

Class: 11
Subject: Mathematics
Topic: Linear Equations in one variable
No. of Questions: 20
Duration: 60 Min
Maximum Marks: 60

1. A plane p is perpendicular to the two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$ and passes through the point $(1, -2, 1)$. The distance of p from the point $(1, 2, 2)$ is

- a) 0
b) 1
c) $\sqrt{2}$
d) $2\sqrt{2}$

Ans. D

Solution:

Let the equation of the plane be

$$a(x - 1) + b(y + 2) + c(z - 1) = 0.$$

Since it is perpendicular to the given planes

$$2a - 2b + c = 0, \quad a - b + 2c = 0,$$

$$\Rightarrow c = 0, \quad a = b,$$

$$\Rightarrow \text{Equation of the plane is } x + y + 1 = 0$$

Its distance from the point $(1, 2, 2)$ is

$$\frac{|1 + 2 + 1|}{|\sqrt{1 + 1}|} = 2\sqrt{2}$$

2. Equation of a plane which passes through the line $x + py + q = 0 = rz + s$ and makes equal intercepts on y and z axes is $x + py + q + \lambda(rz + s) = 0$ where λ is equal to

- a) q/s
b) p/r
c) r/s
d) p/q

Ans. B

Solutions:

$$\frac{-\lambda s - q}{p} = \frac{-\lambda s - q}{\lambda r} \Rightarrow p = \lambda r \Rightarrow \lambda = p/r.$$

3. Let P (3, 2, 6) be a point in space and Q be a point on the line $r = (i - j + 2k) + \mu(-3i + j + 5k)$. Then the value of μ for which the vector PQ is parallel to the plane $x - 4y + 3z = 1$ is
- $\frac{1}{4}$
 - $-\frac{1}{4}$
 - $\frac{1}{8}$
 - $-\frac{1}{8}$

Ans. A

Solution:

As Q lies on the given line

$$\text{Let } \quad \mathbf{OQ} = (i - j + 2k) + \mu(-3i + j + 5k)$$

$$\text{Also } \quad \mathbf{OP} = 3i + 2j + 6k$$

As \mathbf{PQ} is parallel to the plane $x - 4y + 3z = 1$

$$\mathbf{PQ} \cdot (i - 4j + 3k) = 0$$

$$\Rightarrow (\mathbf{OQ} - \mathbf{OP}) \cdot (i - 4j + 3k) = 0$$

$$\Rightarrow (1 + 4 + 6) + \mu(-3 - 4 + 15) - (3 - 8 + 18) = 0$$

$$\Rightarrow 11 + 8\mu - 13 = 0 \Rightarrow \mu = \frac{1}{4}$$

4. A line with positive direction cosine passes through the point P (2, -1, 2) and makes equal angles with the coordinate axes. The line meets the plane $2x + y + z = 9$ at point Q. The length of the line segment PQ equals.

- 1
- $\sqrt{2}$
- $\sqrt{3}$
- 2

Ans. C

Solution:

The equation of the line is

$$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-2}{1} = k$$

Let the coordinates of the point Q on this line be

Q (k + 2, k - 1, k + 2). Since Q lies on the plane

$$2(k + 2) + (k - 1) + (k + 2) = 9$$

$$\Rightarrow k = 1 \text{ and } PQ = \sqrt{(3-2)^2 + (0+1)^2 + (3-2)^2} = \sqrt{3}$$

5. The foot of the perpendicular from (a, b, c) on the line $x = y = z$ is the point (r, r, r) where

- a) $r = a + b + c$
- b) $r = 3(a + b + c)$
- c) $3r = a + b + c$
- d) none of these

Ans. C

Solution:

Direction ratios of the perpendicular are $r - a$, $r - b$, $r - c$ and those of the lines are 1, 1, 1. So

$$1 \cdot (r - a) + 1 \cdot (r - b) + 1 \cdot (r - c) = 0$$

$$\Rightarrow 3r = a + b + c.$$

6. Equation of a line passing through the point whose position vector is $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and in the direction of the vector $3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$ is

- a) $4x + 3y = 17, 5y - 4z = 1$
- b) $4x - 3y = 17, 5y + 4z = 1$
- c) $4x + 5y = 12, 3y + 4z = 1$
- d) $5y + 4z = 1, 4x + 3z = 1$

Ans. B

Solution:

Equation of the line is $\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k} - \lambda(3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k})$ so the line passes through $(2, -3, 4)$ and the direction ratios are 3, 4, -5 and so its equation is

$$\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-4}{-5}$$

$$\Rightarrow 4x - 3y = 17, 5y + 4z = 1$$

7. Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane

containing the straight lines, $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$

- a) $x + 2y - 2z = 0$
- b) $3x + 2y - 2z = 0$
- c) $x - 2y + z = 0$
- d) $5x + 2y - 4z = 0$

Ans. C

Solution:

Let a, b, c be the direction ratios of the normal

to the plane containing the lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$

$$\text{then } 3a + 4b + 2c = 0$$

$$4a + 2b + 3c = 0$$

$$\Rightarrow \frac{a}{8} = \frac{b}{-1} = \frac{c}{-10}$$

Let α, β, γ be the direction ratios of the normal to the required plane.

$$\text{then } 8\alpha - \beta - 10\gamma = 0$$

$$2\alpha + 3\beta + 4\gamma = 0$$

$$\Rightarrow \frac{\alpha}{26} = \frac{\beta}{-52} = \frac{\gamma}{26}$$

$$\Rightarrow \frac{\alpha}{1} = \frac{\beta}{-2} = \frac{\gamma}{1}$$

As the required plane contains the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$, the

point $(0, 0, 0)$ lying on it also lies on the plane and hence equation of the required plane is $x - 2y + z = 0$

8. YZ-plane divides the line joining the points $(3, 5, -7)$ and $(-2, 1, 8)$ in the ratio

- a) 1: 2
- b) 2: 3
- c) 3: 2
- d) 2: 1

Ans. C

Solution:

Let the ratio be $m : n$, then x -coordinate of the point of division is zero

$$\Rightarrow \frac{-2m + 3n}{m + n} = 0 \Rightarrow \frac{m}{n} = \frac{3}{2}$$

9. The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2}, z=2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is
- $\pi/3$
 - $\pi/6$
 - $\pi/4$
 - $\pi/2$

Ans. D

Solution:

Direction ratios of the lines are 3, -2, 0 and

$$1, 3/2, 2; 3 \times 1 - 2 \times \frac{3}{2} + 0 \times 2 = 0$$

10. If the distance of the point P (1, -2, 1) from the plane $x + 2y - 2z = \alpha$ where, $\alpha > 0$, is 5, then the foot of the perpendicular from P to the plane is

- $\left(\frac{8}{3}, \frac{4}{3}, \frac{-7}{3}\right)$
- $\left(\frac{4}{3}, \frac{-4}{3}, \frac{1}{3}\right)$
- $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$
- $\left(\frac{2}{3}, \frac{-1}{3}, \frac{5}{2}\right)$

Ans. A

Solution:

Let Q be the foot of the perpendicular from P

then equation of PQ is $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-1}{-2} = r$ (say)

Let the coordinates of Q be $(r+1, 2r-2, -2r+1)$ Since it lies on the plane, $r+1+2(2r-2)-2(-2r+1) = \alpha$

$$\Rightarrow 9r-5 = \alpha \quad \Rightarrow r = (1/9)(5 + \alpha)$$

Now $PQ = 5$

$$\Rightarrow r^2 + 4r^2 + 4r^2 = 25 \Rightarrow r = \pm 5/3$$

$$\text{So } \frac{\alpha+5}{9} = r = \frac{\pm 5}{3} \Rightarrow \alpha = 10 \quad (\because \alpha > 0)$$

and $r = 5/3$

Hence the coordinates of Q are $\left(\frac{8}{3}, \frac{4}{3}, \frac{-7}{3}\right)$

11. If the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$ meets the coordinate axes in A, B, C , and the centroid of the triangle ABC is at $P(2, 4, 8)$, then a, b, c are in

- A.P
- G.P
- H.P
- None of these

Ans. B

Solution:

Coordinate of A are $(3a, 0, 0)$ $B(0, 3b, 0)$ and $C(0, 0, 3c)$ so the centroid of the triangle ABC is $(a, b, c) = (2, 4, 8)$

12. The points $(1, -2, 3), (2, 3, -4), (0, -7, 10)$ are the vertices of

- a right angled triangle
- isosceles triangle
- equilateral triangle
- none of these

Ans. B

Solution:

Lengths of the sides are

$$\sqrt{(2-1)^2 + (3+2)^2 + (-4-3)^2} = \sqrt{75}, \sqrt{75} \text{ and}$$

$\sqrt{300}$. So the triangle is isosceles.

13. The lines $r = i - j + \lambda(2i + k)$ and $r = 2i - j + \mu(i + j - k)$

- intersect each others
- do not intersect
- intersect at $r = 3i - j + k$
- are parallel

Ans. B

Solution:

$$\text{Since } \begin{vmatrix} 2-1 & -1+1 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -1 \neq 0, \text{ the lines do}$$

not intersect.

