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Class: IX
Subject: Math
Topic: Complex Numbers
No. of Questions: 25

1. If z_1 and z_2 are two complex numbers, and a and b are two real numbers, then $|az_1 - bz_2|^2 + |bz_1 + az_2|^2$ equals

(A) $(a^2 + b^2)|z_1 z_2|$
(B) $(a^2 + b^2)(z_1^2 + z_2^2)$
(C) $(a^2 + b^2)(|z_1|^2 + |z_2|^2)$
(D) $2ab|z_1 z_2|$

Sol. C

We have

$$|az_1 - bz_2|^2 + |bz_1 + az_2|^2$$
$$= a^2|z_1|^2 + b^2|z_2|^2 - abz_1\bar{z}_2 - ab\bar{z}_1 z_2$$
$$+ b^2|z_1|^2 + a^2|z_2|^2 + ba\, z\bar{z}_2 + ba\,\bar{z}_1 z_2$$
$$= (a^2 + b^2)(|z_1|^2 + |z_2|^2)$$

2. If $z \neq 0$ is a complex number such that Re(z) = 0, then

(A) $\text{Re}(z^2) = 0$
(B) $\text{Im}(z^2) = 0$
(C) $\text{Re}(z^2) = \text{Im}(z^2)$
(D) None of these

Sol. B

As $\text{Re}(z) = 0$, $z = ib$ where $b \in \mathbf{R}$

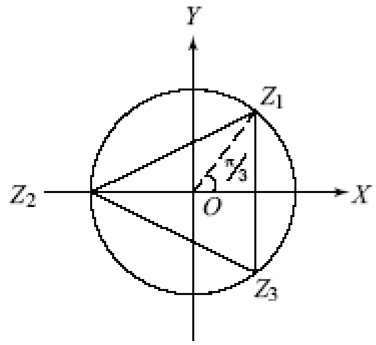
$$\Rightarrow \quad z^2 = -b^2 \quad \Rightarrow \quad \text{Im}(z^2) = 0$$

3. Suppose z_1, z_2, z_3 are the vertices of an equilateral triangle inscribed in the circle | z | = 2. If $z_1 = 1 + \sqrt{3}\,i$, then z_2 and z_3 are respectively

(A) 2, $1 - \sqrt{3}$ i
(B) $1 + \sqrt{3}$ i, – 2
(C) 2, $-1 + \sqrt{3}$ i
(D) 2, $2 + \sqrt{3}$ i

Sol. A

$z_1 = 1 + \sqrt{3}i = 2e^{\pi i/3}$.



Since $\angle z_1 o z_3 = \frac{2\pi}{3}$, and $\angle z_2 o z_3 = \frac{2\pi}{3}$,

$z_2 = z_1\, e^{2\pi i/3}$

and $z_3 = z_2\, e^{2\pi i/3}$

$\Rightarrow$ $z_2 = 2e^{\pi i} = 2(\cos\pi + i\sin\pi) = -2$

and $z_3 = 2e^{5\pi i/3}$

$$= 2\left[\cos\left(2\pi - \frac{\pi}{3}\right) + i\sin\left(2\pi - \frac{\pi}{3}\right)\right]$$

$$= 2\left[\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right] = 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$

$$= 1 - \sqrt{3}i$$

4. Let z and w be two complex numbers such that | z | = | w | = 1 and | z + iw | = | z – i $\overline{w}$ | = 2. Then z equals

(A) 1 or i
(B) i or – i
(C) 1 or – 1
(D) i or – 1

Sol. C

We have $|-iw| = |-i|\,|w| = 1$

and $|i\overline{w}| = |i|\,|\overline{w}| = 1$

$\Rightarrow$ $-iw$ and $i\overline{w}$ lie on the circle $|z| = 1$.

As $|z - (-iw)| = |z - i\overline{w}| = 2$ we get z and $-iw$, as well as z and $i\overline{w}$ are the end points of the same diameter, with one end point at z.

$\therefore$ $-iw = i\overline{w} \Rightarrow w + \overline{w} = 0$

$\Rightarrow$ w is purely imaginary.

Let $w = ik$ where $k \in \mathbf{R}$.

As $|w| = 1$, we get $|ik| = 1$

$\Rightarrow$ $|k| = 1 \Rightarrow k = \pm 1$.

$\therefore$ $w = \pm i \Rightarrow -iw = i\overline{w} = \pm 1$

Thus, $|z - 1| = 2$ and $|z + 1| = 2$.

Out of the choices, we get $z = -1$ or 1.

