Class: 11
Subject: Math's
Topic: Binomial Theorem
No. of Questions: 25

Q1. If the coefficients of x^7 and x^8 in $\left(2+\frac{x}{3}\right)^n$ are equal, then value of n is

A. 56
B. 55
C. 47
D. 19

Sol. B

T_{r+1}, the $(r+1)$th term in the expansion of $\left(2+\frac{x}{3}\right)^n$ is given by

$$T_{r+1} = {}^nC_r\, 2^{n-r}\left(\frac{x}{3}\right)^r = {}^nC_r\left(\frac{2^{n-r}}{3^r}\right)x^r$$

According to the given condition

$${}^nC_7\left(\frac{2^{n-7}}{3^7}\right) = {}^nC_8\left(\frac{2^{n-8}}{3^8}\right) \Rightarrow \frac{{}^nC_7}{{}^nC_8} = \frac{2^{n-8}}{3^8}\cdot\frac{3^7}{2^{n-7}}$$

$$\Rightarrow \frac{n!}{7!\,(n-7)!}\,\frac{8!\,(n-8)!}{n!} = \frac{1}{6} \Rightarrow \frac{8}{n-7} = \frac{1}{6}$$

$$\Rightarrow 48 = n-7 \Rightarrow n = 55$$

Q2. If a is real and the 4th term in the expansion of $\left(ax + \frac{1}{x}\right)^n$ is 5/2, for each x ∈ R – {0}, then the values of a and n, respectively, are

A. 5, 1/2
B. 6, – 1/2
C. 3, 1/3
D. 6, ½

Sol. D

We have

$$T_4 = T_{3+1} = {}^nC_3\,(ax)^{n-3}\left(\frac{1}{x}\right)^3$$

$$= {}^nC_3\,a^{n-3}x^{n-6} = 5/2$$

As this is true for each $x \in \mathbf{R} - \{0\}$, we get

$$n - 6 = 0 \text{ and } {}^nC_3\,a^{n-3} = 5/2$$

$$\Rightarrow \quad n = 6 \text{ and } {}^6C_3\,a^3 = 5/2$$

$$\therefore \quad a^3 = \frac{5}{2}\times\frac{3!\,3!}{6!} = \frac{5}{2}\times\frac{1}{20} = \frac{1}{8}$$

$$\Rightarrow \quad a = 1/2.$$

Thus, $n = 6, a = 1/2.$

Q3. If (r + 1)th term in the expansion of $\left(\frac{a^{1/3}}{b^{1/6}} + \frac{b^{1/2}}{a^{1/6}}\right)^{21}$ has equal exponents of both a and b, then the value of r is

A. 8
B. 9
C. 10
D. 11

Sol. B

We have

$$T_{r+1} = {}^{21}C_r \left(\frac{a^{1/3}}{b^{1/6}}\right)^{21-r} \left(\frac{b^{1/2}}{a^{1/6}}\right)^r$$

$$= {}^{21}C_r \frac{a^{7-r/3}}{b^{7/2-r/6}} \cdot \frac{b^{r/2}}{a^{r/6}} = {}^{21}C_r a^{7-r/2} b^{2r/3-7/2}$$

Since exponents of a and b in the $(r+1)$th term are equal,

$$7 - \frac{r}{2} = \frac{2r}{3} - \frac{7}{2} \Rightarrow \frac{21}{2} = \frac{7}{6} r \Rightarrow r = 9.$$

Q4. If x^{2k} occurs in the expansion of $\left(x + \frac{1}{x^2}\right)^{n-3}$, then

A. n – 2k is a multiple of 2
B. n – 2k is a multiple of 3
C. k = 0
D. none of these

Sol. B

T_{r+1} the $(r+1)$th term in the expansion of $\left(x + \frac{1}{x^2}\right)^{n-3}$ is given by

$$T_{r+1} = {}^{n-3}C_r (x)^{n-3-r} \left(\frac{1}{x^2}\right)^r = {}^{n-3}C_r x^{n-3-3r}$$

As x^{2k} occurs in the expansion of $\left(x + \frac{1}{x^2}\right)^{n-3}$, we must have $n - 3 - 3r = 2k$ for some non-negative integer r.

$\Rightarrow \quad 3(1+r) = n - 2k \quad \Rightarrow \quad n - 2k$ is a multiple of 3.

