

Class: 11

Subject: Physics

Topic: Gravitation

No. of Questions: 20

Duration: 60 Min

Maximum Marks: 60

1. Two spherical bodies of mass M and $5M$ and radii R and $2R$ respectively are released in free space with initial separation between their centers equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is
- A. $2.5R$
 B. $4.5R$
 C. $7.5R$
 D. $1.5R$
- Sol: C

(c) : Let the spheres collide after time t , when the smaller sphere covered distance x_1 and bigger sphere covered distance x_2 .

The gravitational force acting between two spheres depends on the distance which is a variable quantity. The gravitational force,

$$F(x) = \frac{GM \times 5M}{(12R - x)^2}$$

$$\text{Acceleration of smaller body, } a_1(x) = \frac{G \times 5M}{(12R - x)^2}$$

$$\text{Acceleration of bigger body, } a_2(x) = \frac{GM}{(12R - x)^2}$$

From equation of motion,

$$x_1 = \frac{1}{2} a_1(x) t^2 \text{ and } x_2 = \frac{1}{2} a_2(x) t^2 \Rightarrow \frac{x_1}{x_2} = \frac{a_1(x)}{a_2(x)} = 5 \Rightarrow x_1 = 5x_2$$



We know that $x_1 + x_2 = 9R$

$$x_1 + \frac{x_1}{5} = 9R$$

$$\therefore x_1 = \frac{45R}{6} = 7.5R$$

Therefore the two spheres collide when the smaller sphere covered the distance of $7.5R$.

2. An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth. Based on above information, answer the following question: The height of the satellite above the surface of the earth is (R is the radius of the earth)
- A. $2/R$
 B. R
 C. $3R/2$
 D. $2R$

Sol: B

$$v_e = \sqrt{2gR}$$

$$v_s = \frac{V_e}{2} = \sqrt{\frac{gR}{2}}$$

The orbital velocity of the satellite is

$$\sqrt{\frac{GM}{(R+h)}} = \sqrt{\frac{gR^2}{R+h}}$$

$$\Rightarrow \sqrt{\frac{gR}{2}} = \sqrt{\frac{gR^2}{R+h}} \text{ or } h = R.$$

3. Newton's law of gravitation is valid for
- A. small bodies only
 B. planets only
 C. both small and big bodies
 D. only valid for solar system

Sol: C

(c): Many students have misconception that Newton's law of gravitation is valid only for big (heavenly) bodies. But this is wrong. Actually gravitational force is the attractive force that one body exerts on another body because of its mass.

Therefore, gravitation exists because of masses of bodies and does not depend on size of the bodies.

4. The question contains statement-1 (Assertion) and Statement-2 (Reason). It has four choices. You have to select the correct choice.

Assertion: A satellite moves around the earth in a circular orbit under the action of gravity.

A person in the satellite experience a zero gravity field in the satellite.

Reason: The contact force by the surface on the person is zero.

- A. if statement-1 is true but statement 2 is false.
 B. if statement-1 is false and statement-2 is true.
 C. if both statement-1 and statement-2 are true and statement-2 is the correct explanation of statement-1.

D. if both statement-1 and statement-2 are true but statement-2 is not the correct explanation of statement-1.

Sol: B

The person experiences zero net force as the force of gravity is balanced by the centrifugal force inside the satellite.

5. The density inside an isolated large solid sphere of radius $a = 4$ km is given by $\rho = \rho_0 \left(1 - \frac{r}{a}\right)$ where ρ_0 is the density at the surface and equals to 10^9 kg/m³ and r denotes the distance from the centre. The gravitational field in m/s² due to this sphere at a distance $2a$ from its centre is (Take $G = 6.65 \times 10^{-11}$ Nm²/kg²)

- A. 418 m/s²
- B. 382 m/s²
- C. 258 m/s²
- D. 948 m/s²

Sol: A

(a) : The gravitational field at the given point is

$$E = \frac{GM}{(2a)^2} - \frac{GM}{(4a)^2} \dots\dots\dots(i)$$

The mass M may be calculated as follows.

