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Class: 11
Subject: Mathematics
Topic: Sequence and Series
No. of Questions: 20
Duration: 60 Min
Maximum Marks: 60

1. If $a_1, a_2, \ldots, a_n$ are in H.P ., then the expression $a_1 a_2 + a_2 a_3 + \ldots + a_{n-1} a_n$ is equal to
   a. $n(a_1 - a_n)$
   b. $(n-1)(a_1 - a_n)$
   c. $n a_1 a_n$
   d. $(n-1) a_1 a_n$

   Sol: D

   Given $a_1, a_2, \ldots, a_n$ are in H.P.

   $$\Rightarrow \frac{1}{a_1}, \frac{1}{a_2}, \ldots \frac{1}{a_n} \in A.P.$$

   $$\Rightarrow \frac{1}{a_2} - \frac{1}{a_1} = d \Rightarrow a_1 a_2 = \frac{a_1 - a_2}{d} = \frac{a_1}{d} - \frac{a_2}{d} \qquad \ldots(i)$$

   $$a_2 a_3 = \frac{a_2}{d} - \frac{a_3}{d} \qquad \ldots(ii)$$

   .

   .

   $$a_{n-1} a_n = \frac{a_{n-1}}{d} - \frac{a_n}{d} \qquad \ldots(n)$$

   Adding (i), (ii)...... (n) equations we get

   $$a_1 a_2 + a_2 a_3 + a_3 a_4 + \ldots a_{n-1} a_n = \frac{a_1}{d} - \frac{d_n}{d}$$

   Also $\frac{1}{a_n} = \frac{1}{a_1} + (n-1)d \Rightarrow \frac{a_1 - a_n}{d} = (n-1) a_1 a_n$

   $$\therefore a_1 a_2 + a_2 a_3 + \ldots a_{n-1} a_n = (n-1) a_1 a_n.$$

2. If $1.3 + 3.3^2 + 5.3^3 + 7.3^4 + \ldots$ up to n terms is equal to $3 + (n-1).3^b$, then b =
   a. n
   b. n – 1
   c. 2n – 1
   d. n + 1

   Sol: D

$$S = 1.3 + 3.3^2 + 5.3^3 + 7.3^4 + \ldots + (2n-1)3^n$$
$$3S = 1.3^2 + 3.3^3 + \ldots + (2n-3)3^n + (2n-1)3^{n+1}$$
$$-2S = 1.3 + 2[3^2 + 3^3 + \ldots + 3^n] - (2n-1)3^{n+1}$$
$$S = \left(\frac{2n-1}{2}\right)3^{n+1} - \frac{2.3(3^n-1)}{2(3-1)} + \frac{1.3}{2}$$
$$= 3 + (n-1)3^{n+1}$$
But given $= 3 + (n-1)3^b \therefore b = n+1$

3. Statement -1 : If three positive numbers in G.P. represent the sides of a triangle then the common ratio of the G.P. must lie between $\frac{\sqrt{5}-1}{2}$ and $\frac{\sqrt{5}+1}{2}$

Statement -2 : Three positive numbers can form sides of triangle if sum of any two sides is greater than the third side.

a. if both statement-1 and statement-2 are true and statement-2 is the correct explanation of statement-1
b. if both statement-1 and statement-2 care true but statement-2 is not the correct explanation of statement-1
c. if statement-1 is true but statement-2 is false
d. if statement-1 is false and statement-2 is true.

Sol: B

Let a, ar, ar^2 be the three terms of a GP

(which forms a triangle) .

We have $(a, r > 0)$

$a + ar > ar^2$ (assume $r \geq 1$

(Sum of two sides > third side)

$\Rightarrow 1 + r > r^2$

$\Rightarrow r^2 - r - 1 < 0$

$\Rightarrow \frac{-1-\sqrt{5}}{2} < r < \frac{1+\sqrt{5}}{2}$

But $r > 1$ hence $1 < r < \frac{1+\sqrt{5}}{2}$

Also r can be less then 1 so that gives $r > \frac{\sqrt{5}-1}{2}$

4. If $a_1, a_2, a_3, \ldots, a_n \in$ GP, then the value of the determinant $\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$