14. Mirror image of the point P (2, 1, 6) in the plane passing through the points. (2, 1, 0), (5, 0, 1) and (4, 1, 1) is the point

- (-2, -1, -6)
- (6, 5, -2)
- (1, 1, 4)
- (2, 5, 4)

Ans. B

Solution:

Equation of the plane through the given points

is

$$\begin{vmatrix} x-2 & y-1 & z-0 \\ 5-2 & 0-1 & 1-0 \\ 4-2 & 1-1 & 1-0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-1 & z-0 \\ 3 & -1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = 0$$

$$\Rightarrow x + y - 2z = 3$$

Equation of the line through P perpendicular to this plane

is

$$\frac{x-2}{1} = \frac{y-1}{1} = \frac{z-6}{-2} = r \text{ say}$$

Let $Q(r+2, r+1, -2r+6)$ be the image of P in this plane,

then the mid-point $R\left(\frac{r}{2}+2, \frac{r}{2}+1, -r+6\right)$ of PQ lies on the plane.

$$\text{So } \frac{r}{2} + 2 + \frac{r}{2} + 1 - 2(-r+6) = 3$$

$$\Rightarrow 3r = 12 \Rightarrow r = 4$$

and the coordinates of Q are $(6, 5, -2)$

15. The variable plane $(2\lambda + 1)x + (3 - \lambda)y + z = 4$ always passes through the line

- a) $\frac{x}{2} = \frac{y}{1} = \frac{z+4}{1}$
 b) $\frac{x}{1} = \frac{y}{2} = \frac{z}{-3}$
 c) $\frac{x}{1} = \frac{y}{2} = \frac{z-4}{-7}$
 d) none of these

Ans. C

Solution:

Plane passes through the line

$$x + 3y + z - 4 = 2x - y = 0$$

$$\text{or } \frac{x}{1} = \frac{y}{2} = \frac{z-4}{-7}$$

16. A line with direction ratios 2, 7, -5 is drawn to intersect the lines

$\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1}$ and $\frac{x+3}{-3} = \frac{y-3}{2} = \frac{z-6}{4}$ at P and Q respectively. Length of PQ is

a) $\sqrt{78}$

b) $\sqrt{77}$

c) $\sqrt{54}$

d) $\sqrt{74}$

Ans. A

Solution:

Right Answer Explanation:

Let $P(3r_1 + 5, -r_1 + 7, r_1 - 2)$ and $Q(-3r_2 - 3, 2r_2 + 3, 4r_2 + 6)$, direction ratios of PQ are 2, 7, -5

$$\text{So } \frac{3r_1 + 3r_2 + 8}{2} = \frac{-r_1 - 2r_2 + 4}{7} = \frac{r_1 - 4r_2 - 8}{-5} \quad r_1 = r_2 = -1$$

$$P(2, 8, -3), Q(0, 1, 2) \Rightarrow PQ = \sqrt{78}$$

17. Parametric form of the equation of the line $3x - 6y - 2z - 15 = 0$ and $2x + y - 2z - 5 = 0$ is

a) $\frac{x-5}{14} = \frac{y}{2} = \frac{z}{12}$

b) $\frac{x-1}{14} = \frac{y-5}{2} = \frac{z-1}{15}$

c) $\frac{x-3}{14} = \frac{y+1}{2} = \frac{z}{15}$

d) none of these

Ans. C

Solution:

If l, m, n are the direction ratios of the line then
 $3l - 6m - 2n = 0$ and $2l + m - 2n = 0$

$$\Rightarrow \frac{l}{14} = \frac{m}{2} = \frac{n}{15}$$

Equation of the line is

$$\frac{x-\alpha}{14} = \frac{y-\beta}{2} = \frac{z-\gamma}{15}$$

where (α, β, γ) lies on the line. only $(\alpha, \beta, \gamma) = (3, -1, 0)$ satisfies the condition.

18. The plane passing through the points $(-2, -2, 2)$ and containing the line joining the points $(1, 1, 1)$ and $(1, -1, 2)$ makes intersections on the coordinates axes, the sum of whose length is
- 3
 - 4
 - 6
 - 12

Ans. D

Solution:

Equation of the plane be $a(x + 2) + b(y + 2) + c(z - 2) = 0$. As it passes through $(1, 1, 1)$ and

$(1, -1, 2)$, $\frac{a}{1} = \frac{b}{-3} = \frac{c}{-6}$. Equation of the plane is

$$\frac{x}{-8} + \frac{y}{8/3} + \frac{z}{8/6} = 1 \text{ and the required sum} = 12.$$

19. If the foot of the perpendicular from the origin to a plane is (a, b, c) , then the equation of the plane is
- $ax + by + cz = 1$
 $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$
 - $ax + by + cz = a^2 + b^2 + c^2$
 - $ax + by + cz = 0$

Ans. C

Solution:

Let $P(a, b, c)$, d.r. of the normal to the plane are a, b, c and as it passes through (a, b, c) its equation is $a(x - a) + b(y - b) + c(z - c) = 0$

20. If $\mathbf{r} \cdot \mathbf{n} = q$ is the equation of a plane normal to the vector \mathbf{n} , the length of the perpendicular from the origin on the plane is

- a) $\frac{q}{n}$
- b) $q|\mathbf{n}|$
- c) $q/|\mathbf{n}|$
- d) $q/|q|$

Ans. C

Solution:

Equation of the plane is $\mathbf{r} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} = \frac{q}{|\mathbf{n}|}$, $\frac{\mathbf{n}}{|\mathbf{n}|}$ is a unit

vector, So the required length = $q/|\mathbf{n}|$.