5. If $\omega(\neq 1)$ is a complex cube root of unity, the least value of n $\in$ N for which $(1 + \omega^2)^n = (1 + \omega^4)^n$ i is

(A) 6
(B) 5
(C) 3
(D) 2

Sol. C

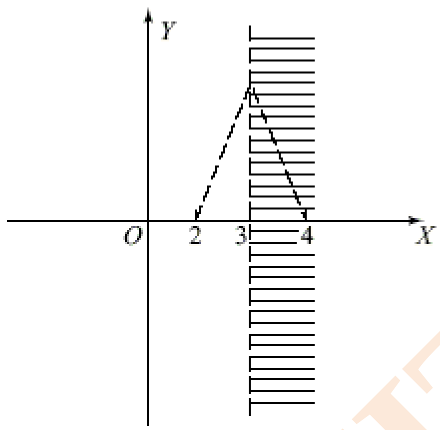
$(1+\omega^2)^n = (1+\omega^4)^n$

$\Rightarrow \quad (-\omega)^n = (1+\omega)^n = (-\omega^2)^n$

$\Rightarrow \quad (-\omega)^n = (-\omega)^n\,\omega^n \quad \Rightarrow \quad \omega^n = 1.$

$\therefore$ the least value of $n \in \mathbf{N}$ for which this happens is 3.

6. The inequality | z – 4| < | z – 2 | represents the region given by



(A) Re(z) $\geq$ 0
(B) Re(z) < 3
(C) Re(z) $\leq$ 0
(D) Re(z) > 3

Sol. D

If z satisfies $|z-4| = |z-2|$, then z lies on the perpendicular bisector of the segment joining $z = 2$ and $\text{Re}(z) = 3$.

i.e., $|z-4| = |z-2| \quad \Rightarrow \quad \text{Re}(z) = 3.$

As $z = 0$ does not satisfy $|z-4| < |z-2|$, we get $|z-4| < |z-2|$ represents the region $\text{Re}(z) > 3$.

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7. For any complex number z, the minimum value of |z| + |z – 2i| is

(A) 0
(B) 1
(C) 2
(D) None of these

Sol. C

We have, for $z \in C$

$|2i| = |z + (2i - z)| \le |z| + |2i - z|$

$\Rightarrow \quad 2 \le |z| + |z - 2i|$

Thus, minimum value of $|z| + |z - 2i|$ is 2 and it is attained when $z = i$.

8. If z = x + iy and w = $\frac{1 - iz}{z - i}$, then |w| = 1 implies that in the complex plane

(A) z lies on the imaginary axis
(B) z lies on the real axis
(C) z lies on the unit circle
(D) none of these

Sol. B

$|w| = 1 \quad \Rightarrow \quad \left|\frac{1 - iz}{z - i}\right| = 1$

$\Rightarrow \quad |-i^2 - iz| = |z - i| \quad \Rightarrow \quad |(-i)(z + i)| = |z - i|$

$\Rightarrow \quad |z + i| = |z - i|$

$\Rightarrow$ z lies on the perpendicular bisector of the segment joining i and $-i$.

$\Rightarrow$ z lies on the real axis.

9. If the imaginary part of $\frac{2z+1}{iz+1}$ is – 4, then the locus of the point representing z in the complex plane is
(A) a straight line
(B) a parabola
(C) a circle
(D) an ellipse

Sol. C

Let $z = x + iy$, then

$$\frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1} = \frac{(2x+1)+2iy}{(1-y)+ix}$$

$$= \frac{[(2x+1)+2iy][(1-y)-ix]}{(1-y)^2+x^2}$$

As $\text{Im}\left(\frac{2z+1}{iz+1}\right) = -4$, we get

$$\frac{2y(1-y)-x(2x+1)}{x^2+(1-y)^2} = -4$$

$\Rightarrow$ $2x^2+2y^2+x-2y = 4x^2+4(y^2-2y+1)$

$\Rightarrow$ $2x^2+2y^2-x-6y+4=0$

which represents a circle.

10. If ω is complex cube root of unity, then a root of the equation $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0$ is

(A) x = 1
(B) x = ω
(C) x = ω^2
(D) x = 0

Sol. D

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Let us denote the given determinant by Δ. Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\Delta = \begin{vmatrix} x+1+\omega+\omega^2 & \omega & \omega^2 \\ x+1+\omega+\omega^2 & x+\omega^2 & 1 \\ x+1+\omega+\omega^2 & 1 & x+\omega \end{vmatrix} = \begin{vmatrix} x & \omega & \omega^2 \\ x & x+\omega^2 & 1 \\ x & 1 & x+\omega \end{vmatrix}$$

Clearly $\Delta = 0$ for $x = 0$.

