Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

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Class: 11
Subject: Mathematics
Topic: Limits And Derivatives
No. of Questions: 20
Duration: 60 Min
Maximum Marks: 60

1. $\lim_{x\to\infty}\left[\frac{1}{1-n^2}+\frac{2}{1-n^2}+\ldots\ldots+\frac{n}{1-n^2}\right]=$

   a. $-\frac{1}{2}$
   b. 0
   c. ∞
   d. 1

   Ans. A

   Solution:

   $$\lim_{n\to\infty}\left[\frac{1}{1-n^2}+\frac{2}{1-n^2}+\ldots\ldots+\frac{n}{1-n^2}\right]=\lim_{n\to\infty}\frac{n(n+1)}{2(1-n^2)}$$

   $n \to \infty$

   the

   $\frac{1}{n}\to 0$

   $=-1/2$

2. $\lim_{x\to 0}\frac{[(a-n)\,nx-\tan x]\sin nx}{x^2}=0$, when $n\neq 0$ is real, then a =

   a. n
   b. $n-\frac{1}{n}$
   c. $n+\frac{1}{n}$
   d. $\frac{1}{n_s}$

   Ans. C

$$\lim_{x \to 0} \frac{[(a-n)\,nx - \tan x]\sin nx}{x^2}$$

$$= \lim_{x \to 0} \frac{[a\,nx\sin nx - n^2 x\sin nx - \tan x\sin x]}{x^2} = an^2 - n(n^2+1) = 0$$

$$a = n + \frac{1}{n}$$

3. If $f(x) = \frac{\sin [x]}{[x]},\ [x] \neq 0, f(x) = 0, [x] = 0, \lim_{x \to 0} f(x) =$

   a. 1
   b. 0
   c. -1
   d. None of these

   Ans. A

   $$\lim_{x \to 0} \frac{\sin [x]}{[x]} = 1$$

4. Let $f(x) = \begin{cases} x, & \text{if } x \text{ is irrational} \\ 0, & \text{if } x \text{ is rational} \end{cases}$ then f is

   a. continuous everywhere
   b. discontinuous everywhere
   c. continuous only at 0
   d. continuous at all rational numbers

   Ans. D

   Continuous at all rational numbers

5. If 2 f(x) - 3f$\left(\frac{1}{x}\right) = x^2, x \neq 0$, then f(2) =

   a. $\frac{5}{4}$
   b. $-\frac{7}{4}$
   c. $\frac{7}{2}$
   d. $-\frac{7}{2}$

   Ans. B

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Solution:

$$\text{If } 2f(x) - 3f\left(\frac{1}{x}\right) = x^2,\ x \neq 0$$

$$putting...x = 2$$

$$2f(2) - 3f(\tfrac{1}{2}) = 4.....................(1)$$

$$and$$

$$when..x = 1/2$$

$$2f(1/2) - 3f(2) = 1/4.......................(2)$$

$$solving (1) and (2)$$

$$we........get\ f(2) = -7/4$$

6. $\lim_{n \to \infty} \frac{1}{n^3}\left[1 + 3 + 6 + ... + \frac{n(n+1)}{2}\right] =$

a. 0
b. $\frac{1}{6}$
c. $\frac{1}{3}$
d. None of these

Ans. B

Solution:

$$\left[1 + 3 + 6 + ... + \frac{n(n+1)}{2}\right] = \frac{1}{2}\Sigma n^2 + \frac{1}{2}\Sigma n$$

$$= \frac{2n^3 + 6n^2 + 4n}{12}$$

$$\lim_{n \to \infty} \frac{1}{n^3}\left[1 + 3 + 6 + ... + \frac{n(n+1)}{2}\right] = \frac{1}{6}$$

7. If f' (x) is a differentiable function and f'' (0) 10

then $\lim\limits_{x \to 0} \dfrac{2f(x) - 3f(2x) + f(4x)}{x^2} =$

a. 10
b. 20
c. 30
d. 40

Ans. C

Solution:

$$\lim_{x \to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2} = \lim_{x \to 0} \frac{2f'(x) - 6f'(2x) + 4f'(4x)}{2x}$$
$$= \lim_{x \to 0} \frac{2f''(x) - 12f''(2x) + 16f''(4x)}{2} = \frac{2f''(0) - 12f''(2x) + 16f''(4x)}{2} = 30$$