Q5. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, then the value of $C_0 + 2C_1 + 3C_2 + \ldots + (n + 1)C_n$ is

A. 2^{n-1}
B. $n(2^{n-1})$
C. $n(2^{n-1}) + 2^n$
D. $(n + 1)2^n$

Sol. C

Let E be $C_0 + 2C_1 + 3C_2 + \ldots + nC_{n-1} + (n + 1)C_n$. (1)
Using $C_r = C_{n-r}$, we can rewrite (1) as
$E = (n + 1) C_0 + nC_1 + (n - 1)C_2 + \ldots + 2C_{n-1} + C_n$ (2)
Adding (1) and (2), we get
$2E = (n + 2)C_0 + (n + 2)C_1 + (n + 2)C_2 + \ldots + (n + 2)C_n$
$= (n + 2) \{C_0 + C_1 + \ldots + C_n\} = (n + 2)2^n$
$\Rightarrow \quad E = (n + 2)2^{n-1}$

Alternative Solution:

We have
$C_0 x + C_1 x^2 + C_2 x^3 + \ldots + C_n x^{n+1} = x (1 + x)^n$
Differentiating both the sides, we get
$C_0 + 2C_1 x + 3C_2 x^2 + \ldots + (n + 1)C_n x^n$
$= (1 + x)^n + nx(1 + x)^{n-1}$ (1)
Putting x = 1, we get
$C_0 + 2C_1 + 3C_2 + \ldots + (n + 1)C_n$
$= 2^n + n(1) 2^{n-1} = (n + 2)2^{n-1}$

Q6. If $n \in N$, $n > 1$, then value of $E = a - {}^nC_1 (a - 1) + {}^nC_2 (a - 2) + \ldots + (-1)^n (a - n) ({}^nC_n)$ is

A. a
B. 0
C. a^2
D. 2^n

Sol. B

We can write E as
$a [{}^nC_0 - {}^nC_1 + {}^nC_2 - \ldots + (-1)^n ({}^nC_n)] + [{}^nC_1 - (2) ({}^nC_2) + (3) ({}^nC_3) - \ldots - (-1)^n (n) ({}^nC_n)]$
= 0 + F where
$F = {}^nC_1 - (2) ({}^nC_2) + (3) ({}^nC_3) - \ldots - (-1)^n (n) ({}^nC_n)$
We have $(1 + x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + \ldots + {}^nC_n x^n$
Differentiating, we get
$n(1 + x)^{n-1} = {}^nC_1 + 2({}^nC_2)x + 3({}^nC_3)x^2 + \ldots + n({}^nC_n)x^{n-1}$
Putting x = – 1, we get
$0 = {}^nC_1 - 2({}^nC_2) + 3({}^nC_3) - \ldots - (-1)^{n-1} (n)({}^nC_n)$
$\Rightarrow \quad 0 = F.$
Thus, E = 0 + 0 = 0.

Q7. For n $\geq$ 2, let $a_n = \sum_{r=0}^{n} \frac{1}{C_r^2}$, then value of $b_n = \sum_{r=1}^{n} \frac{1}{r^2\, C_r^2}$ equals

A. $\frac{1}{n^2} a_n$

B. $\frac{1}{n^2} a_{n-1}$

C. a_n

D. a_n^2

Sol. B

For r $\geq$ 1, r C_r = r (${}^nC_r$) = n (${}^{n-1}C_{r-1}$)

Thus, $b_n = \sum_{r=1}^{n} \frac{1}{n^2\,({}^{n-1}C_r)^2} = \frac{1}{n^2} a_{n-1}$

Q8. Sum of the coefficients of x^3 and x^6 in the expansion of $\left(x^2 - \frac{1}{x}\right)^9$ is

Sol.

$$T_{r+1} = {}^9C_r\,(x^2)^{9-r}\left(-\frac{1}{x}\right)^r = (-1)^r\,({}^9C_r)x^{18-3r}$$

For coefficient of x^3, x^6, we set $18 - 3r = 3,6$

$\Rightarrow \quad r = 5,4.$

$\therefore$ sum of coefficients of x^3 and x^6

$$= (-1)^5\,({}^9C_4) + (-1)^4\,({}^9C_5) = 0$$

Q9. The coefficient of x^n in the expansion of $\left(1+\frac{1}{1!}x+\frac{1}{2!}x^2..+\frac{1}{n!}x^n\right)^2$ is