equals

a. 2
b. 1
c. 0
d. -2

Sol : C

$$\frac{a_2}{a_1} = \frac{a_2}{a_2} = -\frac{a_n}{a_{n-1}} = r$$

which means $a_n, a_{n+1}, a_{n+2} \in G.P.$

$$\Rightarrow a_{n+1}^2 = a_n a_{n+2}$$

$$\Rightarrow 2\log a_{n+1} - \log a_n - \log a_{n+2} = 0 \qquad ...(i)$$

Similarly

$$2\log a_{n+1} - \log_{n+3} - \log a_{n+5} = 0 \qquad ....(ii)$$

and

$$2\log a_{n+7} - \log a_{n+6} - \log a_{n+8} = 0 \qquad ...(iii)$$

Using $C_1 \to C_1 + C_3 - 2C_2$

we get $\Delta = 0$

5. The value of $2^{1/4}.4^{\frac{1}{8}}.8^{\frac{1}{16}}.16^{\frac{1}{32}}...\infty$ equals
   a. 1
   b. 2
   c. $\frac{3}{2}$
   d. 4

   Sol: B

$$S_\infty = 2^{\frac{1}{4}}.2^{\frac{2}{8}}.2^{\frac{3}{16}}.2^{\frac{4}{32}}.....\infty = 2^\lambda (say)...(*)$$

Where $\lambda = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + ...\infty \qquad (A)$

$$\frac{\lambda}{2} - 0 + \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + ...\infty \qquad ...(B)$$

Now (A) - (B) $\Rightarrow \frac{\lambda}{2} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + ... \frac{\lambda}{2} = \frac{a}{1-r} = \frac{1}{4} x \frac{2}{1} \qquad \therefore \lambda = 1$

so $S_\infty = 2^1$

6. If one G.M., is g and two A.M.s are p and q, are inserted between two number a and b, then $\frac{(2\,p - q)(p - 2\,q)}{g^2} =$
   a. 1
   b. -1

c. 2
d. -3

Sol: B

$g = \sqrt{ab}$

a, p, q, b are in A.P.

Common difference d is $\frac{\text{b - a}}{3}$

$\therefore p = a + d = a + \frac{b-a}{3} = \frac{2a+b}{3}$

$q = b - d = b - \frac{b-a}{3} = \frac{a+2b}{3}$

$(2p-q)(p-2q)$

$= \frac{(4a+2b-a-2b)}{3} . \frac{(2a+b-2a-4b)}{3}$

$= -ab = -g^2.$

7. If 1, log $_{81}$ (3^x +48) and log$_9$ $\left(3^x - \frac{8}{3}\right)$ are in A.P then x is equal to
   a. 1
   b. 2
   c. 9
   d. 3

Sol: B

The Three numbers are $\log_{\theta} 9, \log_{9_2}(3^x + 48)$

and $\log_9\left(3^x - \frac{8}{3}\right)$,

$i.e. \log_9 9, \frac{1}{2}\log_9(3^x + 48), \log_0\left(3^x - \frac{8}{3}\right)$ are in A.P.

$\Rightarrow \left\{(3^x + 48)^{\frac{1}{2}}\right\}^2 = 9\left(3^x - \frac{8}{3}\right) \Rightarrow 8.3^x = 72$

$\Rightarrow 3^x = 9 \Rightarrow x = 2.$

8. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. The common ratio of the G.P., is
   a. $\frac{\sqrt{5}}{2}$
   b. $\sqrt{5}$
   c. $\frac{\sqrt{5}-1}{2}$
   d. $\frac{1-\sqrt{5}}{2}$

Sol: C

9. Let S be the sum, P be the product and R be the sum of reciprocal of n terms of a G.P. Then
   a. $R = S. P^{1/n}$
   b. $R = S. P^{2/n}$
   c. $S = R. P^{1/n}$
   d. $S = R. P^{2/n}$

   Sol: D

   Let $a, ar, ar^2, ...$ be the G.P.

   $$S = \frac{a(1-r^n)}{1-r}$$

   The reciprocal s are in G.P.

   $$\frac{1}{a}, \frac{1}{ar}, \frac{1}{ar^2}, ...$$

   $$R = \frac{\frac{1}{a}\left(1-\frac{1}{r^n}\right)}{1-\frac{1}{r}} = \frac{1}{a}\left(\frac{1-r^n}{1-r}\right)\frac{1}{r^{n-1}}$$

   $$\therefore \frac{S}{R} = a^2 r^{n-1} \qquad ...(1)$$

   $$P = a.ar.ar^2...ar^{n-1}$$

   $$= a^n r^{(1+2+...+(n-1))} = (a^2 r^{n-1})^n = \left(\frac{S}{R}\right)^n (by (1))$$

   $$\Rightarrow S = R.P^{2/n}.$$

10. The sum of the first n terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ Is $\frac{n(n+1)^2}{2}$, when n is even. When n is odd, the sum is
   a. $\frac{n^2(n+1)}{2}$
   b. $\frac{n(n+1)(2n+1)}{6}$
   c. $\frac{n(n+1)^2}{2}$
   d. $\frac{n^2(n+1)^2}{2}$

   Sol: A

If n is odd, n - 1 is even . Sum of (n - 1) terms will be

$$\frac{(n-1)(n-1+1)^2}{2} = \frac{n^2(n-1)}{2}$$

The n^{th} term will be n^2.

