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Class: 11
Subject: Mathematics
Topic: Complex Number
No. of Questions: 20
Duration: 60 Min
Maximum Marks: 60

1. The value of the expression $1(2-\mu)(2-\mu^2)+2(3-\mu)(3-\mu^2)+....+(n-1)(n-\mu)(n-\mu^2)$ where μ is not real cube root of unity, is
   a. $\left[\frac{n(n+1)}{2}\right]^2$
   b. $\frac{n^2(n+1)^2}{4}-n$
   c. $\frac{n^2(n+1)^2}{4}+n$
   d. None of these

   Sol: B

$$T_r=(r-1)(r-\omega)(r-\omega^2)r=2,3,...n$$
$$=(r-1)[(r^2-(\omega+\omega^2)r+1)]$$
$$=(r-1)(r^2+r+1)$$
$$T_r=r^3-1$$
$$\therefore \sum_{r=2}^{n} T_r=\sum_{r=2}^{n}\left(r^3-1\right)=\frac{n^2(n+1)^2}{4}-n$$

2. If 1, μ , μ^2 are cube roots of unity, then the value of $\frac{a\mu+b\mu^2+c\mu^3+d\mu}{c\mu+d\mu^2+a\mu^2+b}$ *equals*
   a. μ
   b. μ^2
   c. 0
   d. None of these

   Sol: B

$$\frac{a\omega + b\omega^2 + c\omega^3 + d\omega}{c\omega + d\omega^2 + a\omega^2 + b}$$
$$= \frac{\omega(a\omega + b\omega^2 + c\omega^3 + d\omega)}{\omega(c\omega + d\omega^2 + a\omega^2 + b)}$$

$(Multiply \quad and \quad Divide \quad by \quad \omega)$

$$= \frac{1}{\omega}\left(\frac{a\omega^2 + b + c\omega + d\omega^2}{a\omega^2 + b + c\omega + d\omega^2}\right) = \frac{1}{\omega} = \omega^2$$

3. If z = (t+3) + i $\sqrt{5-t^2}$, then the locus of z is the circle
   a. |z-3| = 5|
   b. |z-5| = $\sqrt{2}$
   c. |z-3| = $\sqrt{5}$
   d. |z-3| = 1

   Sol: D

$$z = (t+3) + i\sqrt{5-t^2}$$
$$x + iy = (t+3) + i\sqrt{5-t^2}$$
$$x = t+3 \Rightarrow x-3 = t \Rightarrow t^2 = (x-3)^2 \;......(1)$$
$$y = \sqrt{5-t^2} \Rightarrow y^2 = 5-t^2 \;.........(2)$$
$$then$$
$$(x-3)^2 = 5-y^2$$
$$x^2 + y^2 - 6x = -4$$
$$which..is..equivalent..to|z-3| = 1$$

4. If $x^2 - x + 1 = 0$, then $\left(x^5 + \frac{1}{x^5}\right)^2 =$
   a. 1
   b. 4
   c. 2
   d. None of these

   Sol: D

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Here $x^2 - x + 1 - 0$

then

$$x + \frac{1}{x} = 1$$

$$\left(x + \frac{1}{x}\right)^5 - 1 - \left[x^5 + \frac{1}{x^5} + 10(x + \frac{1}{x}) + 5(x^3 + \frac{1}{x^3})\right] - x^5 + \frac{1}{x^5}$$

$$\left(x^5 + \frac{1}{x^5}\right)^2 = x^5 + 2 + \frac{1}{x^5} = 1 + 2 = 3$$

5. If $x + i\,y = \frac{(3-4i)\,(7+24i)}{4-3i}$, *then* $x^2 + y^2 =$
    a. 25
    b. 625
    c. 5
    d. $\sqrt{5}$

    Sol: B

$$x + i\,y = \frac{(3-4i)\,(7+24i)}{4-3i}$$

$$= \frac{21 - 28i + 72i + 96}{4-3i} = \frac{(117-44i)(4+3i)}{(4-3i)(4+3i)}$$

$$= \frac{600 + 175i}{25} = 24 + 7i$$

$$x = 24$$

and

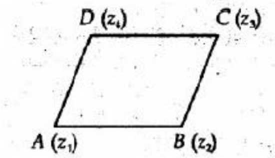
$$y = 7$$

$$x^2 + y^2 = 24^2 + 7^2 = 625$$

6. The Points Z_1, Z_2, Z_3, Z_4 in the complex plane are the vertices of a parallelogram if and only if
    a. $z_1 + z_2 = z_2 + z_3$
    b. $z_1 + z_3 = z_2 + z_4$
    c. $z_1 + z_2 = z_3 + z_4$
    d. None of these