A. $\frac{2^n}{n!}$
B. $\frac{2^n}{n}$
C. n!
D. $\frac{1}{n!}$

Sol. A

Coefficient of x^n in

$$\left(1+\frac{1}{11}x+\frac{1}{2!}x^2+.\cdots+\frac{1}{n!}x^n\right)$$

$$\left(1+\frac{1}{11}x+\frac{1}{2!}x^2+\cdots+\frac{1}{n!}x^n\right)$$

$$=\frac{1}{n!}+\frac{1}{1!(n-1)!}+\frac{1}{2!(n-2)!}+.\cdots+$$

$$\frac{1}{(n-1)!1!}+\frac{1}{n!}$$

$$=\frac{1}{n!}\left[{}^nC_0+{}^nC_1+{}^nC_2+\cdots{}^nC_{n-1}+{}^nC_n\right]=\frac{2^n}{n!}$$

Q10. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is

A. 132
B. 144
C. 132
D. 144

Sol. D

$(1 - x - x^2 + x^3)^6 = (1 - x)^6 (1 - x^2)^6$

Coefficient of x^7 in $(1 - x)^6 (1 - x^2)^6$

$= \text{Coefficient of } x^7 \text{ in } [1 - {}^6C_1 x + {}^6C_2 x^2 - + {}^6C_6 x^6]$

$\times\ [1 - {}^6C_1 x^2 + {}^6C_2 x^4 - + {}^6C_6 x^{12}]$

$= (-{}^6C_1)(-{}^6C_3) + (-{}^6C_3)({}^6C_2) + (-{}^6C_5)(-{}^6C_1)$

$= -144$

Q11. Suppose F is the fractional part of M = $(\sqrt{13} + \sqrt{11})^6$, then value of M (1 – F) is

A. 128
B. 64
C. 32
D. 16

Sol. B

Let $N = (\sqrt{13} - \sqrt{11})^6$,

As $(\sqrt{13} - \sqrt{11}) = \dfrac{2}{\sqrt{13} + \sqrt{11}}$, we get $0 < N < 1$

Also, $M + N = (\sqrt{13} + \sqrt{11})^6 + (\sqrt{13} - \sqrt{11})^6$,

$$= 2\Big[{}^6C_0 (\sqrt{13})^6 + {}^6C_2 (\sqrt{13})^4 (\sqrt{11})^2 + {}^6C_4 (\sqrt{13})^2 (\sqrt{11})^4 + {}^6C_6 (\sqrt{11})^6\Big]$$

$= M + N$ is an integer, say, J

Let $M = K + F$, where K is the greatest integer contained in M.

We have

$$J = M + N = K + N + F$$

As $0 < N < 1$, $0 < F < 1$ we get, $0 < N + F < 2$

Also, $N + F$ is an integer

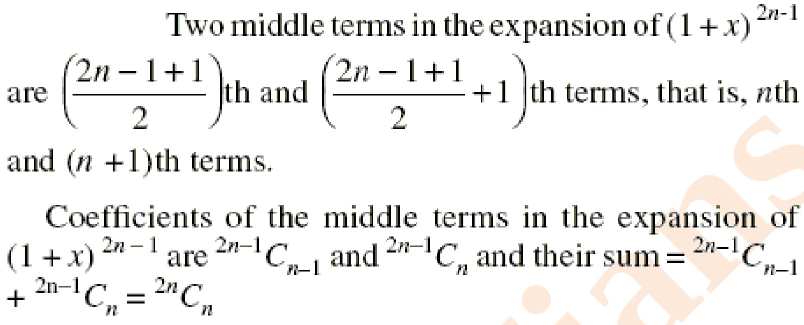
$\Rightarrow \quad N + F = 1 \Rightarrow 1 - F = N$

Thus, $M(1 - F) = (\sqrt{13} + \sqrt{11})^6 (\sqrt{13} - \sqrt{11})^6 = 2^6 = 64$

Q12. Sum of two middle terms in the expansion of $(1 + x)^{2n-1}$ is

A. ${}^{2n-1}C_{n-1}$
B. ${}^{2n-1}C_n$
C. ${}^{2n}C_n$
D. ${}^{2n}C_{n+1}$

Sol. C



Two middle terms in the expansion of $(1+x)^{2n-1}$ are $\left(\frac{2n-1+1}{2}\right)$th and $\left(\frac{2n-1+1}{2}+1\right)$th terms, that is, nth and $(n+1)$th terms.

Coefficients of the middle terms in the expansion of $(1+x)^{2n-1}$ are ${}^{2n-1}C_{n-1}$ and ${}^{2n-1}C_n$ and their sum $= {}^{2n-1}C_{n-1} + {}^{2n-1}C_n = {}^{2n}C_n$

Q13. What will be the sixth term in the expansion of $(x^{1/3} + y^{1/2})^n$, if the binomial coefficient of the third term from the end is 45?

