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Class: 11
Subject: Mathematics
Topic: Principles of mathematical Induction
No. of Questions: 20
Duration: 60 Min
Maximum Marks: 60

Q1. Let P(n): $1+\frac{1}{4}+\frac{1}{9}+\cdots+\frac{1}{n^2}<2-\frac{1}{n}$ *is true for*

A. ∀n
B. For n =1
C. N > 1∀ n ∈ N
D. None of these

Sol: C

For n − 1
L.H.S of P(1) = 1
R.H.S of P(1) = 1
L.H.S = R.H.S
∴ 1 cannot be < 1
∴ P(1) is not true.
as L.H.S. of P(1) = 1 < 1(false)
Again n − 2
L.H.S. of P(2) = 1 + 1/4 = 5/4
R.H.S of P(2) = $2 - \frac{1}{2} = 3/2 = 6/4$
∴ L.H.S P(2) < R.H.S of P(2) ∴ P(2) is true.

Q2. P(n) = $7^n - 3^n$ is divisible by 4. A ≠ b, then expansion $a^{n-} - b^n$ is divisible by (a-b) if n is odd or even.

A. If both statement-1 and statement -2 are true and statement -2 is the correct explanation of statement- 1.
B. If both statement – 1 and statement -2 are true but statement -2 is not the correct explanation of statement -1
C. If statement-1 is true but statement -2 is false

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D. If statement-1 is false and statement-2 is true

Sol: A

$P(n) = 7^n - 3^n$, n is either even or odd $(a = 7, b = 3)$ divisible by $(a - b) = 7 - 3 = 4$.
Hence, statement -1 followed by statement -2.

Q3. Let P(n) = 111 1 (91 times) then P(n) is a prime number.

Every prime number has at most and at least two factors.

A. If both statement-1 and statement -2 are true and statement -2 is the correct explanation of statement- 1
B. If both statement – 1 and statement -2 are true but statement -2 is not the correct explanation of statement -1
C. If statement-1 is true but statement -2 is false
D. If statement-1 is false and statement-2 is true

Sol: D

$$P(n) = 1 + 10 + 10^2 + \ldots + 10^{90}$$
$$= \frac{10^{91} - 1}{9} = \frac{(10^7)^{13} - 1}{9}, \text{ where } t = 10^7$$
$$= \frac{(t-1)(t^{12} + t^{11} + \ldots. + t + 1)}{9}$$
$$= \frac{10^7 - 1}{10 - 1}(t^{12} + t^{11} + \ldots. + t + 1), t = 10^7$$
$$= (1 + 10 + 10^2 + \ldots. + 10^6)(1 + t + t^2 + \ldots\ldots + t^{12})$$

$\therefore P(n) = 111\ldots1$(91 times) is not a prime as P(n) has more than two factors so statement -1 is false.

Q4. For each natural number, the statement P(n) = 2^{3n} - 1 is divisible by

A. 8
B. 16
C. 7
D. None of these

Sol: C

Let $P(n) = 2^{3n} - 1$

Putting $n = 1$

$\therefore P(1) = 2^3 - 1 = 7$

$Now\ P(k) = 2^{3k} - 1$

To prove $P(k+1)$ is divisible by 7 whenever

$P(k) = 7M$

$\therefore P(k+1) = 2^{5(k+1)} - 1 = 2^{3k} x 2^3 - 1$

$- 2^{3k}(7+1) - 1 - 7.2^{3k} + 2^{3k} - 1 - 7.2^{3k} + 7M$

$= 7(2^{3k} + M)$

Q5. **Statement 1:** If A = $(300)^{600}$, B = 600, C = $(200)^{600}$! , Then A > B > C.

