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B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Class: IX
Subject: Maths
Topic: Conic Section
No. of Questions: 25

Q1. Above x-axis, the equation of the common tangent to the circle $(x - 3)^2 + y^2 = 9$ and parabola $y^2 = 4x$ is

(A) $\sqrt{3}y = 3x + 1$
(B) $\sqrt{3}y = -(x + 3)$
(C) $\sqrt{3}y = x + 3$
(D) $\sqrt{3}y = -(3x + 1)$

Sol. C

Q2. The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is

(A) $x = -1$
(B) $x = 1$
(C) $x = -\frac{3}{2}$
(D) $x = \frac{3}{2}$

Sol. D

On completing the square, we get:
$(y + 2)^2 = - 4 (x - 1/2)$
And the equation of directrix is $x = 3/2$

Q3. The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix

(A) $x = -a$
(B) $x = -a/2$
(C) $x = 0$
(D) $x = a/2$

Sol. C

Let the point be $(at^2, 2at)$ on the parabola, the mid points between the points (a, 0) and the parabola will be $(a(1 + t^2)/2, at)$, and the point lies on the parabola $y^2 = 2a\,(x - a/2)$, having directrix x = 0

Q4. If the focal chord of $y^2 = 16x$ touches $(x – 6)^2 + y^2 = 2$, then the slope of such chord is

(A) 1, –1
(B) 2, $-\frac{1}{2}$
(C) $\frac{1}{2}$, – 2
(D) 2, – 2

Sol. A

Let the equation of focal chord be

y =m(x-4) (here a= 4) which touches the circle if $\left|\frac{m(6) - 0 - 4m)}{\sqrt{1 + m^2}}\right| = \sqrt{2}$
$m^2 = 1$ or m = 1, -1

Q5. The angle between the tangents drawn from (1, 4) to the parabola $y^2 = 4x$ is

(A) $\frac{\pi}{2}$
(B) $\frac{\pi}{3}$
(C) $\frac{\pi}{6}$
(D) $\frac{\pi}{4}$

Sol. B

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Q6. A tangent at point P(1, 7) of the parabola $y = x^2 + 6$ touching the circle $x^2 + y^2 + 16x + 12y + c = 0$ at point

(A) (– 6, – 7)
(B) (– 9, – 7)
(C) (– 6, – 3)
(D) (– 10, – 15)

Sol. A

7) Direction: The following question has four choices, out of which ONE or More is/ are correct. P And Q are two points on a parabora, if tangents P and Q intersecting at right angle,

(A) chord PQ always passes through a fixed point
(B) chord PQ always intersects the letus rectum
(C) the minimum angle subtended by chord PQ at the vertex is $\tan^{-1}\left(\frac{4}{3}\right)$
(D) the minimum length of the chord PQ > length of letus rectum

Sol. A B C

Let us the consider the parabola $y^2 = 4ax$
$\because$ perpendicular tangents intersect on the directrix
Let so be take their point of intersection as (-a, k)
Now, equation of chord PQ $ky - 2a(x - a) = 0$ which of the form $x - a + \lambda y = 0$
all such lines passes through a fixed point (a, 0), which is focus of the parabola
Late AB is a focal chord where A $A(at^2, 2at),\ B\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$
Slope of OA = $\frac{2}{t}$, Slope of OB = -2t, If θ is acute angle AOB, then

$$\tan\theta = \left|\frac{\frac{2}{t} + 2t}{1-4}\right| = \frac{2}{3}\left(t + \frac{1}{t}\right) \geq \frac{4}{3}$$

$\therefore$ the least angle = $\tan^{-1}\frac{4}{3}$ and for this t = 1
$\Rightarrow$ the chord AB becomes latus rectum

8) Directions: The following question has four choices, out of which ONLY ONE is correct. Let us consider a family of trajectory $y^2 = 4ax$ where a is a parameter, P, Q, R are three points on it, such that normal at Q and R meet at P.Locus of circum centre of triangle PQR is

