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Class: 11
Subject: Mathematics
Topic: Conic Section
No. of Questions: 20
Duration: 60 Min
Maximum Marks: 60

1. The asymptotes of the hyperbola xy = 4x + 3y are
   a. x = 3, y = 4
   b. x = 4, y = 3
   c. x = 2, y = 6
   d. x = 6, y = 2

   Ans. A

   Solution:

   Let the equation of the asymptote be y=mx+c
   Now x(mx+c)=4x+3(mx+c)..............(1)
   Line y=mx+c is an asymptote to the given hyperbola
   Thus ,
   It touches the hyperbola at infinite
   Therefore, roots of (1)must be infinity
   Then m=0,c-4-3m=0
   Or c=4
   Asymptote is y=4
   Other is x=3

2. The slope m of the common tangent to the parabola $y^2 = 4ax$ and the rectangular hyperbola $x^2 - y^2 = a^2$ satisfies the relation

   a. $m^2 + m^4 = 1$
   b. $m^4 = m^2 + 1$
   c. $m^2 = m^4 + 1$
   d. $m^2 = m^4 + 2$

   Ans. B

   Solution:

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Let the tangent be y=mx+c
For parabola c=a/m
For hyparbola

$$c = \pm\sqrt{a^2m^2 - a^2} = \pm a\sqrt{m^2 - 1}$$

$$\frac{a}{m} = \pm a\sqrt{m^2 - 1}$$

$$1 = m^2(m^2 - 1) = m^4 - m^2$$

$$m^4 = 1 + m^2$$

3. If the distance between foci of a hyperbola is 36 and the transverse axis is 9, then its eccentricity is
   a. 2
   b. 4
   c. 3
   d. None of these

   Ans. B

   Solution:

   Length of transverse axis =2a =9
   Then a=9/2
   As ,

$$foci(\pm ae, 0)$$

$$dis\tan ce..between..2..foci = 2ae = 36$$

$$e = 4$$

4. If a≠0 and the line 2bx + 3cy + 4d = 0 passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then
   a. $d^2 + (3b - 2c)^2 = 0$
   b. $d^2 + (3b + 2c)^2$
   c. $d^2 + (2b - 3c)^2 = 0$
   d. $d^2 + (2b + 3c)^2 = 0$

   Ans. D
   Solution:
   Their points of intersections (0,0), (4a,4a)
   Given line passes through (0,0) then d=0
   And 2bx +3cy=0

To make it x=y we have 2b= -3c
Therefore, required condition: $d^2 + (2b + 3c)^2 = 0$

5. x = 4 (1 + cos θ) and y = 3(1 + sin θ) are the parametric equations of
   a. $\frac{(x-4)^2}{16} + \frac{(y-3)^2}{9} = 1$
   b. $\frac{(x-4)^2}{16} - \frac{(y-3)^2}{9} = 1$
   c. $\frac{(x+4)^2}{16} + \frac{(y+3)^2}{9} = 1$
   d. $\frac{(x-3)^2}{9} + \frac{(y-4)^2}{16} = 1$

   Ans. A
   Solution:

   Here (x- 4)/4 = cos θ
   and (y -3)/3= sin θ
   Required equation $\frac{(x-4)^2}{16} + \frac{(y-3)^2}{9} = 1$

6. The tangents to the parabola $y^2 = 4ax$ at t1 and t2 intersect on its axis. Then
   a. $t_1 t_2 = 1$
   b. $t_1 t_2 = t - 1$
   c. $t_1 + t_2 = 0$
   d. $t_1 + t_2 = -1$

   Ans. C
   Solution:
   Tangents are $y=t_1x+a/t_1$
   And $y=t_2x+a/t_2$
   Axis of the given parabola is x axis then y coordinate=0, $t_1x+a/t_1+t_2x+a/t_2=0$
   Or $(t_1+t_2)\,x + (t_1+t_2)\,a/t_1t_2=0$
   $(t_1+t_2)=0$