A. $252\, x^{5/3}\, y^{5/2}$
B. 45
C. $45\, x^{5/3}\, y^{5/2}$
D. None of these

Sol. A

Binomial coefficients of the third term from the end in the expansion of $(x^{1/3} + y^{1/2})^n$, is
${}^nC_2 = 45 \Rightarrow n(n-1) - 90 = 0$
$\Rightarrow n = 10$
$\therefore$ Sixth term in the expansion of $(x^{1/3} + y^{1/2})^n$ is
${}^nC_5\, (x^{1/3})^{n-5}\, (y^{1/2})^5 = 252\, x^{5/3}\, y^{5/2}$

Q14. Coefficient of $\frac{1}{x}$ in the expansion of $(1 + x)^n (1 + 1/x)^n$ is

A. ${}^{2n}C_{n-1}$
B. ${}^{2n}C_n$
C. 1
D. 0

Sol. A

We have

$$(1+x)^n\left(1+\frac{1}{x}\right)^n = \frac{(1+x)^{2n}}{x^n}$$

$$\therefore \text{ Coefficient of } x^{-1} \text{ in } (1+x)^n\left(1+\frac{1}{x}\right)^n$$

$$= \text{Coefficient of } x^{n-1} \text{ in } (1+x)^{2n}$$

$$= {}^{2n}C_{n-1}$$

Q15. The last term in the binomial expansion of $\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)^n$ is $\left(\frac{1}{3.9^{\frac{1}{3}}}\right)^{\log_3 8}$, then the 5th term from the beginning is

A. ${}^{10}C_6$
B. $2({}^{10}C_4)$
C. $\frac{1}{2}({}^{10}C_4)$
D. $-{}^{10}C_6$

Sol. B

$$ {}^{n}C_{n}\left(-\frac{1}{\sqrt{2}}\right)^{n} = \left(3^{-1-2/3}\right)^{\log_3{}^{8}} = 3^{-5\log_3{}^{2}} = \frac{1}{2^5} $$

$$ \Rightarrow \qquad n = 10. $$

$\therefore$ 5th term from the beginning = ${}^{10}C_4\,(\sqrt{2})^{10-4}$

$$ \left(-\frac{1}{\sqrt{2}}\right)^{4} = 2({}^{10}C_4) $$

Q16. Let S (K) be1 + 3 + 5 + … + (2K – 1) = 3 + K^2. Which of the following is true?

A. S (K) $\nRightarrow$ S (K + 1)
B. S (K) $\Rightarrow$ S (K + 1)
C. S (1) is correct.
D. Principle of mathematical induction can be used to prove the formula.

Sol. B

S (1) : 1 = 3 + $1^{2,}$ which is not true.
Suppose S(K) is true, then
1 + 3 + 5 + … + (2K – 1) = 3 + K^2
Adding (2K + 1) to both the sides, we get
1 + 3 + 5 + … + (2K – 1) + (2K + 1) = 3 + K^2 + 2K + 1 = 3 + $(K + 1)^2$
which is S (K + 1).
Thus, S (K) $\Rightarrow$ S (K + 1)

Q17. The interval in which x (> 0) must lie so that the greatest term in the expansion of $(1 + x)^{2n}$ has
the greatest coefficient is

A. $\left(\frac{n-1}{n}, \frac{n}{n-1}\right)$
B. $\left(\frac{n}{n+1}, \frac{n+1}{n}\right)$
C. $\left(\frac{n}{n+2}, \frac{n+2}{n}\right)$
D. none of these

Sol. B

The greatest coefficient in the expansion of $(1+x)^{2n}$ is ${}^{2n}C_n$. We are given ${}^{2n}C_n\,x^n$ is the greatest term.

$$\therefore \quad {}^{2n}C_{n-1}\,x^{n-1} < {}^{2n}C_n\,x^n \text{ and}$$

$$ {}^{2n}C_{n+1}\,x^{n+1} < {}^{2n}C_n\,x^n$$

$$\Rightarrow \quad \frac{{}^{2n}C_{n-1}}{{}^{2n}C_n} < x < \frac{{}^{2n}C_n}{{}^{2n}C_{n+1}}$$

$$\Rightarrow \quad \frac{n}{n+1} < x < \frac{n+1}{n}.$$

Q18. Sum of the series $S = 3^{-1}\,({}^{10}C_0) - {}^{10}C_1 + (3)({}^{10}C_2) - 3^2({}^{10}C_3) + \ldots + 3^9\,({}^{10}C_{10})$ is

