

Class: XI
 Subject: Physics
 Topic: Heat and Thermodynamics-B
 No. of Questions: 20
 Duration: 60 Min
 Maximum Marks: 60

Two moles of Helium gas ($\gamma = 5/3$) are initially at temperature 27°C and occupy a volume of 20 liters. The gas is first expanded at constant pressure until the volume is doubled. Then it undergoes an adiabatic change until the temperature returns to its initial value.

1. What is the final volume?

- (A) $113.13 \times 10^{-3} \text{ m}^3$ (B) $213.13 \times 10^{-3} \text{ m}^3$
 (C) $313.13 \times 10^{-3} \text{ m}^3$ (D) $13.13 \times 10^{-3} \text{ m}^3$

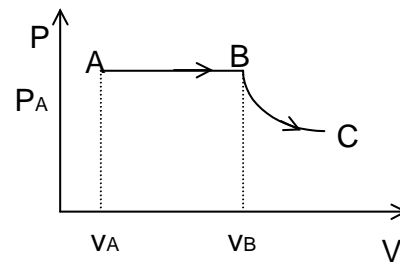
Ans. (A)

Solution:1. From ideal gas equation

$$PV = nRT$$

$$\text{initial pressure } P = \frac{nRT}{V} = \frac{2 \times 8.3 \times 300}{20 \times 10^{-3}}$$

$$= 2.49 \times 10^5 \text{ N/m}^2$$



When volume of gas is doubled at constant pressure, its temperature is also doubled. This process is shown on P-V curve by line AB. The gas then cools to temperature T adiabatically. This is shown by curve BC. The whole process is represented by curve ABC.

At point B, pressure $P_B = P_A = 2.49 \times 10^5 \text{ N/m}^2$. Volume $V_B = 2V_A = 40 \times 10^{-3} \text{ m}^3$, Temperature $T_B = 600\text{K}$.

Now from adiabatic equation $TV^{\gamma-1} = \text{constant}$

We have $T_A V_A^{(\gamma-1)} = T_C V_C^{(\gamma-1)}$

$$\therefore \left(\frac{V_C}{V_B}\right)^{\gamma-1} = \frac{T_B}{T_C} = \frac{600}{300} = 2$$

$$\therefore \frac{V_C}{V_B} = 2^{1/(\gamma-1)} = 2^{3/2}$$

Final volume

$$\begin{aligned}\therefore V_C &= 2\sqrt{2}V_B \\ &= 2 \times 1.414 \times 40 \times 10^{-3} = 113.13 \times 10^{-3} \text{ m}^3\end{aligned}$$

2. What is the final pressures of gas?

- (A) $0.44 \times 10^5 \text{ N/m}^2$ (B) $0.84 \times 10^5 \text{ N/m}^2$
(C) $0.94 \times 10^5 \text{ N/m}^2$ (D) $0.34 \times 10^5 \text{ N/m}^2$

Ans. (a)

final pressure

$$P_C = \frac{nRT_C}{V_C} = \frac{2 \times 8.3 \times 300}{113.13 \times 10^{-3}} = 0.44 \times 10^5 \text{ N/m}^2$$

3. What is the work done by the gas? (Gas constant $R = 8.3 \text{ T/mole K}$)

- (A) 13450 J (B) 14450 J
(C) 16450 J (D) 12450 J

Ans. (d)

The work done by gas in isobaric process AB

$$= 2.49 \times 10^5 \times (40 - 20) \times 10^{-3} = 4980 \text{ J}$$

The work done by gas during adiabatic process BC

$$W_2 = \frac{nR}{1-\gamma} [T_2 - T_1] = \frac{2 \times 8.3}{1 - \left(\frac{5}{3}\right)} [300 - 600] = 7470 \text{ J.}$$

$$\therefore \text{Net work done by gas } W = W_1 + W_2 \\ = 4980 + 7470 = 12450 \text{ J}$$