Hence the required sum

$$= \frac{n^2(n-1)}{2} + n^2 = \frac{n^2(n+1)}{2}$$

11. The value of the expression $\frac{1}{2^2-1} + \frac{1}{4^2-1} + \frac{1}{6^2-1} + \cdots + \frac{1}{20^2-1}$ is

a. $\frac{11}{21}$
b. $\frac{10}{21}$
c. $\frac{10}{19}$
d. $\frac{2}{19}$

Sol: B

$$\frac{1}{(2n)^2-1} = \frac{1}{(2n-1)(2n+1)} = \frac{1}{2}\left[\frac{1}{(2n-1} - \frac{1}{2n+1}\right]$$

Set n = 1,2,….,10

$$\frac{1}{2^2-1} = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{3}\right]$$

$$\frac{1}{4^2-1} = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{5}\right]$$

$$\frac{1}{20^2-1} = \frac{1}{2}\left[\frac{1}{19} - \frac{1}{21}\right]$$

Adding we get

$$S = \frac{1}{2}\left(1 - \frac{1}{21}\right) = \frac{1}{2} x \frac{20}{21} = \frac{10}{21}.$$

12. Statement – 1 : Let $2^{1/4}.4^{1/8}.8^{1/16}.16^{1/32} \ldots \infty = 2^x$, then the value of x is equal to 1.
Statement -2 : The sum of series $\frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} = \ldots \infty$ is equal to 1.

a. if both statement-1 and statement-2 are true and statement-2 is the correct explanation of statement-1
b. if both statement-1 and statement-2 care true but statement-2 is not the correct explanation of statement-1
c. if statement-1 is true but statement-2 is false
d. if statement-1 is false and statement-2 is true.

Sol: A

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$$2^{1/4}4^{1/8}8^{1/16}(16)^{1/32}\ldots\infty = 2^x$$

$$\Rightarrow 2^x = 2^{1/4}.2^{2/8}.2^{3/16}.2^{4/32}\ldots.\infty \Rightarrow 2^x = 2^{\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+\ldots\infty}$$

$$\Rightarrow x = \frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+\ldots.\infty \qquad \ldots..(i)$$

$$\Rightarrow x = \frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+\ldots.\infty \qquad \ldots.(ii)$$

$\therefore (i)-(ii)$ we have

$$\frac{x}{2} = \frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots.\infty$$

$$\frac{x}{2} = \frac{1}{4}\left(\frac{1}{1-\frac{1}{2}}\right)$$

$$x = 1.$$

13. (2n +1) G.M.'s are inserted between 4 and 2916. Then the (n + 1)th G.M. is equal to
    a. 36
    b. 54
    c. 108
    d. 324

    Sol : C

    The (n + 1)th G.M. = middle G.M. = G.M. of 4 and 2916 = $\sqrt{4x2916}$ = 108

14. If a_1, a_2, a_3, a_4 and b are real numbers such that $\left(a_1^2+a_2^2+a_3^2\right)b^2 - 2\left(a_1a_2+a_2a_3+a_3a_4\right)b+\left(a_2^2+a_3^2+a_4^2\right) \le 0$, then a_1, a_2, a_3, a_4 are
    a. In A.P
    b. In G.P.
    c. In A.G.P.
    d. Such that $(a_1+a_2)(a_3-a_4) = (a_1+a_3)(a_2-a_4)$

    Sol: B

    The given condition

$$\Rightarrow (a_1b-a_2)^2+(a_2b-a_3)^2+(a_3b-a_4)^2 \le 0$$

$\Rightarrow$ only the equality holds goods

$$\Rightarrow a_1b = a_2, a_2b = a_3, a_3b = a_4$$

$$\Rightarrow b = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3}$$

$\Rightarrow a_1, a_2, a_3, a_4$ are in G.P.