7. The equation $\sqrt{(x+4)^2 + (y+2)^2} - \sqrt{(x-4)^2 + (y-2)^2} = 8$
   a. a line segment
   b. a parabola
   c. an ellipse
   d. a hyperbola

   Ans. D
   Solution:
   This equation represents hyperbola

8. The equation of the chord of the ellipse $\frac{x^2}{36}+\frac{y^2}{9}=1$, bisected at (2,1) is
    a. 2x + y = 5
    b. x + y = 3
    c. x - y = 1
    d. x + 2y = 4

    Ans. D
    Solution:
    Here, we take all the options and calculate their points of intersections.
    Taking option 4 we get the points of intersections
    $y-1\pm\sqrt{6}$
    $x=4-2(1\pm\sqrt{6})=2+2\sqrt{6}$
    $po\text{int}\ s..are(2-2\sqrt{6},1+\sqrt{6})$
    $and(2+2\sqrt{6},1-\sqrt{6})$
    $their.mid..po\text{int}(2,1)$
    Therefore option 4 is correct

9. The locus of the point of intersection of mutually ⊥tangents to the conic $x^2 + 2x - 4y + 9 = 0$ is
    a. y = 1
    b. y + 1 = 0
    c. x + 1 = 0
    d. x = 1

    Ans. A
    Solution:
    We know that the locus of the point of intersection of mutually ⊥ tangents to the parabola is its directrix
    From the given equation we have $(x + 1)^2=4y-8=4(y-2)$
    Equation of the directrix is y - 2= -1
    Or y=1

10. The equation of the line of symmetry of the parabola $4x^2 - 20x - 12y + 49 = 0$ is
    a. 2x = 5
    b. x = 5
    c. y = 5
    d. none of these

    Ans. A
    Solution:
    parabola $4x^2 - 20x - 12y + 49 = 0$
    or $(x - 5/2)^2=3(y-2)$
    axis x - 5/2 =0

or 2x = 5

11. The angle between the asymptotes of the hyperbola $x^2 - 3y^2 = 12$ is
    a. 30º
    b. 60º
    c. 75º
    d. 45º

Ans. B
Solution:
Hyperbola $x^2 - 3y^2 = 12$

$$\frac{x^2}{12} - \frac{y^2}{4} = 1$$

$$eccentricity = e = \sqrt{1+\frac{4}{12}} = \sqrt{\frac{16}{12}} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}} = \sec\theta$$

$$where\, \theta = 30^\circ$$

Angle between 2 asymptote =2*30=60

12. The coordinates of foci of the hyperbola $4x^2 - 45y^2 = 180$ are
    a. $(0, \pm\sqrt{41})$
    b. $(0, \pm 7)$
    c. $(\pm 7, 0)$
    d. $(\pm, \sqrt{41}, 0)$

Ans. C
Solution:

$$\frac{x^2}{180/4} - \frac{y^2}{180/45} = 1$$

$$a = \sqrt{\frac{180}{4}} = \sqrt{45}$$

$$e = \sqrt{1+\frac{4}{45}} = \frac{7}{\sqrt{45}}$$

$$foci(\pm ae, 0) = (\pm 7, 0)$$

13. y = x + 5 is not a tangent to the conic
    a. $y^2 = 20x$
    b. $\frac{x^2}{16} + \frac{y^2}{9} = 1$
    c. $\frac{x^2}{29} + \frac{y^2}{4} = 1$
    d. $x^2 + y^2 = 25$

Ans. D

Circle $x^2 + y^2 = 25$,centre(0,0),radius=5
Perpendicular distance from (0,0) to the line y=x+5 is

$$\left|\frac{-5}{\sqrt{1^2+1^2}}\right| \neq radius$$

This line is not the tangent of the circle

14. The locus of the centre of a circle which touches externally two given circles is
    a. a hyperbola
    b. straight line
    c. a parabola
    d. another circle

    Ans. B
    Solution:
    Here locus is a straight line.