A. 2^9
B. $2^{10} - 1$
C. $\frac{1}{3}(2^{11} - 2)$
D. $\frac{1}{3}(2^{10})$

Sol. D

We have

$$S = 3^{-1}\,[{}^{10}C_0 - {}^{10}C_1\,(3) + {}^{10}C_2\,(3^2) - {}^{10}C_3\,(3^3) + \ldots + {}^{10}C_{10}(3^{10})]$$

$$= \frac{1}{3}\,(1-3)^{10} = \frac{1}{3}\,(2^{10})$$

Q19. Let $(1 + x)^{3n}$ be $C_0 + C_1x + C_2x^2 + \ldots + C_{3n}x^{3n}$, and $\omega \neq 1$ be a cube root of unity.
Statement-1: $C_0 + C_1\omega + C_2\omega^2 + C_3 + C_4\omega + C_5\omega^2 + \ldots = (-1)^n$
Statement-2: Cube roots unity form a triangle of area 3 square units.

A. Statement – 1 is true, statement – 2 is true and statement – 2 is the correct explanation for statement – 1.
B. Statement – 1 is true, statement – 2 is true and statement – 2 is not the correct explanation for Statement – 1.
C. Statement – 1 is true, statement – 2 is false.
D. Statement – 1 is false, statement – 2 is true.

Sol. C

$$C_0 + C_1\omega + C_2\omega^2 + C_3 + C_4\omega + C_5\omega^2 + \cdots$$
$$= \sum_{k=0}^{3n} C_k\,\omega^k = (1+\omega)^{3n}$$
$$= (-\omega^2)^{3n} = (-1)^{3n}\,\omega^{6n}$$
$$= (-1)^n\,(1) = (-1)^n.$$

Statement-2 is false as area of triangle formed by cube roots of unity is $3\sqrt{3}/4$ square units.

Q20. Let $(1 + x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$.

Statement - 1: For m $\geq$ 2, $C_0 - C_1 + C_2 - \ldots + (-1)^{m-1} C_{m-1} = (-1)^{m-1} ({}^{n-1}C_{m-1})$

Statement - 2: ${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$ for $1 \leq r \leq n - 1$

A. Statement – 1 is true, statement – 2 is true and statement – 2 is the correct explanation for statement – 1.
B. Statement – 1 is true, statement – 2 is true and statement – 2 is not the correct explanation for Statement – 1.
C. Statement – 1 is true, statement – 2 is false.
D. Statement – 1 is false, statement – 2 is true.

Sol. A

$$C_0 = {}^{n-1}C_0$$
$$-C_1 = -{}^{n-1}C_0 - {}^{n-1}C_1$$
$$C_2 = {}^{n-1}C_1 + {}^{n-1}C_2$$
$$-C_3 = -{}^{n-1}C_2 - {}^{n-1}C_3$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$(-1)^{m-1}(C_{m-1}) = (-1)^{m-1}({}^{n-1}C_{m-2} + {}^{n-1}C_{m-1})$$

Adding the above equations, we get

$$C_0 - C_1 + C_2 - C_3 + \cdots + (-1)^{m-1}C_{m-1}$$
$$= (-1)^{m-1}\,({}^{n-1}C_{m-1})$$

Q21. Let $(1 + t)^n = C_0 + C_1t + C_2t^2 + \ldots + C_nt^n$.

$$\frac{C_0}{1.2} + \frac{C_1}{2.3} + \frac{C_2}{3.4} + \ldots + \frac{C_n}{(n+1)(n+2)} = \frac{1}{n+1}\left[\frac{2^{n+2}}{n+2} - 1\right]$$

$$\frac{C_0}{2} - \frac{C_1}{3} + \frac{C_2}{4} - \ldots + (-1)^n \frac{C_n}{n+2} = 0$$

Statement - 1:

Statement - 2:

A. Statement – 1 is true, statement – 2 is true and statement – 2 is the correct explanation for statement – 1.

B. Statement – 1 is true, statement – 2 is true and statement – 2 is not the correct explanation for Statement – 1.

C. Statement – 1 is true, statement – 2 is false.

D. Statement – 1 is false, statement – 2 is true.

Sol. B

From (1)

$$\int_0^x (1+t)^n \, dt = \int_0^x [C_0 + C_1 t + C_2 t^2 + \cdots + C_n t^n] \, dt$$

$$\Rightarrow \quad \frac{1}{n+1}[(1+x)^{n+1} - 1]$$

$$= \frac{C_0}{1}x + \frac{C_2}{2}x^2 + \cdots + \frac{C_n}{n+1}x^{n+1} \qquad (2)$$

