol: A

    Circle $x^2 + y^2 + 2x + y + 1 = 0$,centre(-1,-1/2)
    and $x^2 + y^2 - x + 2y + 3 = 0$,centre(1/2,-1)

    $slope..of..the..lines..joining..2..centres = -1/3$

    $its..perpendicular..slope = 3$

    $in..the..given..options$

    $3x - y = 0 has..slope = 3$

14. If the equation of a circle is $ax^2 + (2a-3)\,y^2 - 4x - 1 = 0$, then its centre is
    a. (2,0)
    b. (2/3, 0)
    c. (-2/3, 0)
    d. None of these

    Sol: B

    A circle is $ax^2 + (2a - 3)\,y^2 - 4x - 1 = 0$, then its centre is (2/a,0) and from the equation of the circle we can say (2a-3)/ a=1 therefore 2a-3 = a or a=3 centre (2/3,0)

15. If the circles $x^2 + y^2 + a\ y + 4y + 1 = 0$ and $x^2+y^2 - 2x + a\ y + 1 = 0$ are orthogonal, then a =
    a. 1
    b. 2
    c. 0
    d. None of these

    Sol: B

$$centre..of..the..first..circle(-g,-f) = (-\lambda/2,-2), c = 1$$
$$2nd..circle(-g1,-f1) = (1,-\lambda/2), c1 = 1$$
$$we.know..that..the..condition..for..orthogonal..cut$$
$$2gg1 + 2ff1 = c + c1$$
$$we..get..\lambda = 2$$

16. If $3x - 4y + k = 0$ is tangent to the circle $(x-1)^2 + (y-1)^2 = 2^2$, then k =
    a. $\pm$ 10
    b. 9,-11
    c. 11
    d. None of these

    Sol: D

(1,1) is the centre of the circle
Perpendicular distance from (1,1) to the line
$$\frac{3-4+k}{\sqrt{9+16}} = \pm 2$$
$$k = 11,-9$$

17. The equation of the tangent to the circle $2x^2 + 2y^2 - x - 47 = 0$ at (5, 1) is
    a. 19x + 14y =99
    b. 20x + 4y =99
    c. 19x + 4y = 99
    d. None of these

    Sol: C

circle $2x^2 + 2y^2 - x - 47 = 0$

diff..wrt...x..we..get

$$4x + 4y\frac{dy}{dx} - 1 = 0$$

$$\frac{dy}{dx} = \frac{1-4x}{4y}$$

slope..at(5,1)

$$= -\frac{19}{4}$$

equation..of..the..tan gent

$$y - 1 = -\frac{19}{4}(x-5)$$

$$19x + 4y = 99$$

18. The no. of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 4x + 2y - 4 = 0$ is
    a. 1
    b. 2
    c. 3
    d. 4

    Sol: B

$x^2 + y^2 - 4x + 2y - 4 = 0$

$(x-2)^2 + (y+1)^2 = 3^2$

$centre(2,-1)$

$let..equation..of..the..\tan gent$

$y = mx + 2\sqrt{1+m^2}$

$perpendicular..dis\tan ce..from..centre(2,-1)to..the..\tan gent$

$$\frac{-1-2m-2\sqrt{1+m^2}}{\sqrt{1+m^2}} = \pm 3$$

$-1-2m = 5\sqrt{1+m^2}....and....-1-2m = -\sqrt{1+m^2} \Rightarrow 1+4m+4m^2 = 1+m^2$

$1+4m+4m^2 = 25(1+m^2)$

$-24+4m-21m^2 = 0$

$21m^2 - 4m + 24 = 0$

$no..real..solution$

and

$4m + m^2 = 0$

$2..real..solution$

$number...of..commom..\tan gent = 2$

19. The tangent at P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the Y- axis then the length of PQ is
    a. 4
    b. 2
    c. 3
    d. 5

    Sol: D

The line $5x - 2y + 6 = 0$ meets the y axis at Q (0,3)

QP = The length of the tangent from Q to the circle

$= \sqrt{0+9+0+18-2} = \sqrt{25} = 5.$

20. The centre of the circle inscribed in a square formed by the lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 - 0$ is
    a. (4,7)
    b. (7,4)
    c. (9,4)

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d. (4,9)

Sol: A

Lines are y-5=0,y-9=0,x-2=0,x-6=0
Centre of the circle is the point of intersection of the
Lines y-7=0and x-4=0
Centre(4,7)