8. $\lim\limits_{x \to 0} \dfrac{9^x + 9^{-x} - 2}{x^2} =$

a. $(\log 3)^2$
b. $(\log 9)^2$
c. 2 log 9
d. log 9

Ans. B

Solution:

$$\lim_{x \to 0} \frac{9^x + 9^{-x} - 2}{x^2} [0/0 \, form]$$
$$= \lim_{x \to 0} \frac{9^x \log 9 \quad 9^{\quad x} \log 9}{2x} [0/0 \, form]$$
$$= \lim_{x \to 0} \frac{9^x (\log 9)^2 + 9^{-x} (\log 9)^2}{2} = (\log 9)^2$$

9. Which of the following is a false statement?
   a. If f(x) is continuous at x a, the $\underset{x\to a}{\text{Lt}} f(x)$ exists
   b. If $f'(a)$ exists, the f(x) is continuous at x = a
   c. If f(x) is continuous at x a, then $\underset{x\to a}{\text{Lt}} f(x) = f(a)$
   d. If $\underset{x\to a}{\text{Lt}} f(x)$ exists, then f(x) is continuousat x = a

   Ans. D

   Solution:

   (1), (2), (3) are true
   False (4)

   For continuity limit should exist and should be equal to value of function at that point.

10. The integer n for which $\lim_{x\to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ finite non - zero number is
   a. 1
   b. 2
   c. 3
   d. 4

   Ans. C

   Solution:

$$\lim_{x\to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n} = \lim_{x\to 0} \frac{-2\sin^2\frac{x}{2}\left(\cos x - e^x\right)}{x^n} [\frac{0}{0} \, form]$$

$$= \lim_{x\to 0} \frac{2\sin\frac{x}{2}\cos\frac{x}{2}(\cos x - e^x) - 2\sin^2\frac{x}{2}(-\sin x - e^x)}{nx^{n-1}}$$

$$= finite.non..zero.number$$

$$\underset{x\to 0}{\text{Lt}} \frac{1-\cos x}{x^2} = \frac{1}{2}; \underset{x\to 0}{\text{Lt}} \frac{1-\cos x}{x} = 0; \underset{x\to 0}{\text{Lt}} \frac{e^x - 1}{x} = 1$$

$$\therefore \underset{x\to 0}{\text{Lt}} \frac{\cos x - 1}{x^2}\left[\frac{\cos x - 1}{x} - \frac{e^x - 1}{x}\right] \cdot \frac{1}{x^{n-3}} \; will \;\; exist \;\; if \;\; n = 3$$

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11. $\lim_{x \to 0} \left(\frac{1+5x^2}{1+3x^2}\right)^{\frac{1}{x^2}} -$

a. e2
b. e^{-2}
c. e^{15}
d. $e^{5/3}$

Ans. A

Solution:

$putting..\frac{1}{x^2} - v$

$$\lim_{x \to 0} \left(\frac{1+5x^2}{1+3x^2}\right)^{\frac{1}{x^2}} = \lim_{v \to \infty} \left(1+\frac{2}{3+v}\right)^{v} = e^2$$

12. $\mathop{Lt}_{x \to 0} [e^{2x} + x]^{1/x} =$

a. e^2
b. e^3
c. e^4
d. none of these

Ans. B

Solution:

$k = \mathop{Lt}_{x \to 0} [e^{2x} + x]^{1/x}$

$taking..\log..both..the..sides..we..get$

$\log k = \mathop{lt}_{x \to 0} \frac{1}{x} \log[e^{2x} + x] ....[\frac{0}{0} form]$

$$= \mathop{lt}_{x \to 0} \frac{\frac{2e^{2x}+1}{e^{2x}+x}}{1} = 3$$

$k = e^3$

13. The value of $\lim_{x \to \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ where α and β are the distinct root of $ax^2 + bx + c = 0$, is

a. $\frac{1}{2}(\alpha - \beta)^2$

b. $-\frac{a^2}{2}(\alpha - \beta)^2$

c. 0

d. $\frac{a^2}{2}(\alpha - \beta)^2$

Ans. A

Solution:

$$\lim_{x \to \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$$

$$= \lim_{x \to \alpha} \frac{1 - \cos(x - \alpha)(x - \beta)}{(x - \alpha)^2} = \lim_{x \to \alpha} \frac{2\sin^2 \frac{(x-\alpha)(x-\beta)}{2}}{(x-\alpha)^2}$$

$$= \lim_{x \to \alpha} \frac{(x-\beta)^2 \, 2\sin^2 \frac{(x-\alpha)(x-\beta)}{2}}{\frac{2^2 (x-\alpha)^2 (x-\beta)^2}{2^2}} = \frac{2(\alpha-\beta)^2}{4} = \frac{(\alpha-\beta)^2}{2}$$