15. The tangent at P to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the major axis at T. If PN is ⊥ to the major axis, then ON. OT=
    a. b^2
    b. $a^2 - b^2$
    c. a^2e^2
    d. a^2

    Ans. D
    Solution:

    $the.line..y - mx + \sqrt{a^2m^2+b^2} is..the...tan gent..to..the..given..ellipse...at..the..point$

    $$\left(-\frac{a^2m}{\sqrt{a^2m^2+b^2}}, \frac{b^2}{\sqrt{a^2m^2+b^2}}\right)$$

    $$tan gent..meet..the..xaxis..at..\left(-\frac{\sqrt{a^2m^2+b^2}}{m}, 0\right)$$

    $$OT = \frac{\sqrt{a^2m^2+b^2}}{m}, ON = \frac{a^2m}{\sqrt{a^2m^2+b^2}}$$

    $$ON.OT = a^2$$

16. The locus of the midpoint of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with the directrix
    a. x = - a
    b. x = - a/2
    c. x = 0
    d. x = a/2

Ans. C
Solution:
The parabola $y^2 = 4ax$ focus(a,0), vertex= (0,0)
Focus of the new parabola (a,0) vertex(a/2,0)
new parabola $y^2=4A(x-a/2)$
A=a-(a/2) =a/2
Directrix x-a/2+A=0
Or x=0

17. The equation of the tangent to the hyperbola $4x^2 - 9y^2 - 16x + 18y + 12 = 0$ at (1, 2) is
    a. 9x - 4y - 1 = 0
    b. 4x + 9y = 22
    c. 4x + 9y = 3
    d. none of these

    Ans. B
    Solution:

$$slope..of..the..\tan gent..at(1,2) = \frac{dy}{dx}at(1,2) = -4/9$$

$$equation..of..the..\tan gent$$

$$y-2 = -\frac{4}{9}(x-1)$$

$$9y + 4x = 22$$

18. If e_1 and e_2 are the eccentricities of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} - 1$ and $\frac{y^2}{b^2} - \frac{x^2}{a^2} - 1$ which of the following is not true?
    a. $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$
    b. $e_1 : e_2 = b : a$
    c. $(e_1^2 - 1)(e_2^2 - 1) = 1$
    d. None of these

    Ans. D
    Solution:

$$e_1^2 = 1 + \frac{b^2}{a^2} = \frac{a^2 + b^2}{a^2}$$

$$e_2^2 = 1 + \frac{a^2}{b^2} = \frac{b^2 + a^2}{b^2}$$

Then

$$\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$$

True

$e_1 : e_2 = b : a$ true

$(e_1^2 - 1)(e_2^2 - 1) = 1$

true

19. The equation to the tangent to the hyperbola $x^2 - y^2 = a^2$, which makes an angle of 45° with positive x-axis, is
    a. y = x
    b. y = x ± 1
    c. y = 0
    d. y = x + 1

    Ans. A
    Solution:
    Equation of the tangent is y=tan45 x +c

    $c = \sqrt{a^2 m^2 - a^2} = 0..[m = \tan 45 = 1]$

    $reqd..equation..y = x$

20. The normal to the parabola $y^2 = 4x$ at a point whose ordinate is equal to abscissa subtends a right angle at the
    a. Vertex
    b. Focus
    c. an end point of LR
    d. none of these

    Ans. B
    Solution:
    If the normal to the parabola $y^2=4x$ at $P(t_1^2,2t_1)$
    Focus at S(1,0)
    Normal meets the parabola again at the point $Q=(t_2^2,2t_1)$
    Then we have $t_2=-t_1-2/t_1$
    In P abscissa=ordinate then $t_1^2=2t_1$

Or $t_1=2$
Then $t_2=-3$
Coordinate of P=(4,4) Q=(9,-6)
Slope of PS=4/3
Slope of QS=-3/4<PSQ=90 degree
Hence the result