(A) $2y^2 = a(x+a)$

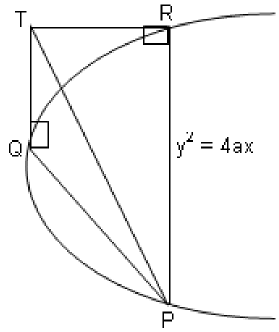
(B) $2y^2 = 9a(x-a)$

(C) $2y^2 = a(x+2a)$

(D) $2y^2 = a(x-a)$

Sol. D

Let Q (t_1), R (t_2), P (t_3) and T is the point of intersects of tangents at Q and R



$\therefore\ t_3 = -t_1 - \frac{2}{t_1} = -t_2 - \frac{2}{t_2} \quad \Rightarrow \quad t_1 t_2 = 2 \quad \Rightarrow T(2a, a(t_1+t_2))$

also P is the point of intersection normals at Q and R

$\therefore\ P\left(a(t_1+t_2)^2, -2a(t_1+t_2)\right)$

$\because \angle$ PRT = angle PQT = $\frac{\pi}{2}$

$\therefore$ PT is the diameter of the circle passing through P, Q, R and T

$\Rightarrow$ circum centre of triangle PQR is mid point of PT, whose coordinates are

$x = \frac{2a + a(t_1+t_2)^2}{2},\ y = \frac{-a(t_1+t_2)}{2}$

$\therefore$ locus is $2y^2 = a(x-a)$

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9) Directions: The following question has four choices, out of which ONLY ONE is correct.Let us consider a family of trajectory $y^2 = 4ax$ where a is a parameter, P, Q and R are three points on it, such that normal at Q and R meet at P.If tangents drawn at the points P, Q, R, taken in pairs, meet at the points A, B and C, $\frac{\text{area of triangle } ABC}{\text{area of triangle } PQR}$ is

(A) $\frac{1}{2}$
(B) 2
(C) 1
(D) depending on the positions of the points P, Q and R

Sol. A

Let $P(t_1), Q(t_2), R(t_3)$, area of the triangle PQR = Δ_1, area of triangle ABC = Δ_2

$$\Delta_1 = \frac{1}{2}\begin{vmatrix} at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \\ at_3^2 & 2at_3 & 1 \end{vmatrix} = a^2\left|(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)\right|$$

$A(at_1t_2, a(t_1+t_2)), B(at_2t_3, a(t_2+t_3)), C(at_3t_1, a(t_3+t_1))$

$$\Delta_2 = \frac{1}{2}\begin{vmatrix} at_1t_2 & a(t_1+t_2) & 1 \\ at_2t_3 & a(t_2+t_3) & 1 \\ at_3t_1 & a(t_3+t_1) & 1 \end{vmatrix} = \frac{a^2}{2}\left|(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)\right|$$

$$\Rightarrow \frac{\Delta_2}{\Delta_1} = \frac{1}{2}$$

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10) Directions: The following question has four choices, out of which ONLY ONE is correct.
Let us consider a family of trajectory $y^2 = 4ax$ where *a* is a parameter, P, Q and R are three points on it, such that normals at Q and R meet at P.
If tangents drawn at the points Q and R intersect at t and the chord QR touches $y^2 = 4bx$, what is the locus of the point t?

(A) $by^2 = 4a^2x$
(B) $y^2 = 4ax$
(C) $4b^2x = ay^2$
(D) circle

Sol. A

If $Q(t_1),\ R(t_2)$ then $T(at_1t_1,\ a(t_1+t_2))$

Now equation of QR as the chord contact is $y.a(t_1+t_2) - 2a(x + at_1t_2) = 0$

$\Rightarrow \quad y = \dfrac{2x}{t_1+t_2} + \dfrac{2at_1t_2}{t_1+t_2}$

by condition of tangency $\dfrac{2at_1t_2}{t_1+t_2} = b\left(\dfrac{t_1+t_2}{2}\right)$

$\therefore$ locus of the point T is $by^2 = 4a^2x$

11) Directions: The answer to the following question is a single digit integer, ranging from 0 to 9. Enter the correct digit in the box given below.