Consider a concentric shell of radius r and thickness dr .

Its volume is $dV = (4\pi r^2)dr$

$$\text{and its mass is } dM = \rho dV = \left(\rho_0 \frac{a}{r}\right)(4\pi r^2 dr)$$

$$= 4\pi \rho_0 ar dr.$$

The mass of the whole sphere is

$$M = \int_0^a 4\pi \rho_0 ar dr = 2\pi \rho_0 a^3$$

Thus, by (i) the gravitational field is

$$E = \frac{2\pi G \rho_0 a^3}{4a^2} = \frac{1}{2} \pi G \rho_0 a$$

$$\therefore E = \frac{1}{2} \times \frac{22}{7} \times 6.65 \times 10^{-11} \times 10^9 \times 4 \times 10^3 \times m/s^2$$

$$\Rightarrow E = 418 \text{ m/s}^2$$

6. Acceleration due to gravity g and the mean density ... of the Earth are related by the relation G is the gravitational constant and R_e is the radius of the Earth

A. $\frac{g}{G} = \frac{4f}{3} R_e^3 = \dots$

B. $\frac{g}{G} = \frac{4f}{3} R_e = \dots$

C. $\frac{g}{G} = \frac{4f}{3} R_e^3 = \dots$

D. $\frac{g}{G} = \frac{4f}{3} R_e^3 = \dots$

Sol: B

$$g = \frac{GM}{R_e^2} = \frac{G}{R_e^2} \left(\frac{4}{3} \pi R_e^3 \cdot \rho \right)$$

$$g = \frac{4}{3} \pi R_e \cdot G \cdot \rho \Rightarrow \rho = \frac{g}{\frac{4\pi}{3} \cdot R_e G}$$

7. The question contains statement-1 (Assertion) and Statement-2 (Reason). It has four choices. You have to select the correct choice.
 Assertion: The time period of geostationary satellite is 24 hours.
 Reason: Geostationary satellite must have the same time period as the time taken by the earth to complete one revolution about its axis.
- A. if statement-1 is true but statement 2 is false.
 B. if statement-1 is false and statement-2 is true.
 C. if both statement-1 and statement-2 are true and statement-2 is the correct explanation of statement-1.
 D. if both statement-1 and statement-2 are true but statement-2 is not the correct explanation of statement-1.

Sol: C

As the geostationary satellite is established in an orbit in the plane of the equator at a particular place, so it move in the same sense as the earth and hence its time period of revolution is equal to 24 hours, which is equal to time period of revolution of earth about its axis.

8. An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth. Based on above information, answer the following question: The additional energy required by the satellite to escape the earth is:

- A. $\frac{2GMm}{R}$
 B. $\frac{GMm}{R}$
 C. $\frac{GMm}{2R}$
 D. $\frac{3GMm}{2R}$

Sol: C

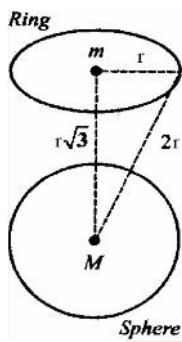
(c) : Energy at h + ΔE = Energy at infinity

$$\Rightarrow \frac{-GMm}{R+h} + \Delta E = 0$$

$$\Delta E = \frac{GMm}{R+h}$$

$$\text{But } h = R \text{ so } \Delta E = \frac{GMm}{2R}$$

9. A uniform ring of mass m and radius r is placed directly above a uniform sphere of mass M and of equal radius. The centre of the ring is directly Ring above the centre of the sphere at a distance $r\sqrt{3}$ as shown in the figure, the change in potential energy is



- A. $\frac{GM_E m}{2R_E}$
 B. $\frac{GM_E m}{4R_E}$
 C. $\frac{GM_E m}{8R_E}$

$$D. \frac{GM_E m}{16R_E}$$

Sol: B

(b) : The change in potential energy is twice the change in the total energy.

$$\Rightarrow U = 2E = 2 \left[\frac{GM_E m}{8R_E} \right] = \frac{GM_E m}{4R_E}$$