    Sol: B

We know that in a parallelogram the diagonals bisect each other



∴ Midpoint of AC is the same as mid point of BD

$\therefore \frac{z_1+z_3}{2} = \frac{z_2+z_4}{2} \qquad \therefore z_1+z_3 = z_2+z_4$

7. The value of the sum $\sum_{n=1}^{13}(\mathrm{i}^n+\mathrm{i}^{n+1})$, *where* $i=\sqrt{-1}$ *equals*

   a. i
   b. i -1
   c. –i
   d. 0

Sol: B

$$\sum_{n=1}^{13}(i^n+i^{n+1}) = \sum_{n=1}^{13} i^n(1+i)$$

$$= (1+i)\left\{\frac{i(1-i^{13})}{1-i}\right\} = (1+i)\left\{\frac{i(1-i)}{1-i}\right\}$$

$$= (1+i)i = -1+i$$

8. *If* $i=\sqrt{-1}$, *then* $4+5\left(\frac{-1}{2}+\frac{i\sqrt{3}}{2}\right)^{334}+3\left(\frac{-1}{2}+\frac{i\sqrt{3}}{2}\right)^{365}$ *is equal to*

   a. $1-i\sqrt{3}$
   b. $-1+i\sqrt{3}$
   c. $i\sqrt{3}$
   d. $-i\sqrt{3}$

Sol: C

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Given that

$$4+5\left(\frac{-1}{2}+\frac{i\sqrt{3}}{2}\right)^{334}+3\left(\frac{-1}{2}+\frac{i\sqrt{3}}{2}\right)^{365}$$

$$=4+5\omega^{334}+3\omega^{365}$$

$$=4+5(\omega^3)^{111}.\omega+3(\omega^3)^{121}.\omega^2$$

$$=4+5.\omega+3\omega^2 \qquad (\therefore \omega^3=1)$$

$$=1+2\omega+3(1+\omega+\omega^2)=1+2\omega+3(0)$$

$$=1-1+\sqrt{3}i=\sqrt{3}i$$

9. The real part of log (3-4i)is
   a. Log 3
   b. $\frac{1}{2}\log 5$
   c. Log 5
   d. None of these

   Sol: C

$$let$$

$$z=3-4i$$

$$real..part=\log|z|=\log\left|\sqrt{3^2+4^2}\right|=\log 5$$

10. If $w=\text{cis}\frac{2\pi}{3}$, the common roots of the equations $z^3+2z^2+2z+1=0$ & $z^{1000}+z^{2000}+1=0$ are
    a. -1 , μ , μ^2
    b. -1, μ
    c. μ , μ^2
    d. -1, μ^2

    Sol: C

$w = \text{cis}\ \dfrac{2\pi}{3}$

then

$1 + w + w^2 = 0$

$w^3 = 1$

In the given equation $z^3 + 2z^2 + 2z + 1 = 0$
Putting z=w
We get

$w^3 + 2w^2 + 2w + 1 = 1 + 2(w + w^2) + 1 = 2 - 2 = 0$

So w is one root.it satisfy $z^{1000} + z^{2000} + 1 = 0$
Putting $z = w^2$
We get $w^3 + 2w^2 + 2w + 1 = 0$
And satisfy the other equation
Common roots are w, w^2