When 1 gm of water changes from liquid to vapour phase at constant pressure of 1 atmosphere, the volume increases from 1 cm³ to 1671 c.c. The heat of vaporization at this pressure is 540 cal/gm. Find

4. The work done (in J) in change of phase

- | | |
|------------------|------------------|
| (A) 170.78 Joule | (B) 200.67 Joule |
| (C) 190.78 Joule | (D) 168.67 Joule |

Ans. (d)

As the process is isobaric

$$\Delta W = \int P dV = P[V_F - V_i] \\ = 1.01 \times 10^6 [1671 - 1] = 1688.7 \times 10^6 \text{ erg} \\ = 168.67 \text{ Joule} \quad [1 \text{ erg} = 10^{-7}]$$

5. Increase in internal energy of water.

- | | |
|---------------|---------------|
| (A) 2099.33 J | (B) 3099.33 J |
| (C) 4099.33 J | (D) 5099.33 J |

Ans. (a)

From 1st law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$$\Delta Q = mL = 1 \times 540 \text{ cal}$$

$$= 2268 \text{ J}, [1 \text{ cal} = 4.2]$$

$$\text{so, } \Delta U = \Delta Q - \Delta W$$

$$= (2268 - 168.67) \text{ J}$$

$$= 2099.33 \text{ J}$$

6. A blackbody is at a temperature of 527°C. To radiate twice as much energy per second, what temperature must be increased?

(A) 951 K (B) 451 K

(C) 551 K (D) 651 K

Solution: (A) Since $P \propto T^4$, the temperature must be increased to $2^{1/4}(800 \text{ K}) = 951 \text{ K}$.

7. Use Stefan's law to calculate the total power radiated per square meter by a filament at 1727°C having an absorption factor of 0.4.

(A) 0.16 MW/m² (B) 0.26 MW/m²

(C) 0.36 MW/m² (D) 0.46 MW/m²

Solution: (C) Stefan's law gives $R = \epsilon \sigma T^4 = 0.4(5.67 \times 10^{-8}) (2000)^4 = 0.36 \text{ MW/m}^2$

8. A blackbody is at a temperature of 527°C . To radiate twice as much energy per second, how many times of increase in radiated power when the temperature of a blackbody is increased from 7 to 287°C .

- (A) 5 times (B) 16 times
(C) 6 times (D) 20 times

Solution: (B) Since $P \propto T^4$, the temperature must be increased to $2^{1/4}(800\text{ K}) = 951\text{K}$.

$$\text{And } \frac{P(560\text{ K})}{P(280\text{ K})} = \left(\frac{560}{280}\right)^4 = 16 \text{ times}$$

9. The initial and final temperature of water as recorded by an observer are $(40.6 \pm 0.2)^{\circ}\text{C}$ and $(78.3 \pm 0.3)^{\circ}\text{C}$. Calculate the rise in temperature with proper error limit.

- (A) $(27.7 \pm 0.5)^{\circ}\text{C}$ (B) $(17.7 \pm 0.5)^{\circ}\text{C}$
(C) $(37.7 \pm 0.9)^{\circ}\text{C}$ (D) $(37.7 \pm 0.5)^{\circ}\text{C}$

Solution: (D)

$$\text{Let } \theta_1 = 40.6^{\circ}\text{C}, \Delta\theta_1 = \pm 0.2^{\circ}\text{C}$$

$$\theta_2 = 78.3^{\circ}\text{C}, \Delta\theta_2 = \pm 0.3^{\circ}\text{C}$$

$$\Rightarrow \theta = \theta_2 - \theta_1 = 78.3 - 40.6 = 37.7^{\circ}\text{C}$$

$$\& \Delta\theta = \pm (\Delta\theta_1 + \Delta\theta_2) = \pm (0.2 + 0.3) = \pm 0.5^{\circ}\text{C}$$

$$\text{Hence rise in temperature} = (37.7 \pm 0.5)^{\circ}\text{C}$$

10. A cylinder of radius R made of a material of thermal conductivity k_1 is surrounded by a cylindrical sheet of inner radius R and outer radius $2R$ made of material of thermal conductivity k_2 . The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. Calculate the effective thermal conductivity of the system.