14. $\operatorname{Lt}_{n \to \infty} \frac{2^n + 3^n}{4^n + 5^n} =$

a. 0

b. 1

c. $\frac{5}{9}$

d. None of these

Ans. A

Solution:

$$\underset{n\to\infty}{\mathrm{Lt}}\ \frac{2^{n}+3^{n}}{4^{n}+5^{n}} - \underset{n\to\infty}{lt}\ \frac{3^{n}\left[\left(\frac{2}{3}\right)^{n}+1\right]}{5^{n}\left[\left(\frac{4}{5}\right)^{n}+1\right]}$$

$$\frac{2}{3},\frac{4}{5},\frac{3}{5}all..are..<1$$

$$then..\lim it=0$$

15. $\underset{x\to 0}{\mathrm{Lt}}\ \frac{\tan(\sin^{\ 1}3x)}{\sin^{-1}(2\tan 2x)}-$

a. $\frac{3}{4}$
b. $\frac{3}{2}$
c. $\frac{4}{3}$
d. None of these

Ans. A

$$\underset{x\to 0}{\mathrm{Lt}}\ \frac{\tan(\sin^{-1}3x)}{\sin^{-1}(2\tan 2x)}[0/0\,form]=\underset{x\to 0}{lt}\ \frac{\sec^{2}(\sin^{-1}3x)\frac{3}{\sqrt{1-9x^{2}}}}{\frac{2}{\sqrt{1-4\tan^{2}2x}}2\sec^{2}2x}=3/4$$

16. $\lim_{x\to 1}\frac{x^{x}-1}{x\log x}=$

a. 0
b. 1
c. e
d. none of these

Ans. B

Solution:

$$\lim_{x\to 1}\frac{x^{x}-1}{x\log x}[\frac{0}{0}form]$$

$$=\lim_{x\to 1}\frac{x^{x}(\log x+1)}{\log x+1}=\frac{1}{1}=1$$

17. $\lim_{x \to \infty} \frac{(1+x)^{40}(4+x)^{5}}{(2\ \ x)^{45}} =$

a. 0
b. 1
c. -1
d. None of these

Ans. C

Solution:

$$\lim_{x \to \infty} \frac{(1+x)^{40}(4+x)^{5}}{(2-x)^{45}} = \lim_{x \to \infty} \frac{(1+\frac{1}{x})^{40}(1+\frac{4}{x})^{5}}{(\frac{2}{x}-1)^{45}}$$

$\frac{1}{x} \to 0 ... as .. x \to \infty$

$= \frac{1^{40}(1)^{5}}{(-1)^{45}} = -1$

18. If $\lim_{x \to \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$ then the values of a and b are

a. $a \in R, b \in R$
b. $a = 1, b \in R$
c. $a \in R, b = 2$
d. $a = 1, b = 2$

Ans. B

Solution:

$\lim_{n\to\infty} e^{2x(1+\frac{a}{x}+\frac{b}{x^2}-1)} = e^{2a}$

True for a=1 and any b.

19. For $x > 0$, $\lim_{x \to 0} \left[(\sin x)^{\frac{1}{x}} + \left(\frac{1}{x}\right)^{\sin x} \right]$ is

a. 0
b. -1
c. 1

d. 2

Ans. C

sinx = 0 for x=0

$(\sin x)^{1/x}$ =0 irrespective of power. (Remember 0^{∞} is not indeterminate form.)

For $(1/x)^{\sin x}$ solve by taking log and applying L-Hospital's rule it will simplify to 1.

20. Which of the following functions is not continuous for all real x?
    a. e^x
    b. tan x
    c. $\tan^{-1}x$
    d. sinh x

Ans. B

Solution:

Here 'tan x' is not continuous for all real x