11. *Let* $\mu = -\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2}$ *then the value of* $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\mu^2 & \mu^2 \\ 1 & \mu^2 & \mu^4 \end{vmatrix}$ *is*

a. $2\mu^2$
b. $3\mu(1-\mu)$
c. $3\mu(\mu-1)$
d. None of these

Sol: C

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1\ \omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{matrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega^2 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} 3 & 0 & 0 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} \quad \left(\begin{matrix} \because 1 + \omega^2 = -\omega \\ \omega^3 - 1 \end{matrix} \right)$$

$= 3[\omega^2 - \omega]$

$= 3\omega(\omega - 1)$

12. I f $\left(\frac{1+i}{1-i}\right)^x = 1\ then$

a. $x = 2n, n \in I^+$
b. $x = 4n+1, n \in I^+$
c. $x = 4n, n \in I^+$
d. $x = 4n, n \in I^+$

Sol: D

$$\text{Given } \left(\frac{1+i}{1-i}\right)^x = 1$$

$$\Rightarrow \left(\frac{1+i}{1-i} x \frac{1+i}{1+i}\right)^x = 1$$

$$\Rightarrow \left(\frac{2i}{2}\right)^x = 1$$

$$\Rightarrow i^x = 1$$

$$\Rightarrow i^x = (i)^{4n}$$

$$\Rightarrow x = 4n, n \in I^+$$

13. $If\ x^2 - x\sqrt{3} + 1 = 0,\ then \int_0^x \sum_{n=1}^{36} \left(x^n - \frac{1}{x^n}\right)^2 dx$ is equal to

a. 0
b. -72π
c. $72\ \pi$
d. None of these

Sol: B

$$x = \frac{\sqrt{3} \pm i}{2} = \frac{\sqrt{3}}{2} \pm \frac{i}{2}$$

$$x = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} \quad \therefore x^{2n} = \cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}$$

$$\therefore \left(x^? - \frac{1}{x^n}\right)^2 = x^{?n} + \frac{1}{x^{2n}} - 2 = x^{?n} + x^{-?n} - 2$$

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$$\therefore \left(x^n - \frac{1}{x^n}\right)^2 = x^{2n} + x^{-2n} - 2$$

$$= -2 + 2\cos\frac{n\pi}{3}$$

$$Now\ \sum_{n=1}^{36}\left(-2 + 2\cos\frac{n\pi}{3}\right)$$

$$= -2x36 + 2\left[\cos\left(\frac{\pi}{3}\right) + \cos\frac{2\pi}{3} + ... + \cos\frac{36\pi}{3}\right]$$

$$= -72 + 2\frac{\cos\left(\frac{\pi}{3} + \frac{35\pi}{6}\right).\sin\left(\frac{36\pi}{3x2}\right)}{\sin\frac{\pi}{6}}$$

$$= -72 + 0$$

$$\left[\begin{array}{l}\cos\alpha + \cos(\alpha+\beta) + ..upto\ \ n\ \ term \\ = \dfrac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\cos\left(\alpha + \frac{(n-1)\beta}{2}\right)\end{array}\right]$$

$$\therefore \int_0^x \sum_{n=1}^{36}\left(x^n - \frac{1}{x^n}\right)^2 dx = -\int_0^x 72dx$$

$$= -72[\pi - 0]$$

$$= -72\pi$$

14. Let two non-zero distinct complex numbers z_1 and z_2 are represented by the points A and B in the complex plane with O origin. Then OA is ⊥ to OB if

a. $z_1 + \bar{z}_2 = z_2 + \bar{z}_1$

b. $z_1 - \bar{z}_2 = z_2 - \bar{z}_1$

c. $z_1\bar{z}_2 + z_2\bar{z}_1$

d. $z_1\bar{z}_2 - z_2\bar{z}_1 = 0$

Sol: C

If $z_1\bar{z}_2 + z_2\bar{z}_1 = 0$

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15. The amplitude of $\frac{(1+i)\,(2+i)}{1+3i}$ is
    a. 0
    b. $\tan^{-1}\frac{1}{2}$
    c. $\frac{\pi}{4}$
    d. None of these

