

Class: 11
Subject: Mathematics
Topic: Statistics and mathematical reasoning
No. of Questions: 20
Duration: 60 Min
Maximum Marks: 60

1. If p : 4 is an even prime number q : 6 is a divisor of 12 and r : the HCF of 4 and 6 is 2, then which one of the following is true?
- A. $(P \wedge q)$
 - B. $(p \wedge q) \wedge \sim r$
 - C. $\sim (p \wedge q) \vee p$
 - D. $\sim p \vee (q \wedge r)$

Sol: D

After seeing the given statements we can say that p is false and q and r are true. In the given options only (d) can be true because it is $\sim p \vee (q \wedge r)$ which means not p or (q and r) which means p is not correct and q and r are correct.

2. Let S be a non-empty subset of \mathbb{R} . Consider the following statement:

P : There is a rational number $x \in S$ such that $x > \theta$.

Which of the following statements is the negation of the statement P ?

- A. There is a rational number $x \in S$ such that $x \leq \theta$.
- B. There is no rational number $x \in S$ such that $x \leq \theta$.
- C. Every rational number $x \in S$ such that $x \leq \theta$.
- D. $x \in S$ and $x \leq \theta \Rightarrow x$ is not rational.

Sol: C

The given statement is P : at least one rational $x \in S$ such that $x > 0$. The negation would be: There is no rational number $x \in S$ such that $x > 0$ Which is equivalent to all rational numbers $x \in S$ satisfy $x \leq 0$.

3. The statement $p \rightarrow (q \rightarrow p)$ is equivalent to
- A. $p \rightarrow (p \leftrightarrow q)$
 - B. $p \rightarrow (p \rightarrow q)$
 - C. $p \rightarrow (p \vee q)$
 - D. $p \rightarrow (p \wedge q)$

Sol: C

Let's simplify the statement $p \rightarrow (q \rightarrow p) = \sim p \vee (q \rightarrow p) = \sim p \vee (\sim q \vee p) = \sim p \vee p \vee \sim q = p \rightarrow (p \vee q)$

4. Consider the statements
- (i) Two plus three is five
 - (ii) Every square is a rectangle
 - (iii) Sun rises in the east
 - (iv) The earth is a star.
- Which of the above' statements having truth value (T)?
- A. (i) and (ii)
 - B. (ii) and (iii)
 - C. (iii) and (iv)
 - D. All of these

Sol: D

Each of the statement is a true declarative statement so each statement have truth value (T).

5. Which of the following sentences are not a statement?
- A. 9 is less than 7
 - B. The sun is a star
 - C. There is no rain without clouds
 - D. Mathematics is fun.

Sol: D

A sentence is called mathematically acceptable statement if it is either true or false but not both. Now, "9 is less than 7" is false sentence because 9 is greater than 7 whose truth value is (F) so 9 is less than 7 is a statement. For the sentence "The sun is a star". It is scientifically established fact that sun is a star, therefore, this sentence is always true with truth value (T). Hence "The sun is a star" is a statement. For sentence "There is no rain without clouds". It is a scientifically established natural phenomenon that clouds is formed before it rains, this sentence is always true and hence it is a statement. The sentence "Mathematics is fun" is subjective in the sense that for those who like mathematics, it may be fun but for others it may not be which means that the sentence "mathematics is fun" is not always true so it is not a statement.

6. Which one of the following statement is not a false statement?
- A. p: Each radius of a circle is a chord of the circle.
 - B. q: Circle is a particular case of an ellipse
 - C. r: $\sqrt{13}$ is a rational number
 - D. s: The centre of a circle bisect each chord of the circle.

Sol: B

We know that equation of an ellipse is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if we take $a = b$ then we get $x^2 + y^2 = a^2$ which satisfies all the conditions of circle.
 \therefore Circle is the particular case of an ellipse.