(A) $4K = 2K_1 + 3K_2$

(B) $4K = 5K_1 + 3K_2$

(C) $4K = 6K_1 + 4K_2$

(D) $4K = K_1 + 3K_2$

Solution: (D)

Two cylinders are in parallel, therefore equivalent thermal resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

But $R = \frac{\lambda}{kA}$

$$\therefore \frac{kA}{\lambda} = \frac{k_1 A_1}{\lambda_1} + \frac{k_2 A_2}{\lambda_2}$$

Here $\lambda_1 = \lambda_2 = \lambda, A_1 = \pi R^2$

$$A_2 = \pi(2R)^2 - \pi R^2 = 3\pi R^2$$

and $A = \pi(2R)^2 = 4\pi R^2$

$$\therefore \frac{K4\pi R^2}{\lambda} = \frac{K_1\pi R^2}{\lambda} + \frac{K_2 3\pi R^2}{\lambda}$$

i.e. $4K = K_1 + 3K_2$

11. If an anisotropic solid has coefficients of linear expansion α_x , α_y and α_z for three mutually perpendicular directions in the solid, what is the coefficient of volume expansion for the solid?

- (A) $\beta \approx \alpha_x + \alpha_y + \alpha_z$ (B) $\beta \approx \alpha_x^2 + \alpha_y + \alpha_z$
(C) $\beta \approx \alpha_x^2 + \alpha_y^2 + \alpha_z^2$ (D) $\beta \approx \alpha_x^3 + \alpha_y^3 + \alpha_z^3$

Solution: (A)

Consider a cube, with edges parallel to X, Y, Z of dimension L_0 at $T = 0$. After a change in temperature $\Delta T = (T - 0)$, the dimensions change to

$$L_x = L_0 (1 + \alpha_x T) \qquad L_y = L_0 (1 + \alpha_y T) \qquad L_z = L_0 (1 + \alpha_z T)$$

And the volume of the parallelepiped is

$$V = V_0 (1 + \alpha_x T) (1 + \alpha_y T) (1 + \alpha_z T) \approx V_0 [1 + (\alpha_x + \alpha_y + \alpha_z) T]$$

Where $V_0 = L_0^3$. Therefore, the coefficient of volume expansion is given by $\beta \approx \alpha_x + \alpha_y + \alpha_z$.

12. In aluminum sheet there is a hole of diameter 2m and is horizontally mounted on a stand. Onto this hole an iron sphere of radius 2.004 m is resting. Initial temperature of this system is 25° C. Find at what temperature, the iron sphere will fall down through the hole in sheet. The coefficients of linear expansion for aluminum and iron are 2.4×10^{-4} and 8.6×10^{-5} respectively.

- (A) 82°C (B) 43°C
(C) 45°C (D) 15°C

Solution: (B)

As value of coefficient of linear expansion for aluminum is more than that for iron, it expands faster than iron. So at some higher temperature

when diameter of hole will exactly become equal to that of iron sphere, the sphere will pass through the hole. Let it happen at some higher temperature T. Thus we have at this temperature T,

$$(\text{Diameter of hole})_{Al} = (\text{diameter of sphere})_{iron}$$

$$2[1 + \alpha_{Al} (T - 25)] = 2.004[1 + \alpha_{iron} (T - 25)]$$

$$2\alpha_{Al} (T - 25) = 0.004 + 2.004 \alpha_{iron} (T - 25)$$

$$\text{Or } T = \left(\frac{0.004}{2\alpha_{Al} - 2.004\alpha_{iron}} + 25 \right) \text{ } ^\circ\text{C}$$

$$\text{Or } T = \frac{0.004}{2 \times 2.4 \times 10^{-4} - 2.004 \times 8.6 \times 10^{-5}} + 25$$

$$\text{Or } T = 43^\circ\text{C}$$

13. An iron ball has a diameter of 6 cm and is 0.010 mm too large to pass through a hole in a brass plate when the ball and plate are at a temperature of 30°C . At what temperature, the same for ball and plate, will the ball just pass through the hole?
- (A) 23.8°C (B) 13.8°C
 (C) 53.8°C (D) 83.8°C