11. Let z_1 and z_2 be two non-zero complex numbers such that $\frac{z_1}{z_2} + \frac{z_2}{z_1} = 1$. Then, the origin and points represented by z_1 and z_2

(A) lie on a straight line
(B) form a right triangle
(C) form an equilateral triangle
(D) none of these

Sol. C

Let $z = \frac{z_1}{z_2}$, then $z + \frac{1}{z} = 1 \quad \Rightarrow \quad z^2 - z + 1 = 0$

$$\Rightarrow \quad z = \frac{1 \pm \sqrt{3}\, i}{2} \quad \Rightarrow \quad \frac{z_1}{z_2} = \frac{1 \pm \sqrt{3} i}{2}$$

If z_1 and z_2 are represented by A and B respectively and O be the origin, then

$$\frac{OA}{OB} = \frac{|z_1|}{|z_2|} = \left|\frac{1 \pm \sqrt{3}i}{2}\right| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$$

$\Rightarrow \quad OA = OB$

Also, $$\frac{AB}{OB} = \frac{|z_2 - z_1|}{|z_2|} = \left|1 - \frac{z_1}{z_2}\right|$$

$$= \left|1 - \left(\frac{1}{2} \pm \frac{\sqrt{3}}{2} i\right)\right| = \left|\frac{1}{2} \mp \frac{\sqrt{3}}{2} i\right| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$$

$\Rightarrow \quad AB = OB$

Thus, $\quad OA = OB = AB$.

$\therefore \Delta OAB$ is an equilateral triangle.

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12. The roots of $z^5 = (z-1)^5$ are represented in the argand plane by the points that are

(A) collinear
(B) concyclic
(C) vertices of a parallelogram
(D) None of these

Sol. A

Let z be a complex number satisfying $z^5 = (z-1)^5$

$\Rightarrow$ $|z^5| = |(z-1)^5| \Rightarrow |z|^5 = |z-1|^5$

$\Rightarrow$ $|z| = |z-1|$.

Thus, z lies on the perpendicular bisector of the segment joining the origin and $A(1 + i0)$ i.e. z lies on $\text{Re}(z) = 1/2$.

13. If z = $\sqrt{20i-21}+\sqrt{20i+21}$, then one of the possible value of arg (z) equals

(A) $\pi/4$
(B) $\pi/2$
(C) $3\pi/8$
(D) π

Sol. A

$20i + 21 = (5 + 2i)^2$

and $20i - 21 = (2 - 5i)^2$

$\therefore$ $z = \pm(5 + 2i) \pm (2 + 5i)$

$\Rightarrow$ $z = 7(1 + i),\ 3(1 - i),\ 3(-1 + i),\ 7(-1 - i)$

$\Rightarrow$ $\arg(z) = \pi/4, -\pi/4, 3\pi/4, -3\pi/4$

Thus, one of the possible value of $|\arg(z)|$ is $\pi/4$, $3\pi/4$

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14. If $(4 + i)(z + \bar{z}) – (3 + i)(z – \bar{z}) + 26i = 0$, then the value of $|z|^2$ is

(A) 13
(B) 17
(C) 19
(D) 11

Sol. B

Let $z = x + iy$, then

$$2(4 + i)\,x – (3 + i)\,2iy + 26i = 0$$

$$\Rightarrow \quad 4x + y = 0,\ x – 3y + 13 = 0$$

$$\Rightarrow \quad x = –1,\ y = 4$$

$$\therefore \quad |z|^2 = 17$$

15. If $3^{49}(x + iy) = (3/2 + \sqrt{3}\,i/2)^{100}$, $y \in \mathbf{N}$, and $x = ky$, then value of k is

(A) –1/3
(B) $+2\sqrt{2}$
(C) $-1/\sqrt{3}$
(D) $+1\sqrt{3}$

Sol. B

We have

$$3^{49}\,|x + iy| = |3/2 + \sqrt{3}i/2|^{100}$$

$$\Rightarrow \quad 3^{49}\sqrt{x^2+y^2} = (9/4 + 3/4)^{50}$$

$$\Rightarrow \quad \sqrt{x^2+y^2} = 3 \Rightarrow (y)\sqrt{1+k^2} = 3$$

$$\Rightarrow \quad \sqrt{1+k^2}\ \ 3, 3/2, 1, \text{ as } y \in \mathbf{N}$$

$$\Rightarrow \quad k = \pm 2\sqrt{2},\ \pm\sqrt{5}/2,\ 0$$

Out of the given values, we have $k = \pm 2\sqrt{2}$.