7. Identify the false statement
- A. $\sim [p \vee (\sim q)] = (\sim p) \vee q$
 - B. $[p \vee q] \vee (\sim p)$ is a tautology
 - C. $\sim (p \vee q) = (\sim p) \wedge (\sim q)$
 - D. $\sim [p \wedge (\sim p)]$ is a tautology

Sol: C

Since $\sim (p \vee q) = \sim p \wedge \sim q$

(By De - Morgan's law)

$\therefore \sim (p \vee q) = \sim p \wedge \sim q$

8. Consider the following compound statements:
- (i) Mumbai is the capital of Rajasthan or Maharashtra
 - (ii) $\sqrt{3}$ is a rational number or an irrational number
 - (iii) 125 is a multiple of 7 or 8
 - (iv) A rectangle is a quadrilateral or a regular hexagon
- Which of the above statement is not true?
- A. (i)
 - B. (ii)
 - C. (iii)
 - D. (iv)

Sol: C

(i) The component statements of 'Mumbai is the capital of Rajasthan or q: Mumbai is the capital of Maharashtra' We note that p is false and q is true, so the compound statement is true.

(ii) The component statements of ' $\sqrt{3}$ is a rational or an irrational' are P: ' $\sqrt{3}$ is a rational number' q: ' $\sqrt{3}$ is an irrational number' We note that p is false and q is true, so compound statement is true.

(iii) The component statements of "125 is a multiple of 7 or 8" are

P: 125 is a multiple of 7 q: 125 is a multiple of 8

(iv) the component statements of "A rectangle is a quadrilateral or a regular hexagon" are P: A rectangle is a quadrilateral q: A rectangle is a regular hexagon as p is true.

9. Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number", and r be the statement 'x is a rational number if y is a transcendental number".
- Statement-1: r is equivalent to either q or p.
- Statement-2: r is equivalent or $\sim (p \leftrightarrow \sim q)$.
- A. Statement-1 is true, Statement-2 is false
 - B. Statement-1 is false, Statement-2 is true
 - C. Statement-1 is true, Statement-2 is true and Statement-2 is a correct explanation for Statement-1
 - D. Statement-1 is true, Statement-2 is true but Statement-2 is not a correct explanation for Statement-1

Sol: A

The given statement $r \equiv \sim p \leftrightarrow q$ The Statement -1 is $r_1 \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$ The Statement -2 is $r_2 \equiv (p \leftrightarrow \sim q) = (p \wedge q) \vee (\sim q \sim p)$ We can establish that $r = r_1$ Thus, Statement-1 is true but Statement-2 is false.

10. Statement- 1: $\sim (p \leftrightarrow \sim q)$ is equivalent to $\leftrightarrow q$.

Statement-2: $\sim (p \leftrightarrow \sim q)$ is a tautology.

- A. Statement-1 is true, Statement-2 is true but Statement-2 is not a correct explanation for Statement-1
- B. Statement-1 is true, Statement-2 is false
- C. Statement-1 is false, Statement-2 is true
- D. Statement-1 is true, Statement-2 is true and Statement-2 is a correct explanation for Statement-1

Sol: B

Let's prepare the truth table

P	q	$\sim q$	$p \leftrightarrow q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow q)$
T	T	F	T	F	T
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	T	F	T

11. Mean of 10 items was found to be 15. On verification, it was found that an item 21 was miscopied as 12. The correct mean is

- A. 15.9
- B. 14.1
- C. 24
- D. none of these

Sol: A

New mean = old mean + $(21-12)/10$

new mean = $15 + 0.9$

new mean = 15.9

12. For a frequency distribution 7th decile is computed by the formula

A. $D_7 = 1 + \left(\frac{\frac{n}{7} - c}{f} \right) x c$

B. $D_7 = 1 + \left(\frac{\frac{n}{10} - c}{f} \right) x c$

C. $D_7 = 1 + \left(\frac{\frac{7}{10}n - c}{f} \right) x c$

D. $D_7 = 1 + \left(\frac{\frac{17}{7}n - c}{f} \right) x c$

Sol: C

$$7^{\text{th}} \text{ decile} = D_7 = 1 + \left(\frac{\frac{7}{10}n - c}{f} \right) x c$$