10. Two satellites are orbiting around the Earth in circular orbits of the same radius. The mass of satellite A is five times greater than the mass of satellite B. Their periods of revolution are in the ratio

- A. 1: 1
 B. 1: 10
 C. 5: 1
 D. 1: 5

Sol: A

$$T^2 = \left(\frac{4\pi^2}{GM} \right) R^3$$

T is independent of the satellite's mass m

$$\Rightarrow T_A : T_B = 1 : 1$$

11. The question contains statement-1 (Assertion) and Statement-2 (Reason). It has four choices. You have to select the correct choice.

Assertion: The difference in the value of acceleration due to gravity at pole and equator is proportional to square of angular velocity of earth.

Reason: The value of acceleration due to gravity is minimum at the equator and maximum at the pole.

- A. if statement-1 is true but statement 2 is false.
 B. if statement-1 is false and statement-2 is true.
 C. if both statement-1 and statement-2 are true and statement-2 is the correct explanation of statement-1.
 D. if both statement-1 and statement-2 are true but statement-2 is not the correct explanation of statement-1.

Sol: D

(d) : Accelerati on due to gravity, $g' = g - R_e \omega^2 \cos^2 \lambda$

At equator, $\lambda = 0^\circ \therefore \cos 0^\circ = 1$

$$\therefore g_e = g - R_e \omega^2$$

At poles, $\lambda = 90^\circ \therefore \cos 90^\circ = 0$

$$\therefore g_p = g$$

$$\text{Thus, } g_p - g_e = g - g + R_e \omega^2 = R_e \omega^2$$

Also, the value of g is maximum at poles and minimum at equator.

12. A particle is projected from the mid-point of the line joining two fixed particles each of mass m . If the separation between the fixed particles is l , the minimum velocity of projection of the particle so as to escape is equal to

- A. $\sqrt{\frac{Gm}{l}}$
 B. $\sqrt{\frac{Gm}{2l}}$
 C. $\sqrt{\frac{2Gm}{l}}$
 D. $2\sqrt{\frac{2Gm}{l}}$

Sol: D

(d) : The gravitational potential at the mid-point P,

$$V = V_1 + V_2 = \frac{-Gm}{(l/2)} - \frac{Gm}{(l/2)} = \frac{4Gm}{l}$$

\Rightarrow The gravitatio nal potential energy

$$U = -\frac{4Gmm_0}{l},$$

$m_0 =$ mass of particle

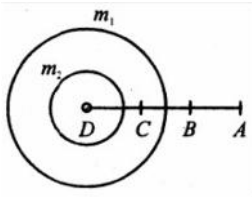
When it is projected with a speed v , it just escapes to infinity, and the potential and kinetic energy will become zero.

$$\Rightarrow \Delta KE + \Delta PE = 0$$

$$\Rightarrow \left(0 - \frac{1}{2} m_0 v^2\right) + \left(-\frac{4Gmm_0}{l}\right) = 0$$

$$\Rightarrow v = 2\sqrt{\frac{2Gm}{l}}$$

13. Figure shows two shells of masses m_1 and m_2 . The shells are concentric. At which point, a particle of mass m shall experience zero force?



- A. A
B. B
C. C
D. D
Sol: D

According to shell theorem, inside a spherical shell the gravitational field due to the shell is zero. Here D is a point inside of both the shells; hence the field due to both the shells will be zero.

14. The question contains statement-1 (Assertion) and Statement-2 (Reason). It has four choices. You have to select the correct choice.

Assertion: Smaller the orbit of the planet around the sun, shorter is the time it takes to complete one revolution.

Reason: According to Kepler's third law of planetary motion, square of time period is proportional to cube of mean distance from sun.

- A. if statement-1 is true but statement 2 is false.
B. if statement-1 is false and statement-2 is true.
C. if both statement-1 and statement-2 are true and statement-2 is the correct explanation of statement-1.
D. if both statement-1 and statement-2 are true but statement-2 is not the correct explanation of statement-1.