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16. If $z \neq 1, \frac{z^2}{z-1}$ is real, then point represented by the complex number z lies

(A) on a circle with centre at the origin
(B) either on the real axis or on a circle not passing through the origin
(C) on the imaginary axis
(D) either on the real axis or on a circle passing through the origin

Sol. B

As $\frac{z^2}{z-1}$ is real, we get

$$\frac{z^2}{z-1} = \frac{\bar{z}^2}{\bar{z}-1}$$

$\Leftrightarrow \quad z^2(\bar{z} - 1) = \bar{z}^2 (z - 1)$

$\Leftrightarrow \quad z\bar{z}(z - \bar{z}) - (z - \bar{z})(z + \bar{z}) = 0$

$\Leftrightarrow \quad (z - \bar{z})(z\bar{z} - z - \bar{z}) = 0$

$\Rightarrow \quad z - \bar{z} = 0$ or $z\bar{z} - z - \bar{z} = 0$

$\Rightarrow \quad z$ lies on the real axis

or z lies on a circle through the origin.

17. For complex numbers $z_1 = x_1 + iy_1$, and $z_2 = x_2 + iy_2$, we write $z_1 \cap z_2$ if $x_1 \leq x_2$ and $y_1 \leq y_2$. Let z be a complex number such that $1 \cap z$, then

Sol. 3

Let $z = x + iy$. As $1 \cap z$, we get $1 \leq x$ and $0 \leq y$

Now, $$\frac{1-z}{1+z} = \frac{(1-x)-iy}{(1+x)+iy} = \frac{[(1-x)-iy][(1+x)-iy]}{(1+x)^2+y^2}$$

$$= \frac{(1-x^2)-y^2-iy(1+x+1-x)}{(1+x)^2+y^2}$$

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$$= \frac{1-(x^2+y^2)-2iy}{(1+x)^2+y^2}$$

As $x \geq 1$, and $y \geq 0$, we get

$$\frac{1-(x^2+y^2)}{(1+x)^2+y^2} \leq 0 \text{ and } \frac{-2y}{(1+x)^2+y^2} \leq 0$$

Thus, $\frac{1-z}{1+z} \cap 0$

18. If w is an imaginary cube root of unity, then value of the expression $1(2-\omega)(2-\omega^2)+2(3-\omega)(3-\omega^2)+\ldots+(n-1)(n-\omega)(n-\omega^2)$ is

Sol. 1

rth term of the given expression is

$$r(r+1-\omega)(r+1-\omega^2)$$
$$=(r+1-1)(r+1-\omega)(r+1-\omega^2)$$
$$=(r+1)^3-1$$
$$[\because (x-1)(x-\omega)(x-\omega^2)=x^3-1]$$

$$\therefore \quad S=\sum_{r=1}^{n-1}\left[(r+1)^3-1\right]=\sum_{r=0}^{n-1}\left[(r+1)^3-1\right]$$

$$=\sum_{r=0}^{n} r^3 - n = \frac{1}{4}n^2(n+1)^2 - n$$

19. If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, then $\frac{(x^2+y^2)^2(c^2+d^2)}{a^2+b^2}$ equals

Sol. 3

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$$(x+iy)^2 = \frac{a+ib}{c+id} \Rightarrow |(x+iy)^2| = \left|\frac{a+ib}{c+id}\right|$$

$$\Rightarrow \quad |x+iy|^2 = \left|\frac{a+ib}{c+id}\right| \Rightarrow (\sqrt{x^2+y^2})^2 = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$$

$$\Rightarrow \quad \frac{(x^2+y^2)^2\,(c^2+d^2)}{a^2+b^2} = 1$$

20. The region of the argand plane defined by $|z - i| + |z + i| \leq 4$ is

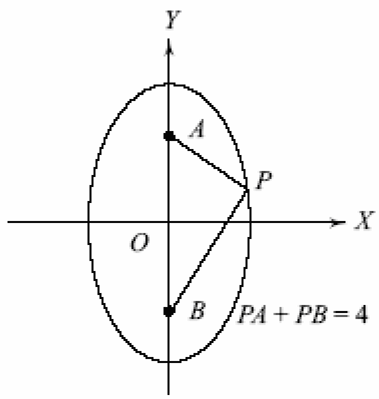
(A) interior of an ellipse
(B) exterior of a circle
(C) interior or on the boundary of an ellipse
(D) none of these

Sol. C

Let A and B be the point $0 + i$ and $0 - i$ and $P(z)$ be any point satisfying $|z - i| + |z + i| \leq 4$.

