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Class: XI
Subject: Math's
Topic: Introduction to 3D Geometry
No. of Questions: 25

1. Two points (a, 3) and (5, b) are the opposite vertices of a rectangle. If the other two vertices lie on the line y = 2x + c which passes through the point (a, b) then the value of c is

A. 7
B. 4
C. 0
D. 7

Sol: A

Mid point of the line joining the given points lie on the given line

$$\frac{3+b}{2} = 2\left(\frac{a+5}{2}\right) + c$$

$\Rightarrow\ 2a + 2c - b + 7 = 0$ (i)

Also since the given line passes through (a, b)

$b = 2a + c$ (ii)

Solving (i) and (ii) we get $c = -7$

2. The line parallel to x-axis passing through the intersection of the lines ax + 2by + 3b = 0 and bx – 2ay – 3a = 0 where (a, b) $\neq$ (0, 0) is

A. above x-axis at a distance 3/2 from it
B. above x-axis at a distance 2/3 from it
C. below x-axis at a distance 3/2 from it
D. below x-axis at a distance 2/3 from it

Sol: C

Eliminating x, we get the $(2b^2 + 2a^2)y + 3b^2 + 3a^2 = 0$

$\Rightarrow$ y = – 3/2 which is the required line and hence below x-axis at a distance 3/2 from it.

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3. Locus of mid point of the portion between the axes of x cos α + y sin α = p, where p is constant is

A. $x^2 + y^2 = 4/p^2$
B. $x^2 + y^2 = 4p^2$
C. $1/x^2 + 1/y^2 = 2/p^2$
D. $1/x^2 + 1/y^2 = 4/p^2$

Sol: D

If (h, k) is the mid-point, then

$$h = p/2 \cos\alpha,\ k = p/2 \sin\alpha$$

so $\quad (p/2h)^2 + (p/2k)^2 = \cos^2\alpha + \sin^2\alpha = 1$

$\Rightarrow \quad 1/h^2 + 1/k^2 = 4/p^2$

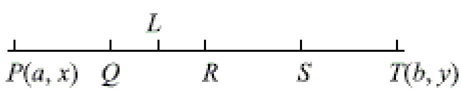
Locus of (h, k) is $1/x^2 + 1/y^2 = 4/p^2$

4. Q, R and S are the points on the line joining the points P (a, x) and T (b, y) such that PQ = QR = RS = ST, then $\left(\frac{5a+3b}{8}, \frac{5x+3y}{8}\right)$ is the mid point of the segment

A. PQ
B. QR
C. RS
D. ST

Sol: B

The point $L\left(\frac{5a+3b}{8}, \frac{5x+3y}{8}\right)$ divides PT in the ratio 3 : 5 and hence is the middle point of QR.



5. An equation of a straight line passing through the inter-section of the straight lines 3x – 4y + 1 = 0 and 5x + y – 1 = 0 and making non-zero, equal intercepts on the axes is

A. 22x + 22y = 13
B. 23x + 23y = 11
C. 11x + 11y = 23
D. 8x – 3y = 0

Sol: B

Equation of any line through the point of intersection of the given lines is

$$(3x - 4y + 1) + k(5x + y - 1) = 0 \qquad (1)$$

or $(3 + 5k)x + (k - 4)y + 1 - k = 0$

or $\dfrac{x}{(k-1)/(3+5k)} + \dfrac{y}{(k-1)/(k-4)} = 1$

Since x-intercept = y-intercept

$\Rightarrow \dfrac{k-1}{3+5k} = \dfrac{k-1}{k-4} \quad \Rightarrow \quad (k - 1)(3 + 5k - k + 4) = 0$

$\Rightarrow \quad k = 1$ or $k = -7/4$

For $k = 1$, (1) becomes $8x - 3y = 0$ which makes zero intercepts on the axes.

$\therefore \quad k = -7/4 \quad \Rightarrow$ The required equation is

$4(3x - 4y + 1) - 7(5x + y - 1) = 0$

$\Rightarrow \quad 23x + 23y = 11.$

6. The lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line for

A. exactly two values of p
B. more than two values of p
C. no value of p
D. exactly one value of p

Sol: D

Two lines will be perpendicular to a common line if these two are parallel.

$$\therefore \quad p(p^2+1) = -\frac{(p^2+1)^2}{p^2+1}$$

$$\Rightarrow \quad (p+1)(p^2+1) = 0 \Rightarrow p = -1$$

7. The area enclosed by $2|x| + 3|y| \leq 6$ is

A. 3 sq. units
B. 4 sq. units
C. 12 sq. units
D. 24 sq. units

Sol: C

The given inequality is equivalent to the following system of inequalities.