Sol: C

According to Kepler's third law of motion, the square of the time period of a planet about the sun is proportional to the cube of the semi major axis of the ellipse or mean distance of the planet from the sun. i.e. $T^2 \propto a^3$, when a is smaller, shorter is the time period.

15. A satellite goes along an elliptical path around earth. The rate of change of arc length a swept by the satellite is proportional to

- A. r
B. r^2
C. $r^{1/2}$

D. r^{-1}

Sol: D

$$\frac{dA}{dt} = \frac{1}{2} r^2 \omega = \text{const} \tan t = k$$

$$\Rightarrow \frac{1}{2} r(r\omega) = k \Rightarrow ur = 2k \Rightarrow v \propto (1/r).$$

16. With what angular velocity the Earth should spin in order that a body lying at 45° latitudes may become weightless?

A. $\sqrt{\frac{g}{R}}$

B. $\sqrt{\frac{2g}{R}}$

C. $2\sqrt{\frac{g}{R}}$

D. None of these

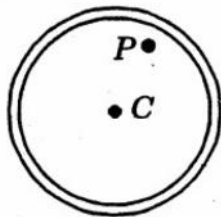
Sol: B

$$g' = g - \omega^2 R \cos^2 \lambda$$

$$\text{if } g' = 0 \text{ at } \lambda = 45^\circ \Rightarrow \omega^2 R \cos^2 45^\circ = g$$

$$\omega^2 R \left(\frac{1}{2}\right) = g \Rightarrow \omega = \sqrt{\frac{2g}{R}}$$

17. The force between a hollow sphere and a point mass at P inside it as shown in the figure



- A. is attractive and constant
 B. is attractive and depends on the position of the point with respect to centre C.
 C. is zero
 D. is repulsive and constant.

Sol: C

Since gravitational field inside hollow sphere is zero. Therefore force acting on the particle p and C are zero.

18. Choose the correct alternative
- A. Acceleration due to gravity increases with altitude
 - B. Acceleration due to gravity increases with increasing depth.
 - C. Acceleration due to gravity is independent of mass of the earth.

D. The formula - $GMm \left[\frac{1}{r_2} - \frac{1}{r_1} \right]$

A. $g(h) = g_o \left(1 - \frac{2h}{R_E} \right)$

B. $g(h) = g_o \left(1 - \frac{d}{R_E} \right)$

C. $g(h) = \frac{GM_e}{r^2}$

D. $mg(r_2 - r_1)$

Sol: D

19. The mass of the Earth is 81 times that of the Moon and the radius of the Earth is 3.5 times that of the Moon. The ratio of the acceleration due to gravity at the surface of the Moon to that at the surface of the Earth is
- A. 0.15
 - B. 0.04
 - C. 1
 - D. 6

Sol: A

$$g_{\text{moon}} = \frac{GM}{R^2}$$

where M and R are mass and radius of moon respective ly.

$$g_{\text{earth}} = \frac{G(81M)}{(3.5R)^2}$$

$$g_{\text{earth}} = 6.61g_{\text{moon}}$$

$$(A) \Rightarrow g_{\text{moon}} = \frac{1}{6.61}g_{\text{earth}} = 0.15g_{\text{earth}}$$

20. The question contains statement-1 (Assertion) and Statement-2 (Reason). It has four choices. You have to select the correct choice.

Assertion: We cannot move even a finger without disturbing all the stars.

Reason: Everybody in this universe attracts every other body with a force which is inversely proportional to the square of distance between them.

- A. if statement-1 is true but statement 2 is false.
- B. if statement-1 is false and statement-2 is true.
- C. if both statement-1 and statement-2 are true and statement-2 is the correct explanation of statement-1.
- D. if both statement-1 and statement-2 are true but statement-2 is not the correct explanation of statement-1.

Sol: B

According to Newton's law of gravitation, everybody in this universe attracts every other body with a force which is inversely proportional to the square of the distance between them. The distance of the finger from the stars is almost infinity for measuring gravitational fields.