    Sol: A

$$\frac{(1+i)\ (2+i)}{1+3i} = \frac{2+2i+i+i^2}{1+3i} = \frac{1+3i}{1+3i} = 1$$

    Amplitude = 0

16. If z and ω are two non-zero complex numbers such that $|z\mu|=1$ and Arg (z) – Arg (μ) $= \frac{\pi}{2}$, then $\bar{z}\,\omega$ is equal to
    a. -1
    b. I
    c. –i
    d. 1

    Sol: C

As $|z\omega| = 1$

$$\Rightarrow |z|\ |\omega| = 1 \quad so \quad |z| = \frac{1}{|\omega|} \qquad \ldots(i)$$

$$Again,\ \ Arg\, z - Arg\ \ \omega = \pi/2$$

$$\therefore \frac{z}{\omega} = \left|\frac{z}{\omega}\right| i = |z|^{2i} \text{ (using (i))}$$

$$\therefore \frac{z}{\omega} = z\bar{z}i = \bar{z}\omega = \frac{1}{i} = -i$$

17. Let z_1, z_2, z_3 be distinct complex numbers and $\mu^3 = 1, \mu \neq 1$ Statement -1: If $z_1 + \mu z_2 + \mu^2 z_3 = 0$, then z_1, z_2, z_3 are the vertices of an equilateral triangle. Statement-2: If $z_3 - z_1 = (z_2 - z_1)e^{i\pi/3}$, then z_1, z_2, z_3 are the vertices of an equilateral triangle.
    a. If both statement-1 and statement -2 are true and statement -2 is the correct explanation of statement -1
    b. If both statement-1 and statement -2 are true but statement-2 is not the correct explanation of statement-1
    c. If statement -1 is true but statement-2 is false

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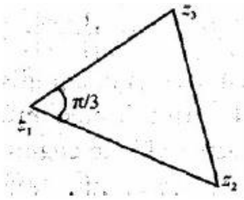
d. If statement-1 is false and statement-2 is true

Sol: A



Subtracting $z_1(1+\omega+\omega^2)=0$ *from*

$$z_1+\omega z_2+\omega^2 z_3=0, \quad we \quad get$$
$$\omega(z_2-z_1)+\omega^2(z_3-z_1)=0$$
$$or \quad z_3-z_1=-\omega(z_2-z_1)=e^{i\left(\pi+\frac{4\pi}{3}\right)}(z_2-z_1)$$
$$or \quad z_3-z_1=(z_2-z_1)e^{i\pi/s}$$

$\therefore z_1, z_2, z_3$ form an equilateral triangle.

18. The modulus of the complex number $(1-\cos\theta+i\sin\theta)^{-2}$ is
    a. $(1-\cos\theta)^{-2}$
    b. $4\,\text{cosec}^2\frac{\theta}{2}$
    c. $\frac{1}{4}\,cosec^2\frac{\theta}{2}$
    d. $\frac{1}{4}\sin^2\frac{\theta}{2}$

Sol: C

$$\text{mod}\,ulus-\{\sqrt{(1-\cos\theta)^2+\sin^2\theta}\}^{-2}-\left(\sqrt{2-2\cos\theta}\right)^{-2}-\left(4\sin^2\frac{\theta}{2}\right)^{-1}$$
$$=\frac{1}{4}\text{cosec}^2\frac{\theta}{2}$$

19. The smallest +ve interger 'n' for which $\left(\frac{1-i}{1+i}\right)^n=i\ is$
    a. 1
    b. 2
    c. 3
    d. 4

Sol: C

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$$\left(\frac{1-i}{1+i}\right)^n = i$$

$$or\left(\frac{(1-i)^2}{(1+i)(1-i)}\right)^n = i$$

$$or(-i)^n = -i$$

$then$

$n = 3$

20. If arg z < 0, then arg (- z) – arg z is equal to
    a. π
    b. -π
    c. -$\pi/2$
    d. $\pi/2$

Sol: A

$if...\theta..be..the....\arg z$

$then$

$\arg(-z) = \pi + \theta$

$here..\arg z < 0$

$then$

$-\pi < \theta < 0$

$\arg z - \arg(-z) = -\pi$

arg (- z) - arg z is equal to ð

Ans is π