Sol. 9

$\sqrt{3}\operatorname{cosec} 20° - \sec 20° = \tan 60° \operatorname{cosec} 20° - \sec 20° = \dfrac{\sin 60°}{\cos 60° \sin 20°} - \dfrac{1}{\cos 20°} = 4$

$\therefore$ the parabola is $y^2 = 4x$

Let y_1, y_2, y_3 are ordinates of the points A, B and C respectively.

$y_1 + y_2 + y_3 = 0$

solving $y^2 = 4x \ \text{and}\ 2x - y = 1,$ we have

$y^2 - 2y - 2 = 0 \quad \Rightarrow \quad y_1 + y_2 = 2$

$\therefore \ y_3 = -2$

coordinates of point C (1, -2)
equation of normal at C

x – y = 3

area of the triangle = $\frac{9}{2}$

12) Directions:The answer to the following question is a single digit integer, ranging from 0 to 9. Enter the correct digit in the box given below. A tangent to the ellipse x2 + 4y2 = 4 meets the ellipse x2 + 2y2 = 6 at P and Q. If is the angle in radian between the tangents at P and Q, find [ ] (where [.] is g.i.f.)

Sol. 1

$\frac{x^2}{4}+\frac{y^2}{1}=1$ …..(1)

$\frac{x^2}{6}+\frac{y^2}{3}=1$ …..(2)

Let tangents P and Q meet at R (x_1, y_1)

As per given information chord of contact of R with respect to (2) touches (1)

$\Rightarrow \frac{xx_1}{6}+\frac{yy_1}{3}=1$ touches (1)

$\Rightarrow \frac{9}{y_1^2}=\frac{x_1^2}{y_1^2}+1$ $\Rightarrow$ $x_1^2+y_1^2=9$ $\Rightarrow$ locus of the point R is $x^2+y^2=9$

which is director circle of (2)

$\therefore$ the required angle $\theta = \frac{\pi}{2}$ $\Rightarrow$ $[\theta] = 1$

13 <u>Directions:</u>The answer to the following question is a single digit integer, ranging from 0 to 9. Enter the correct digit in the box given below.

Sol. 6

Equation of normals at the point "t"

$y-6t=-t\left(x-3t^2\right) \Rightarrow 3t^3+(6-x)t-y=0$

$\Rightarrow t_1+t_2+t_3=0,\ t_1t_2+t_2t_3+t_3t_1=\frac{6-x}{3},\ t_1t_2t_3=\frac{4}{3}$

it is given that $\frac{y_1}{y_2} = \frac{1}{2} \Rightarrow \frac{t_1}{t_2} = \frac{1}{2}$

taken $t_1 = t$ we have $t_2 = 2t,\ t_3 = -3t$

upon eliminating t the locus is obtained as

$$x - 6 = 21\left(\frac{y}{18}\right)^{\frac{2}{3}} \Rightarrow \frac{K}{3} = 6$$

14) Directions: The answer to the following question is a single digit integer, ranging from 0 to 9. Enter the correct digit in the box given below.

Sol. 8

$\because$ the line 2x + y = 4 meets X-axis at R.

$\therefore$ R(2, 0)

Any line through R

$$\frac{x-2}{\cos\theta} = \frac{y-0}{\sin\theta} = r$$

A point on this line (2 + r cos θ, r sin θ)

This point represent S and T both and they lie on xy = 4.

$\therefore$ (2 + r cos θ) r sin θ = 4

$\Rightarrow r^2 \sin\theta \cos\theta + 2r\sin\theta - 4 = 0$

Now, product of roots = $\frac{4}{\sin\theta\cos\theta}$

$$\Rightarrow RS.RT = \frac{4}{\sin\theta\cos\theta} = \frac{8}{2\sin\theta\cos\theta}$$

Hence least value of RS.RT = 8.