Statement 2: $\left(\frac{n}{2}\right)^n > n!\left(\frac{n}{3}\right)^n$ $for\ n > 6.$

A. If both statement-1 and statement -2 are true and statement -2 is the correct explanation of statement- 1
B. If both statement – 1 and statement -2 are true but statement -2 is not the correct explanation of statement -1
C. If statement-1 is true but statement -2 is false
D. If statement-1 is false and statement-2 is true

Sol: A

$$\text{Since, } \left(\frac{n}{2}\right)^n > n! > \left(\frac{n}{3}\right)^n, n > 6$$

Putting $n = 600$

$$\left(\frac{600}{2}\right)^{600} > 600! > (200)^{600} \Rightarrow A > B > C.$$

Q6. Let P(n) = 3^{2n} ∀ n ∈ N , When divided by 8, leaves the remainder

A. 2
B. 1
C. 4
D. 7

Sol: B

$P(n) = 3^{2n}$

$= 9^n = (1+8)^n$ (Using Binomial theorem)

$P(n) = 1 + 8n$ ∴ Remainder is 1.

Q7. Let P(n): $2^n > n$ ∀ n ∈ N and $2^k > K$, ∀ n = K, then which h of the following is true ∀ k ≥ 2

A. $2^k > 5k > 1$
B. $2^{k+1} > 2k > k+1$
C. $2^k > 2(k+1) > k$
D. None of these

Sol: B

$$P(n) = 2^n > n \Rightarrow P(k) = 2^k > k \Rightarrow 2x2^k > 2k$$
$$\Rightarrow 2x2^k > 2k > k+1 \;\; as \;\; k \geq 2$$

Q8. For every natural number n ≥ 2, $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n}$ For every natural number n ≥ 2, $\sqrt{n(n+1) < (n+1)}$

A. Statement (1) is true & statement (2) is false
B. Statement (1) is false & statement (2) is true
C. Statement (1) is true, statement (2) is true & statement (2) is correct explanation for statement (1)
D. Statement (1) is true, statement (2) is true but statement (2) is not the correct explanation for statement (1)

Sol: D

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Let P(n) : $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{n}}>\sqrt{n}$

Step1 : For $n=2$

$L.H.S$ *of* $P(2)$

$=\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}=\frac{\sqrt{2}+1}{\sqrt{2}}=(1.707 \text{ Approx.})$

R.H.S of $P(2)=\sqrt{2}=1.414$

$\therefore L.H.S \text{ of } P(2) > R.H.S \text{ of } P(2)$

Step2 : For $n=k$ *say* $P(n)$ is true

$$P(k) : \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{k}}>\sqrt{k} \quad ...(i)$$

Step3 : For $n=k+1$ we prove that $P(k+1)$

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}>\sqrt{k+1} \quad ...(ii)$$

$L.H.S$ *of* $P(k+1)$

$=\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}$

$=p(k)+\frac{1}{\sqrt{k+1}}$

$$\Rightarrow \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+..+\frac{1}{\sqrt{k+1}}>\sqrt{k}+\frac{1}{\sqrt{k+1}} \quad ...(iii)$$

For n = k

$\sqrt{k(k+1)}<k+1$

$\Rightarrow \sqrt{k}(\sqrt{k+1})<\sqrt{(k+1)}\sqrt{k+1}$

$\Rightarrow \sqrt{k}<\sqrt{k+1}$

$\Rightarrow \sqrt{k+1}>\sqrt{k}$ for $k \geq 2$

$\Rightarrow 1>\frac{\sqrt{k}}{\sqrt{k+1}} \Rightarrow \sqrt{k}>\frac{k}{\sqrt{k+1}}$

$\Rightarrow \sqrt{k}>\frac{(k+1)-1}{\sqrt{k+1}}=\sqrt{k+1}-\frac{1}{\sqrt{k+1}}$

$$\Rightarrow \sqrt{k}+\frac{1}{\sqrt{k+1}}>\sqrt{k+1} \quad ...(iv)$$

$\therefore$ By (iii) & (iv) we have

$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+...+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}>\sqrt{k+1}$

$(\because a>b, b>c \Rightarrow a>c)$

Hence $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+...+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}$ is true for n $=k+1$

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Q9. Let P(n): $a^n + b^n$ such that a,b are even, then P (n) will be divisible by a + b if

A. n>1
B. ∀ n is odd
C. ∀ n is even
D. None of these

Sol: B

$P(n) = a^n + b^n \forall n \in N.$

$n = 1$

$\therefore P(1) = a + b$ which is divisible by $a + b$.