Solution: (C)

We let I stand for the iron ball and B stand for the brass plate. $L_I = 6$ cm and $L_I - L_B = 0.001$ cm at $t = 30^\circ\text{C}$. Since the brass plate expands uniformly, the hole must expand in the same proportion. Then heating both the ball and the plate leads to increases in the diameters of the ball and the hole, with the hole increasing more, since $\alpha_B > \alpha_I$. We require $\Delta L_B - \Delta L_I = 0.001$ cm. $\Delta L_B = \alpha_B L_B \Delta t$; $\Delta L_I = \alpha_I L_I \Delta t$. We can approximate L_B in this formula by 6 cm = L_I . Then

$$\Delta L_B - \Delta L_I = (\alpha_B - \alpha_I) L_I \Delta t = 0.001 \text{ cm}$$

Solving, $\Delta t = 23.8^\circ\text{C}$, and finally $t = 30^\circ\text{C} + 23.8^\circ\text{C} = 53.8^\circ\text{C}$

14. It is desired to put an iron rim on a wooden wheel. The diameter of the wheel is 1.1000m and the inside diameter of the rim is 1.0980 m. If the rim is at 20°C initially, to what temperature must it be heated to just fit onto the wheel?

(A) 52°C

(B) 72°C

(C) 102°C

(D) 152°C

Solution: (D)

α For iron is found to be $1.2 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$.

$$\Delta L = 1.1000 - 1.0980 = 0.0020 \text{ m} = \alpha L \Delta t$$

$$0.0020 = (1.2 \times 10^{-5}) (1.098) \Delta t$$

$$\Delta t + 20 = 172^\circ\text{C}$$

$$\Delta t = 152^\circ\text{C}$$

15. Find the coefficient of volume expansion for an ideal gas at constant pressure.

(A) $\gamma = \frac{1}{T}$

(B) $g = T$

(C) $g = \frac{1}{T^2}$

(D) $g = \frac{1}{T^3}$

Solution: (A)

For an idea gas $PV = nRT$

As P is constant, we have

$$P \cdot dV = nRdT$$

$$\frac{dV}{dT} = \frac{nR}{P}$$

$$\gamma = \frac{1}{V} \frac{dV}{dT} = \frac{nR}{PV} = \frac{nR}{nRT} = \frac{1}{T}$$

$$\gamma = \frac{1}{T}$$

16. What should be the lengths of steel and copper rod so that the length of steel rod is 5cm longer than the copper rod at all the temperatures? Coefficients of linear expansion for copper and steel are 1.7 and 1.1

- (A) 2.17, 14.17 cm (B) 9.17, 14.17 cm
 (C) 9.17, 18.17 cm (D) 3.17, 5.17 cm

Solution: (B)

It is given that the difference in length of the two rods is always 5 cm. Thus the expansion in both the rods must be same for all temperatures. Thus we can say that at all temperature differences, we have

$$\Delta L_{Cu} = \Delta L_{steel}$$

Or $\alpha_{Cu} l_1 \Delta t = \alpha_{st} l_2 \Delta t$ [If l_1 and l_2 are the initial lengths of Cu and steel rods]

$$\text{Or } \alpha_{Cu} l_1 = \alpha_{st} l_2$$

$$\text{Or } 1.7 l_1 = 1.1 l_2 \quad \dots (1)$$

$$\text{It is given that } l_2 - l_1 = 5\text{cm} \quad \dots (2)$$

$$\left(\frac{1.7}{1.1} - 1 \right) l_1 = 5\text{cm}$$

Or $l_1 = \frac{5 \times 1.1}{0.6} = 9.17 \text{ cm}$

Now from equation (2) $l_2 = 14.17 \text{ cm}$

17. A steel wire of cross-sectional area 0.5 mm^2 is held between two rigid clamps so that it is just taut at 20°C . Find the tension in the wire at 0°C . Given that Young's modulus of steel is $Y_{\text{st}} = 2.1 \times 10^{12} \text{ dynes / cm}^2$ and coefficient of linear expansion of steel is $\alpha_{\text{st}} = 1.1 \times 10^{-5} \text{ }^\circ \text{C}^{-1}$.