13. If $\rho(X, Y) = 0.5$, $\text{Cov}(X, Y) = 16$ and $s_x = 4$, then $s_y =$

- A. 4
- B. 8
- C. 16
- D. 64

Sol: B

$$\rho(x, y) = \frac{\text{Cov}(X, Y)}{s_x s_y}$$
$$\Rightarrow s_y = \frac{\text{Cov}(X, Y)}{s_x \rho(x, y)} = \frac{16}{4(0.5)} = 8$$

14. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set

- A. is increased by 2
- B. is decreased by 2
- C. is two times the original median
- D. remains the same as that of the original set

Sol: D

$$n = 9 \text{ then median term} = \left(\frac{9+1}{2} \right)^{\text{th}} = 5^{\text{th}} \text{ term last four observations are increased by 2}$$

\therefore the median is 5th observation which is remaining unchanged.

\therefore There will be no change in median.

15. If in a frequency distribution, the Mean and the Median are 21 and 22 respectively, then its Mode is approximately

- A. 24.0
- B. 25.5
- C. 20.5
- D. 22.0

Sol: A

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean} = 3 \times 22 - 2 \times 21 = 24$$

16. The weighted A.M. of first n natural numbers whose weights are equal to the corresponding numbers, is equal to:

- A. $2n + 1$
- B. $\frac{1}{2} (2n + 1)$
- C. $\frac{1}{3} (2n + 1)$
- D. $\frac{2n+1}{6}$

Sol: C

The required weighted mean is

$$= \left(\frac{x_1 \cdot x_2 \cdot x_n}{y_1 \cdot y_2 \cdot y_n} \right)^{1/n} = \frac{(x_1 \cdot x_2 \dots x_n)^{1/n}}{(y_1 \cdot y_2 \dots y_n)^{1/n}} = \frac{G_1}{G_2} = \frac{1}{3} (2n+1)$$

17. The mean income of a group of workers is \bar{x} and that of another group is \bar{y} . If the number of workers in the second group is 10 times the number of workers in the first group then the mean income of the combined group is:

- A. $\frac{\bar{x}+10\bar{y}}{3}$
B. $\frac{\bar{x}+10\bar{y}}{11}$
C. $\frac{10\bar{x}+\bar{y}}{y}$
D. $\frac{\bar{x}+10\bar{y}}{9}$

Sol: B

Let the series of income (in '000s) in first group be [1, 2] and series of income in second group be [1, 2, 3, 4...20]

$$x = 1.5$$

$$y = 10.5$$

there combined mean = 9.68

which only option 2 satisfies for given set of values

18. The mean of a distribution is 14 and the standard deviation is 5. What is the value of the coefficient of variation?

- A. 60.4%
B. 48.3%
C. 35.7%
D. 27.8%

Sol: C

19. A bus covers first part of its journey of 100 km at a speed of 20 km/hr, second part at a speed of 30 km/hr and third part at a speed of 50 km/hr. Find the approx.. average speed for the whole journey.

- A. 25 km/hr
B. 23 km/hr
C. 33 km/hr
D. None of these

Sol: C

$$= \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \bar{x} = 33 \text{ km / hr.}$$

20. If \bar{X}_1 be the A.M. of n_1 observations, \bar{X} be the A.M. of $(n_1 + n_2)$ observations, what is the A.M. of n_2 observations?

- A. $n_1 / n_2 (\bar{x} - \bar{x}_1) + \bar{x}$
- B. $n_1 / n_2 (\bar{x} + \bar{x}_1) + \bar{x}$
- C. $n_1 / n_2 (\bar{x}_1 - \bar{x}_2)$
- D. none of these

Sol: A

Let \bar{X}_1 be the A.M. of n_2 observations then

$$\bar{X}_2 = \bar{X} \text{ or } (n_1 + n_2)\bar{X}$$

$$= n_1 \frac{70n_1 + 55(150 - n_1)}{150} + n_2 \bar{x}_2$$

$$\text{or } (n_1 + n_2)\bar{x} - n_1 \bar{x}_1 = n_2 \bar{x}_2 \text{ or } \bar{x}_2$$

$$= (n_2 \bar{x} + n_1 \bar{x} - n_1 \bar{x}_1) / n_2, \bar{x}_2 = \frac{n_1}{n_2} (\bar{x} - \bar{x}_1) + \bar{x}$$