$n = 2.$

$\therefore P(2) = a^2 + b^2$ not divisible by $a + b$. $n = 3$

$\therefore P(3) = a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

which is divisible by $a + b$.

with the help of induction we conlude that P(n) will be divisible by $a + b$, if n is odd.

Q10. The statement $P(n) = 9^n - 8^n$, when divided by 8, always leaves the remainder

A. 2
B. 3
C. 1
D. 7

Sol: C

$P(n) = 9^n - 8^n$

$\therefore P(1) = 9 - 8 = 1$

$\therefore P(1) - 1 = 0$ which is divisible by 8.

$\therefore P(1) = 1$ is the remainder when P(n) is divided by 8.

$P(2) = 9^2 - 8^2 = 17 = 16 + 1$

$\Rightarrow$ Remainder is 1, when divided by 8.

Q11. Statement 1: The greatest positive integer which divides (n+11) (n+12) (n +13) (n+14) $\forall n \in N$ is 24

Statement 2: Product of any r consecutive integers is divisible by r!.

A. if both statement -1 and statement -2 are true and statement -2 is the correct explanation of statement -1
B. if both statement -1 and statement -2 are true but statement -2 is not the correct explantion of statement -1
C. if statement -1 is true but statement -2 is false
D. if statement -1 is false and statement -2 is ture

Sol: A

Since the statement -2 is obvious, so the greatest positive integer which divides the product (n + 11) (n + 12) (n+13) (n+14) is 4! = 24.

Q12. Let P(n) be the statement n^2-n+41 is prime, then which of the following is not true?

A. P(2)
B. P(3)
C. P(41)
D. None of these

Sol: C

$$P(n) = n^2 - n + 41$$
$$P(2) = 2^2 - 2 + 41 = 43 \text{ is Prime (true)}$$
$$P(3) = 3^2 - 3 + 41 = 47 \text{ is prime (true)}$$
$$P(41) = 41^2 - 41 + 41 = (41)^2 \text{ is Prime}$$

Q13. The statement P(r) = $P(r) = \sum_{r=1}^{n} r(r!) = (r+1)! - 1$ is true for

A. r > 2
B. true for $\forall$ r $\in$ N
C. for no value of r
D. r > 1

Sol: B

$P(k) = \sum_{k=1}^{n} k(k!) \leftrightarrow (k = 1,...r$ or $k = 1...n$ have same meaning as we have considered $r, n \in N)$

$= \sum_{k=1}^{n} (k+1-1)(k!)$

$= \sum_{k=1}^{n} (k+1)! - \sum_{k=1}^{n} (k!) \qquad k = n, n-1, ...1$

$= (n+1)! - n!$

$n! - (n-1)!$

$(n-1)! - (n-2)!$

— — — — —

— — — — —

3!-2!

2!-1!

By adding, $P(k) = (n+1)!-1!$.

Q14. Let the statement $m^2 > 100$, the statement P(K+1) will be true if

A. P(1) is true
B. P(2) is true
C. P(K) is true
D. None of these

Sol: C

$P(r)$ is true.

$\Rightarrow r^2 > 100 \Rightarrow r^2 + 2r + 1 > 100 + 2r + 1$

$\Rightarrow (r+1)^2 > 100$

$\Rightarrow P(r+1)$ is true as $r^2 + (2r+1) > r^2 > 100$

$\Rightarrow P(k+1)$ is true (say $r = k$)

$P(k+1)$ is true when every $P(k)$ is so. IN order to prove that $P(k+1)$ is true.

