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Class: XI
Subject: math's
Topic: Trigonometry
No. of Questions: 20
Duration: 60 Min
Maximum Marks: 60

1. If $\tan A = \frac{1-\cos B}{\sin B}$, then $\tan 2A =$
   A. $\tan\frac{B}{2}$
   B. $\tan B$
   C. $\tan 2B$
   D. None of these

   Ans. B

   Solution:

$$\tan A = \frac{1-\cos B}{\sin B}$$

$$now..\tan 2A = \frac{2\frac{1-\cos B}{\sin B}}{1-\left(\frac{1-\cos B}{\sin B}\right)^2} = \frac{2(1-\cos B)\sin B}{\sin^2 B-(1-\cos B)^2}$$

$$= \frac{2(1-\cos B)\sin B}{2\cos B(1-\cos B)} = \tan B$$

2. The value of $\cos^2 5^o + \cos^2 15^o + \cdots + \cos^2 90^o =$
   A. $9\frac{1}{2}$
   B. $8\frac{1}{2}$
   C. 9
   D. 10

   Ans. B

   Solution:

value of $\cos^2 5^\circ + \cos^2 10^\circ + \cos^2 15^\circ + \ldots + \cos^2 90^\circ =$

$= \cos^2 5 + \cos^2 10 + \ldots + \cos^2 45 + \cos^2 50 + \ldots + \cos^2 85 + 0$

$= \cos^2 5 + \cos^2 10 + \ldots + \cos^2 45 + \sin^2 40 + \sin^2 35 + \ldots + \sin^2 5 + 0$

$= 1+1+1+1+1+1+1+1+(\frac{1}{\sqrt{2}})^2 = 8\frac{1}{2}$

3. $\cos 12^\circ + \cos 84^\circ \cos 132^\circ + 156^\circ =$
   A. 0
   B. $-\sqrt{5}$
   C. $\frac{1}{2}$
   D. None of these

   Ans. C

   Solution:

$\cos 12^\circ + \cos 84^\circ + \cos 132^\circ + \cos 156^\circ$

$= 2\cos 48 \cos 36 + 2\cos 144 \cos 12$

$= 2\cos 48 \cos 36 - 2\cos 36 \cos 12$

$= -2\cos 36[\cos 12 - \cos 48]$

$= -2\cos 36[2\sin 30 \sin 18]$

$= -2\frac{\sqrt{5}+1}{4}2\frac{1}{2}\frac{\sqrt{5}-1}{4} = -\frac{1}{2}$

4. The angles of elevation of the tops of 2 vertical towers as seen from the middle point of the line joining the feet of the towers are 60o and 30o a respectively. The ratio of the heights of the two towers is
   A. $\sqrt{3:1}$
   B. $1:\sqrt{3}$
   C. 3:1
   D. 2:1

   Ans. C

   Solution:

Let the length of the line joining the feet of the towers = 2a units
Let 2 towers be PR,QS

$$\frac{PR}{a} = \tan 60 = \sqrt{3}$$

$$\frac{QS}{a} = \tan 30 = \frac{1}{\sqrt{3}}$$

$$PR : QS = \sqrt{3}a : \frac{a}{\sqrt{3}} = 3 : 1$$

5. The maximum value of 7 sin x+ 24 cos x is
   A. 7
   B. 24
   C. 31
   D. 25
   Ans. D
   Solution:

$$7\sin x + 24\cos x = 25\left(\frac{7}{25}\sin x + \frac{24}{25}\cos x\right)$$

$$\left[\frac{24}{25} = \sin A, then..\cos A = \frac{7}{25}\right]$$

$$= 25(\sin(A + x)) = 25 = \max ..value..when \sin(A + x) = 1$$

6. log sin 1° . log sin 2°. log sin 3° ... log sin 179°
   A. Is positive
   B. Is negative
   C. Lies between 1 and 180
   D. 0
   Ans. D
   Solution:

log sin 1°. log sin 2°. log sin 3° ... log sin 179°
=log sin 1°. log sin 2°….. log sin 90° ... log sin 179°
=log sin 1°. log sin 2°….. log1 ... log sin 179°
=log sin 1°. log sin 2°….. 0 ... log sin 179°=0