15) A bar of length 20 cm moves along with its extremities on two fixed straight lines (take as axes) at right angles. If a marked point on it is at 4 cm from one end, the eccentricity of ellipse described by the marked point is

(A) 5/4

(B) $\sqrt{(15)/4}$

(C) 6

(D) $\sqrt{(21)/4}$

Sol. B

16) Directions: The answer to the following question is a single digit integer, ranging from 0 to 9. Enter the correct digit in the box given below.

If P is any point on the ellipse $\frac{x^2}{a}+\frac{y^2}{b}=1$ and F_1, F_2 are its foci, find maximum area of the triangle PF_1F_2 where a and b are respectively the greatest and the least values of the function $f(x) = x^3 - 6x^2 + 9x + 1$ on [0, 2].

Sol. 2

$f(x) = x^3 - 6x^2 + 9x + 1, x \in [0, 2]$

$f'(x) = 3x^2 - 12x + 9$

f'(x) is increases if $f'(x) \geq 0$

$\Rightarrow 3x^2 - 12x + 9$

$\Rightarrow x \in [0, 1]$

and f(x) decreases on [1, 2]

$\Rightarrow a = f(1) = 5, b = f(0) = 1$

$\therefore$ equation of the ellipse becomes

$$\frac{x^2}{5}+\frac{y^2}{1}=1$$

we know that greatest area of the triangle PF_1F_2 = abe = $\sqrt{5}.\frac{2}{\sqrt{5}}=2$

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17) Directions: The following question has four choices, out of which ONE or MORE is/are correct.

A hyperbola is lying between the acute angle formed by its pair of asymptotes $ax^2 + 2hyxy + by^2 + 2gx + 2fy + c = 0$, which of the following is (are) not the eccentricity of its conjugate hyperbola?

(A) $\operatorname{cosec}\left(\frac{1}{2}\tan^{-1}\frac{2\sqrt{h^2 - ab}}{|a+b|}\right)$

(B) $\sec\left(\frac{1}{2}\tan^{-1}\frac{2\sqrt{h^2 - ab}}{|a+b|}\right)$

(C) $2\sin\left(\frac{1}{2}\tan^{-1}\frac{2\sqrt{h^2 - ab}}{|a+b|}\right)$

(D) $2\cos\left(\frac{1}{2}\tan^{-1}\frac{2\sqrt{h^2 - ab}}{|a+b|}\right)$

Sol. B

Obtuse angle between asymptotes = $\pi - \tan^{-1}\frac{2\sqrt{h^2 - ab}}{|a+b|} = 2\alpha\,(\text{say})$

So, eccentricity of the conjugate hyperbola = sec α = $\operatorname{cosec}\left(\frac{1}{2}\tan^{-1}\frac{2\sqrt{h^2 - ab}}{|a+b|}\right)$

$\therefore$ the options are correct

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18) Directions: The following question has four choices, out of which ONLY ONE is correct.

$T_n = \frac{\sum_{k=1}^{n} k}{n!}$ and $\sum_{n=0}^{\infty} T_n$ is eccentricity of

(A) a circle
(B) a parabola
(C) an ellipse
(D) a hyperbola

Sol. D

$$T_n = \frac{\sum_{k=1}^{n} k}{n!} = \frac{n^2+n}{2\,n!} = \frac{1}{2}\left(\frac{1}{(n-2)!} + \frac{2}{(n-1)!}\right)$$

$\Rightarrow \sum_{n=0}^{\infty} T_n = \frac{3}{2}e > 1$

which is eccentricity of hyperbola
$\therefore$ the option is correct

19) Directions: The following question has four choices, out of which ONLY ONE is correct.