Multiplying (1) by t and integrating, we get

$$\int_0^x t(1+t)^n \, dt = \int_0^x [C_0 t + C_1 t_2 + \cdots + C_n t^{n+1}] \, dt$$

$$\Rightarrow \quad \frac{x(1+x)^{n+1}}{n+1} - \frac{(1+x)^{n+2}}{(n+1)(n+2)}$$

$$= \frac{C_0}{2}x^2 + \frac{C_1}{3}x^3 + \frac{C_2}{4}x^4 + \cdots + \frac{C_n}{n+2}x^{n+2} \qquad (3)$$

Putting $x = -1$ in (3), we obtain that the second statement is true.

Putting $x = 1$ in (2) and (3) and subtracting, we obtain that statement-1 is also true.

Q22. If the coeff of (r-5)th and (2r-1)th terms in the expansion of $(1+x)^{34}$ are equal find r

Sol:

$$T_{r+1} = {}^{34}\underset{r}{C}(1)^{34-r}.(x)^r$$

$$T_{r+1} = {}^{34}\underset{r}{C}(x)^r \text{.......}(i)$$

Coeff are

$${}^{34}\underset{r-6}{C} \text{ and } {}^{34}\underset{2r-2}{C}$$

$$\text{ATQ } {}^{34}\underset{r-6}{C} = {}^{34}\underset{2r-2}{C}$$

$$r-6 = 2r-2$$
$$r = -4 \ (\text{neglect})$$

$$r-6 = 34-(2r-2) \qquad \left[\begin{array}{l} \because {}^{n}\underset{r}{C} = {}^{n}\underset{p}{C} \\ r = p \ or \ n = r+p \end{array}\right]$$

$$r = 14$$

Q23. Show that the coeff of the middle term in the expansion of $(1+x)^{2n}$ is equal to the sum of the coeff of two middle terms in the expansion of $(1+x)^{2n-1}$

Sol:

As $2n$ is even so the expansion $(1+x)^{2n}$ has only one middle term which is

$$\left(\frac{2n}{2}+1\right)^{th} \ i.e(n+1)^{th} \text{ term}$$

Coeff of x^n is ${}^{2n}\underset{n}{C}$

Similarly $(2n-1)$ being odd the other expansion has two middle term i.e

$$\left(\frac{2n-1+1}{2}\right)^{th}.\ \text{and}\ \left(\frac{2n-1+1}{2}+1\right)^{th}\ \text{term}$$

i.e n^{th} and $(n+1)^{th}$

The coeff are ${}^{2n-1}C_{n-1}$ and ${}^{2n-1}C_{n}$

$${}^{2n-1}C_{n-1}+{}^{2n-1}C_{n}={}^{2n}C_{n} \qquad \left[\because {}^{n}C_{r-1}+{}^{n}C_{r}={}^{n+1}C_{r}\right]$$

Q24. Find the value of r, if the coeff of (2r + 4)th and (r-2)th terms in the expansion of $(1 + x)^{18}$ are equal.

Sol:

$$T_{r+1}={}^{18}C_{r}(1)^{18-r}.(x)^{r}$$

$$T_{r+1}={}^{18}C_{r}\,x^{r}$$

Put $r=r-3$

And $2r+3$

ATQ ${}^{18}C_{2r+3}={}^{18}C_{r-3}$

$$18=2r+\cancel{3}+r-\cancel{3}$$
$$r=6$$

Q25. If three successive coeff. In the expansion of $(1+x)^n$ are 220, 495 and 792 then find n

Sol:

Let coeff are ${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$

ATQ ${}^nC_{r-1} = 220......(i)$

${}^nC_r = 495......(ii)$

${}^nC_{r+1} = 792........(iii)$

Dividing (ii) by (i)

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{495}{220}$$
$$\frac{n-r+1}{r} = \frac{9}{4}$$
$$4n-13r+4=0......(iv)$$

Dividing (iii) by (ii)

$$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{792}{495}$$
$$\frac{n-r}{r+1} = \frac{8}{5}$$
$$5n-13r-8=0......(v)$$

On solving (iv) and (v) we get $n=12$