7. tan 2A + tan A + tan 3A tan 2A tan A =
   A. 2 tan A
   B. 3 tan A
   C. tan 3A
   D. None of these
   Ans. C

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Solution:

$$\tan 3A = \tan(A + 2A) = \frac{\tan A + \tan 2A}{1 - \tan A \tan 2A}$$

$$\tan 3A = \tan A + \tan 2A + \tan 3A \tan 2A \tan A$$

8. If $(1 + \tan\theta)(1 + \tan f) = 2$, then $\theta + f$ is equal to
   A. 30^o
   B. 45^o
   C. 60^o
   D. 75^o
   Ans. B
   Solution:

$$(1 + \tan\theta)(1 + \tan\phi) = 2$$

$$1 + \tan\theta + \tan\phi + \tan\theta \tan\phi = 2$$

$$\tan\theta + \tan\phi = 1 - \tan\theta \tan\phi$$

$$\frac{\tan\theta + \tan\phi}{1 - \tan\theta \tan\phi} = 1$$

$$\tan(\theta + \phi) = \tan 45$$

then $(\theta + \phi) = 45^o$

9. $\frac{\sin 70^c + \cos 40\text{^}o}{\cos 70^o + \sin 40^o}$
   A. $\sqrt{3}$
   B. $\frac{1}{\sqrt{3}}$
   C. 1
   D. None of these
   Ans.A
   Solution:

$$\frac{\sin\ 70^o + \cos\ 40^o}{\cos\ 70^o + \sin\ 40^o} =$$

$$= \frac{\cos 20^o + \cos\ 40^o}{\sin\ 20^o + \sin\ 40^o} = \frac{2\cos 30 \cos 10}{2\sin\ 30 \cos 10} = \cot 30 = \sqrt{3}$$

10. If $\tan\theta = \frac{1}{\sqrt{2}}$, then the value of $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$ is
    A. 0
    B. 1/3
    C. $-\frac{1}{3}$
    D. none of these
    Ans. B
    Solution:
    $$\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} = \cos 2\theta$$
    $$= \frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} = 1/3$$

11. $\sin\frac{\pi}{18}, \sin\frac{5\pi}{18}, \frac{7\pi}{18}$
    A. 1/16
    B. ¼
    C. 1/8
    D. None of these
    Ans.C
    Solution:
    $$\sin\frac{\pi}{18}.\sin\frac{5\pi}{18}.\sin\frac{7\pi}{18} = \sin 10\sin 50\sin 70$$
    $$= \frac{2\cos 10\sin 10\sin 50\sin(90-20)}{2\cos 10} = \frac{2\sin 20\sin 50\cos 20}{2.2\cos 10} = \frac{2\sin 40\cos 40}{8\cos 10}$$
    $$= \frac{\sin 80}{8\cos 10} = 1/8$$

12. The acute angle in radian between the minute hand and hour hand of a clock, when the time is 4 hours 20 minutes, is
    A. 0
    B. π/18
    C. π/6
    D. π/9
    Ans.B
    Solution:

$360^\circ angles..in...60..min\ s$

$in 20 \min\ s..it..completes...120^\circ angles$

$= \frac{2\pi}{3}$

$hour..hand..completes..360 angle..in..12..hrs$

$in 4hrs 20 \min\ s = 4\frac{20}{60} = \frac{13}{3} hrs..it..completes = \frac{30.13}{3} = 130 = \frac{13\pi}{18}$

$required..angle = \frac{13\pi}{18} - \frac{2\pi}{3} = \frac{\pi}{18}$

13. If $\sqrt{\frac{1+\cos A}{1-\cos A}} = \frac{x}{y}$, then the value of $\tan A =$

A. $\frac{x^2+y^2}{x^2-y^2}$

B. $\frac{2xy}{x^2+y^2}$

C. $\frac{2xy}{x^2-y^2}$

D. $\frac{2xy}{y^2-x^2}$

Ans. C

Solution:

$$\sqrt{\frac{1+\cos A}{1-\cos A}} = \frac{x}{y}$$

$$\cot\frac{A}{2} = \frac{x}{y}$$

$$\tan A = \frac{2\frac{y}{x}}{1-(\frac{y}{x})^2} = \frac{2xy}{x^2-y^2}$$

14. If $\cos x + \cos y = \frac{1}{3}$, and $\sin x + \sin y = \frac{1}{4}$, then $\cos(x+y)$

A. 7/25

B. 24/25

C. 7/24

D. None of these

Ans. A

Solution:

$\cos^2 x + \cos^2 y + 2\cos x \cos y = 1/9$.....(1)

$\sin^2 x + \sin^2 y + 2\sin x \sin y = 1/16$.....(2)

adding..we..get

$$2\cos(x-y) = \frac{-263}{144}$$

(1) – (2)

we..get..

$$2\cos(x+y)[1+\cos(x-y)] = \frac{7}{144}$$

$$\cos(x+y) = \frac{7}{25}$$

15. If $\alpha \in (0, \frac{\pi}{2})$ is such that cot (α/2)- tan(α/2)=2, then tan α =
    A. 1
    B. $\sqrt{2-1}$
    C. $\sqrt{2+1}$
    D. $\sqrt{2}$
    Ans. A
    Solution:

$$\cot(\alpha/2) - \tan(\alpha/2) = 2$$

$$\frac{1-\tan^2\frac{\alpha}{2}}{\tan\frac{\alpha}{2}} = 2$$

now

$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = 1$$

16. *If* $0 \le x \le \pi$ *and* $81^{\sin^2 x} = 81^{\cos^2 x} = 30$, *then* $x =$
    A. π/4
    B. 3π/4
    C. π/6
    D. π/2
    Ans C

Solution:

$81^{\cos^2 x} = 27$

$3^{4\cos^2 x} = 3^3$

$now 4\cos^2 x = 3$

$\cos x = \sqrt{\frac{3}{4}} = \sqrt{3}/2$

$x = \pi/6$

17. If $x = y\cos\frac{2\pi}{3} = z\cos\frac{4\pi}{3}$,they $xy + yz + zx =$

A. 0
B. 1
C. 3
D. None of these

Ans. A

Solution:

$x = y\cos\frac{2\pi}{3} = z\cos\frac{4\pi}{3} = k$

$x = -\frac{y}{2} = -\frac{z}{2} = k$

$xy + yz + zx = k(-2k) + (-2k)(-2k) + (-2k)k = 0$

18. In a Δ PQR, $\angle R = \frac{\pi}{2}$ IF $\tan\frac{p}{2}$ and $\tan\frac{q}{2}$ are the roots of the equation $ax^2 + bx + c = 0$, then.

A. a=b+c
B. b=a+c
C. c=a+b
D. none of these

Ans. C

Solution:

$\tan\frac{P}{2}\tan\frac{Q}{2} = c/a$

$$\tan\frac{P}{2}+\tan\frac{Q}{2}=-b/a$$

$$P+Q=90$$

$$\frac{P+Q}{2}=45$$

$$\tan\left(\frac{P+Q}{2}\right)=\tan 45$$

$$\frac{\tan\frac{P}{2}+\tan\frac{Q}{2}}{1-\tan\frac{P}{2}\tan\frac{Q}{2}}=1$$

$$\frac{-b/a}{1-c/a}=1$$

$$c=a+b$$

19. The value of $\cos 1200^{o}+\tan 1485^{o}$ is
    A. ½
    B. 3/2
    C. $-\frac{3}{2}$
    D. $-\frac{1}{2}$
    Ans. A
    Solution:
    Here cos 1200º + tan 1485º=- sin30+tan45=1/2

20. $3\left[\sin^{4}\left(\frac{3\pi}{2}-\alpha\right)+\sin^{4}(3\pi+\alpha)\right]-2\left[\sin^{6}\left(\frac{\pi}{2}+\alpha\right)+\sin^{6}(5\pi-\alpha)\right]=$
    A. 0
    B. 1
    C. 3
    D. None of these
    Ans.B
    Solution:

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$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$$

$$=3[\sin^4(270-\alpha)+\sin^4(540+\alpha)]-2[\cos^6\alpha+\sin^6\alpha]$$

$$=3[\cos^4\alpha+\sin^4\alpha]-2[(\cos^2\alpha+\sin^2\alpha)^3-3\cos^2\alpha\sin^2\alpha(\cos^2\alpha+\sin^2\alpha)]$$

$$=3[1-2\sin^2\alpha\cos^2\alpha]-2[1-3\cos^2\alpha\sin^2\alpha]=1$$