If f(x) is a decreasing function, the set of values of k, for which y = 0 is the major axis of the ellipse $\frac{x^2}{f(k^2+2k+5)} + \frac{y^2}{f(k+1)} = 1$, is

(A) $k \in (-2, 3)$
(B) $k \in (-3, 2)$
(C) $k \in (-\infty, -3) \cup (2, \infty)$
(D) $k \in (-\infty, -3) \cup (3, \infty)$

Sol. B

$\because$ f(x) is a decreases function
$f(k^2 + 2k + 5) > f(k + 1)$
$\Rightarrow k^2 + 2k + 5 < k + 1$

$\Rightarrow$ k $\in$ (– 3, 2)

$\therefore$ the option is correct

20) Directions:The following question has four choices, out of which ONLY ONE is correct.

If (a cos θ + a cos2θ, b sinθ + b sin2θ) is parametric point on $\frac{2x}{a}=\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-k\right)$, Find the value of k =

(A) 0
(B) 1
(C) 2
(D) 3

Sol. D

$$\frac{x}{a}=2\cos\frac{3\theta}{2}\cos\frac{\theta}{2},\ \frac{y}{b}=2\sin\frac{3\theta}{2}\cos\frac{\theta}{2}$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=4\cos^2\frac{\theta}{2}$$

But $\frac{x}{a}=2\cos\frac{\theta}{2}\left(4\cos^3\frac{\theta}{2}-3\cos\frac{\theta}{2}\right)$

$$=2\cos^2\frac{\theta}{2}\left(4\cos^2\frac{\theta}{2}-3\right)$$

$$\Rightarrow\ \frac{2x}{a^2}=\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-3\right)$$

k = 3

21. Find the lengths of axes & length of lat us rectum of the hyperbola, $\frac{y^2}{9} - \frac{x^2}{16} = 1$

Sol:

The given equation is $\frac{y^2}{9} - \frac{x^2}{16} = 1$ means hyperbola

Comparing the given equation with $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we get

$a^2 = 9 \quad \& \quad b^2 = 16$

Length of transverse axis $= 2a = 2 \times 3 = 6$ units

Length of conjugate axis $= 2b = 2 \times 4 = 8$ units

The coordinates of the vertices are $A(0, -a) \quad \& \quad B(0, a)$ i.e $A(0, -3) \quad \& \quad B(0, 3)$

22. Find the eccentricity of the hyperbola of $\frac{y^2}{9} - \frac{x^2}{16} = 1$

Sol: As in above question

A = 3 & b = 4 &

$c^2 = a^2 + b^2 = 9 + 16 = 25$

So , c = 5

Then, e = $\frac{c}{a} = \frac{5}{3}$

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23. Find the equation of the hyperbola with centre at the origin, length of the trans verse axis 6 & one focus at (0,4)

Sol: Let its equation be $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$

Clearly c = 4

Length of transverse axis $=6 \Leftrightarrow 2a=6 \Leftrightarrow a=3.$

Also, $c^2=a^2+b^2 \Leftrightarrow b^2=c^2-a^2=4^2-3^2=16-9=7$

Then $a^2=3^2=9 \quad \& \quad b^2=7$

Hence, the required equation is $\frac{y^2}{9}-\frac{x^2}{7}=1$

24. Find the equation of the ellipse, the ends of whose major axis are $(\pm 3, 0)$ & at the ends of whose minor axis are (0, ± 4)

Sol: Let the required equation be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Its vertices are $(\pm a, 0) \quad \& \quad a=3$

Ends of minor axis are $C(0,-4) \quad \& \quad D(0,4)$

$\therefore CD=8$ ie length of the minor axis = 8 units

Now, $2b=8 \Leftrightarrow b=4$

$\therefore a=3 \quad \& \quad b=4$

Hence the required equation is $\frac{x^2}{9}+\frac{y^2}{16}=1$

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25. Find the equation of the parabola with focus at F (4,0) & directrix x =

Sol: Focus F(4,0) lies on the axis hand side of the origin so, it is a right handed parabola. Let the required equation be $y^2 = 4ax.$

Than, a = 4

Hence, the required equation is $y^2 = 16x$