- (A) 2.31×10^{-4} (B) 2.31×10^{-2}
 (C) 9.31×10^{-6} (D) 2.31×10^{-6}

Solution: (D)

We know that due to drop in temperature, then tension increment in a clamped wire is

$$T = YA \alpha \Delta T = 2.1 \times 10^{12} \times 0.5 \times 10^{-2} \times 1.1 \times 10^{-5} \times 20 = 2.31 \times 10^{-6}$$

18. Two bodies have the same heat capacity. If they are combined to form a single composite body, show that the equivalent specific heat of this composite body is independent of the masses of the individual bodies.

- (A) $\frac{2s_1s_2}{s_2 - s_1}$ (B) $\frac{s_1s_2}{s_2 + s_1}$
 (C) $\frac{2s_1s_2}{s_2 + s_1}$ (D) $\frac{s_2}{s_2 + s_1}$

Solution: (C)

Let the two bodies have masses m_1 , m_2 and specific heats s_1 and s_2 then

$$m_1 s_1 = m_2 s_2 \quad \text{or} \quad m_1/m_2 = s_2/s_1$$

Let s = specific heat of the composite body.

$$\text{Then } (m_1 + m_2) s = m_1 s_1 + m_2 s_2 = 2m_1 s_1$$

$$s = \frac{2m_1 s_1}{m_1 + m_2} = \frac{2m_1 s_1}{m_1 + m_1(s_1/s_2)} = \frac{2s_1 s_2}{s_2 + s_1}$$

19. 20 gm steam at 100°C is let into a closed calorimeter of water equivalent 10 gm containing 100 gm ice at -10°C . Find the final temperature of the calorimeter and its contents. Latent heat of steam is 540 cal/gm, latent heat of fusion of ice = 80 cal/gm, specific heat of ice = 0.5 cal/ $^\circ\text{C}$ gm.

- (A) 13°C (B) 63°C
 (C) 93°C (D) 33°C

Solution: (D)

$$\text{Heat lost by steam} = mL + ms(100 - \theta)$$

Where, θ is the equilibrium temperature

$$\begin{aligned} \text{Heat lost by steam} &= 20 \times 540 + 20 \times 1(100 - \theta) \\ &= 10800 + 2000 - 20\theta \end{aligned}$$

$$\text{Heat gained by (ice + calorimeter)} = 100 \times 80 + 100 \times 0.5 \times 10 + 100 \times \theta$$

$$\text{Now} \quad \text{Heat lost} = \text{Heat gained}$$

$$\therefore 10800 + 2000 - 20\theta = 8000 + 500 + 110\theta$$

$$\text{Or} \quad 1300 = 4300$$

$$\text{Or} \quad \theta = \frac{4300}{130} = 33^\circ\text{C}.$$

20. Victoria Falls in Africa is 122 m in height. Calculate the rise in temperature of the water if all the potential energy lost in the fall is converted to heat.

- (A) 0.29 K (B) 29 K
(C) 0.69 K (D) 0.99 K

Solution: (A)

Consider mass m of water falling.

$$Mgy = mc \Delta t \quad gy = c \Delta t$$

We express both sides in joules by noting

$$c = 1 \text{ kcal/kg} \cdot \text{K} = 4184 \text{ J/kg} \cdot \text{K}$$

$$\text{Then } 9.8(122) = 4184 \Delta t$$

$$\text{And } \Delta t = 0.29 \text{ K}$$

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