$\Rightarrow \quad PA + PB \leq 4$

Thus, P lies in the interior or on the boundary on the ellipse with foci A and B and length of major axis $= 4$.



21. if x + iy = $\sqrt{\frac{1+i}{1-i}}$, prove that $x^2 + y^2 = 1$

Sol:

$$x+iy=\sqrt{\frac{1+i}{1-i}} \quad \text{(i)} \quad \text{(Given)}$$

taking conjugate both side

$$x-iy=\sqrt{\frac{1-i}{1+i}} \quad \text{(ii)}$$

$$\text{(i)} \times \text{(ii)}$$

$$(x+iy)(x-iy)=\sqrt{\frac{1+i}{1-i}}\times\sqrt{\frac{1-i}{1+i}}$$

$$(x)^2-(iy)^2=1$$

$$\boxed{x^2+y^2=1} \quad \text{Proved.}$$

22. Convert in the polar form $\frac{1+7i}{(2-i)^2}$

Sol:

$$\frac{1+7i}{(2-i)^2}=\frac{1+7i}{4+i^2-4i}=\frac{1+7i}{3-4i}$$

$$=\frac{1+7i}{3-4i}\times\frac{3+4i}{3+4i}$$

$$=\frac{3+4i+21i+28i^2}{9+16}$$

$$=\frac{25i-25}{25}=i-1$$

$$=-1+i$$

$r = |z| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

Let α be the acute $\angle$s

ten $\alpha = \left|\frac{1}{-1}\right|$

$\alpha = \pi/4$

since Re (z) < 0, Im (z) > 0

$\theta = \pi - \alpha$

$= \pi - \frac{\pi}{4} = 3\pi/4$

$z = r(\text{Cos}\,\theta + \text{i Sin}\,\theta)$

$= \sqrt{2}\left(\text{Cos}\ \frac{3\pi}{4} + i Sin \frac{3\pi}{4}\right)$

23. Find the real values of x and y if (x - iy) (3 + 5i) is the conjugate of – 6 24 i

Sol:

(x – iy (3 + 5i) = - 6 + 24i

$3x + 5xi - 3yi - 5yi^2 = -6 + 24i$

$(3x+5y)+(5x-3y)i = -6+24i$

$3x+5y = -6$

$5x-3y = 24$

$$\boxed{\begin{aligned} x &= 3 \\ y &= -3 \end{aligned}}$$

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24. If $|z_1| = |z_2| = 1$, prove that $\left|\frac{1}{z_1} + \frac{1}{z_2}\right| = |z_1 + z_2|$

Sol:

$$\text{If } |z_1| = |z_2| = 1 \quad \text{(Given)}$$

$$\Rightarrow |z_1|^2 = |z_2|^2 = 1$$

$$\Rightarrow z_1 \overline{z_1} = 1$$

$$\overline{z_1} = \frac{1}{z_1} \quad (1)$$

$$z_2 \overline{z_2} = 1$$

$$z_2 \overline{z_2} = 1$$

$$\overline{z_2} = \frac{1}{z_2} \quad (2)$$

$$\left[\because z\overline{z} = |z|^2 \right.$$

$$\left|\frac{1}{z_1} + \frac{1}{z_2}\right| = \left|\overline{z_1} + \overline{z_2}\right|$$

$$= \left|\overline{z_1 + z_2}\right|$$

$$= |z_1 + z_2|$$

25) If α and β are different complex number with $|\beta| = 1$ Then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$

Sol:

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|^2 = \left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)\left(\overline{\frac{\beta-\alpha}{1-\bar{\alpha}\beta}}\right) \quad \left[\because |z|^2 = z\bar{z}\right.$$

$$= \left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)\left(\frac{\bar{\beta}-\bar{\alpha}}{1-\alpha\bar{\beta}}\right)$$

$$= \left(\frac{\beta\bar{\beta}-\beta\bar{\alpha}-\alpha\bar{\beta}+\alpha\bar{\alpha}}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+\alpha\bar{\alpha}\beta\bar{\beta}}\right)$$

$$= \left(\frac{|\beta|^2-\beta\bar{\alpha}-\alpha\bar{\beta}+|\alpha|^2}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+|\alpha|^2|\beta|^2}\right)$$

$$= \left(\frac{1-\beta\bar{\alpha}-\alpha\bar{\beta}+|\alpha|^2}{1-\alpha\because\bar{\beta}-\bar{\alpha}\beta+|\alpha|^2}\right) \quad \left[\because |\beta| = 1\right.$$

$$= 1$$

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = \sqrt{1}$$

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = 1$$