15. Statement – 1 : There doesn't exist an A.P. whose three terms are $\sqrt{2}, \sqrt{3}, \sqrt{5}$.
Statement – 2 : There exists distinct real numbers l, m, n such that $\sqrt{2}$ = a +(l-l)d, $\sqrt{3}$= a+(m-l)d and $\sqrt{5}$= a+(n-l) d.

   a. if both statement-1 and statement-2 are true and statement-2 is the correct explanation of statement-1
   b. if both statement-1 and statement-2 care true but statement-2 is not the correct explanation of statement-1
   c. if statement-1 is true but statement-2 is false
   d. if statement-1 is false and statement-2 is true.

   Sol: C

   S_1 is true. It can be shown that S_2 is false. Indeed

   $\sqrt{2}-\sqrt{3}=(1-m)d$

   $\sqrt{3}-\sqrt{5}=(m-n)d$

   On division w e get,

   $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{5}}=\frac{1-m}{m-n}$

   $\Rightarrow$ Irrational $=$ Rational, which is not possible.

16. The maximum value of the sum of the A.P. 50, 48, 46, 44, ..., is
   a. 325
   b. 648
   c. 650
   d. 652

   Sol : C

   The maximum value will correspond to n terms when the n^{th} term is either zero or the smallest positive number of the series.

   i.e., $50+(n-1)(-2)=0$ when $n=26$;

   $S_{26}=\frac{26}{2}(a+b)=13(50+0)=650$

17. Let $S=\frac{4}{19}+\frac{44}{19^2}+\frac{444}{19^3}+\ldots\infty$ The S is equal to
   a. $\frac{38}{81}$
   b. $\frac{4}{19}$
   c. $\frac{36}{171}$
   d. None of these

Sol: A

$$S = \frac{4}{19} + \frac{44}{19^2} + \frac{444}{19^3} + \ldots\infty \qquad \ldots(1)$$

$$\Rightarrow \frac{S}{19} = \frac{4}{19^2} + \ldots\infty \qquad \ldots(2)$$

Subtractin g (2) from (1), we get

$$S.\frac{18}{19} = \frac{4}{19} + \frac{40}{19^2} + \frac{400}{19^3} + \ldots$$

$$= \frac{4}{19}\left[1 + \frac{10}{19} + \left(\frac{10}{19}\right)^2 + \ldots\right] = \frac{4}{19}\left[\frac{1}{1 - \frac{10}{19}}\right] = \frac{4}{9}$$

$$\Rightarrow S = \frac{76}{162} = \frac{38}{81}$$

18. Let T_r be the r^{th} term of an A.P. for r = 1,2,3 ….. If for some positive integers m and n we have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then Tmn=

a. $\frac{1}{mn}$
b. $\frac{1}{m} + \frac{1}{n}$
c. 1
d. 0

Sol: C

Let $a, a+d, a+2d, \ldots$ be the A.P.

$$T_m = \frac{1}{n} \Rightarrow a + (m-1)d = \frac{1}{n}$$

$$T_n = \frac{1}{m} \Rightarrow a + (n-1)d = \frac{1}{m}$$

Solving $a = d = \frac{1}{mn}$

$$T_{mn} = a + (mn-1)d = \frac{1 + mn - 1}{mn} = 1$$

19. If $a_1, a_2, a_3, \ldots, a_n$ are n distinct odd numbers not divisible by any prime greater then 5, then $\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}$

a. <1
b. =1
c. =2
d. <2

Sol: D

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We observe that all the terms of

$\frac{1}{a_1}+\frac{1}{a_2}+\ldots+\frac{1}{a_n}$ are contained in

$\left(1+\frac{1}{3}+\frac{1}{3^2}+\ldots\right)\left(1+\frac{1}{5}+\frac{1}{5^2}+\ldots\right)$.

$$\Rightarrow \frac{1}{a_1}+\frac{1}{a_2}+\ldots+\frac{1}{a_n}<\left(\frac{1}{1-\frac{1}{3}}\right)\left(\frac{1}{1-\frac{1}{5}}\right)=\frac{3}{2}.\frac{5}{4}<2$$

20. The sum of 20 terms of the series 1 + (1+3)+(1+3+5)+(1+3+5+7)+.... Is
    a. 400
    b. 2870
    c. 5740
    d. 1540

Sol: B

$$t_n = 1+3+5+\ldots+(2n-1)=n^2$$
$$S_{20} = 1^2+2^2+3^2+\ldots+20^2 = \frac{20x21x41}{6} = 2870.$$