It is sufficient to consider $P(k)$ is true.

Q15. Statement-1: $S_n = n^3 + 3n^2 + 5n + 3$ is divisible by 3

Statement-2: $t_n = 3^a$

A. If both statement -1 and statement-2 are true and statement-2 is the correct explanation of statement-1
B. If both statement-1 and statement-2 are true but statement-2 is not the correct explanation of statement-1
C. If statement -1 is true but statement -2 is false
D. If statement-1 is false and statement-2 is true

Sol: A

$$S_n = n^3 + 3n^2 + 5n + 3$$
$$\therefore t_n = S_n - S_{n-1}$$
$$= (n^3 + 3n^2 + 5n + 3)$$
$$- \{((n-1)^3 + 3(n-1)^2 + 5(n-1) + 3\}$$
$$= 3n^2 + 3n + 3 = 3(n^2 + n + 1) = 31.$$

Q16. Statement -1: $P(n) = n^2 + n + 1$ is an odd natural number $\forall\, n \in N$.

Statement -2: If 1 added to an even number then it becomes an odd number

A. If both statement -1 and statement-2 are true and statement-2 is the correct explanation of statement-1
B. If both statement-1 and statement-2 are true but statement-2 is not the correct explanation of statement-1
C. If statement -1 is true but statement -2 is false
D. If statement-1 is false and statement-2 is true

Sol: A

$$P(n) = n^2 + n + 1$$
$$\therefore P(1) = 3, P(2) = 7, P(3) = 13$$
$$\therefore P(n) \text{ is odd } \forall \text{ n} \in \text{N}.$$

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Q17. The greatest positive integer which divides n(n+1) (n+2) ...(n+r-1)∀n∈ N is

A. r!
B. (r+1)!
C. n + r
D. n – r + 1

Sol: A

$$\text{Let } P(n) = n(n+1)(n+2)\ldots(n+r-1)$$
$$= (n+0)(n+1)(n+2)\ldots.(n+(r-1))$$
$$= \text{Product of r consecutiv e integers}$$
$$\therefore \text{Divisible by } r!$$

Q18. The inequality $n! > 2^n$ is true for

A. $n \geq 4$
B. $n > 1$
C. $n > 2$
D. $\forall n, n \in N$

Sol: A

$$P(n) = n! > 2^n$$
$$\therefore P(1) = 1! > 2^1 \text{ (false)}$$
$$P(2) = 2! > 2^2 = 4 \text{(false)}$$
$$P(3) = 3! > 2^3$$
$$\text{or } 6 > 8 \text{(false)}$$
$$P(4) = 4! > 2^4$$
$$\text{or } 24 > 16 \text{ true}$$
$$\text{Hence, inequality holds good if } n \geq 4.$$

Q19. Let n ∈ N then P(n) = n(n+1) is an even number Product of two consecutive natural numbers is even

A. If both statement -1 and statement-2 are true and statement-2 is the correct explanation of statement-1
B. If both statement-1 and statement-2 are true but statement-2 is not the correct explanation of statement-1

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C. If statement -1 is true but statement -2 is false
D. If statement-1 is false and statement-2 is true

Sol: A

The number n and (n+1) are consecutive e numbers, so n(n+1) is an even number.

Q20. 3^{2n} $\forall n \in N$ Leaves the remainder 1 when divided by 8 $9^n = 1+ 8l$.

A. If both statement -1 and statement-2 are true and statement-2 is the correct explanation of statement-1
B. If both statement-1 and statement-2 are true but statement-2 is not the correct explanation of statement-1
C. If statement -1 is true but statement -2 is false
D. If statement-1 is false and statement-2 is true

Sol: A

Let P (n) = $3^{2n} = 9^n$

$\therefore 9^n = (1+8)^n = 1+81.$