$2x + 3y \le 6$, when $x \ge 0, y \ge 0$

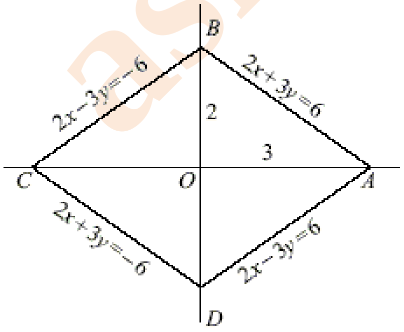
$2x - 3y \le 6$, when $x \ge 0, y \le 0$

$-2x + 3y \le 6$, when $x \le 0, y \ge 0$

$-2x - 3y \le 6$, when $x \le 0, y \le 0$

which represents a rhombus with sides

$2x \pm 3y = 6$ and $2x \pm 3y = -6$



Length of the diagonals is 6 and 4 units along x-axis and y-axis.

$\therefore$ The required area

$$= \frac{1}{2} \times 6 \times 4 = 12 \text{ sq. units.}$$

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8. A straight line through the origin O meets the parallel lines 4x + 2y = 9 and 2x + y + 6 = 0 at points P and Q respectively. The point O divides the segment PQ in the ratio

A. 1 : 2
B. 3 : 4
C. 2 : 1
D. 4 : 3

Sol: B

It is clear that the lines lie on opposite side of the origin O. Let the equation of any line through O be $\frac{x}{\cos\theta} = \frac{y}{\sin\theta}$. If $OP = r_1$ and $OQ = r_2$ then the coordinates of P are $(r_1 \cos\theta, r_1 \sin\theta)$ and that of Q are $(-r_2 \cos\theta, -r_2 \sin\theta)$

Since P lies on $4x + 2y = 9$, $2r_1(2\cos\theta + \sin\theta) = 9$

and Q lies on $2x + y + 6 = 0$, $-r_2(2\cos\theta + \sin\theta) = -6$

so that $\frac{r_1}{r_2} = \frac{9}{12} = \frac{3}{4}$

and the required ratio is thus 3 : 4.

Alternately Let the equation of the line through O be $y = mx$, then coordinates of P and Q are respectively $\left(\frac{9}{4+2m}, \frac{9m}{4+2m}\right)$ and $\left(\frac{-6}{2+m}, \frac{-6m}{2+m}\right)$ so that

$$\frac{OP}{OQ} = \frac{9}{|4+2m|} \times \frac{|2+m|}{6} = \frac{3}{4}$$

9. If area of the triangle formed by the line L perpendicular to 5x – y = 1 and the coordinate axes is 5, then the distance of L from the origin is

A. $5\sqrt{2}$
B. $5/\sqrt{13}$
C. $5\sqrt{13}$
D. None of these

Sol: B

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L: $y = -(1/5)x + c$, meets axes at $(0, c)$ and $(5c, 0)$, $(1/2) \times 5c \times c = 5 \Rightarrow c^2 = 2$ Distance of L from the origin $= \dfrac{c}{\sqrt{1+1/25}} = \dfrac{5\sqrt{2}}{\sqrt{26}}$

10. If the circumcentre of a triangle lies at the point (a, a) and the centroid is the mid-point of the line joining the points (2a + 3, a + 4) and (a – 4, 2a – 3); then the orthocentre of the triangle lies on the line

A. y = x
B. (a – 1) x + (a + 1) y = 0
C. (a – 1) x – (a + 1) y = 0
D. (a + 1) x – (a – 1) y = 2a

Sol: D

Circumcentre, centroid and the orthocentre lie on the same line.

11. If $y = m_i x + \frac{1}{m_i}$ (i = 1, 2, 3) represents three straight lines whose slopes are the roots of the equation. $2m^3 - 3m^2 - 3m + 2 = 0$, A and B are the algebraic sum of the intercepts made by the lines on x-axis and y-axis respectively, then $\alpha A + \beta B = 0$ if (α, β) is

A. (4, 7)
B. (2, 7)
C. (7, 2)
D. (– 1, – 7)

Sol: B

$(m + 1)(2m - 1)(m - 2) = 0$
$\Rightarrow m_1 = -1,\ m_2 = 1/2,\ m_3 = 2$

$$A = -\sum \frac{1}{m_i^2} = -\frac{21}{4}$$

$$B = \sum \frac{1}{m_i} = \frac{3}{2}$$

$$-\frac{21}{4}\alpha + \frac{3}{2}\beta = 0$$

$$\Rightarrow \quad -21\alpha + 6\beta = 0$$

which is satisfied by (*b*)

i.e. option 2

12. If the slope of one of the lines given by $6x^2 + axy + y^2 = 0$ exceeds the slope of the other by one, then a is equal to

A. ± 2
B. 5
C. - 5
D. ± 5

Sol: D

Let the slopes be m and $m + 1$, then
$2m + 1 = -a$, $m(m + 1) = 6$.

13. The line $\frac{x}{3} + \frac{y}{4} = 1$ meets the axis of y and axis of x at A and B respectively. A square ABCD is constructed on the line segment AB away from the origin, the coordinates of the vertex of the square farthest from the origin are

A. (7, 3)
B. (4, 7)
C. (6, 4)
D. (3, 8)

Sol: B

Coordinates of D are $(5 \cos \alpha, 4 + 5 \sin \alpha)$
$\cos \alpha = 4/5$, $\sin \alpha = 3/5$
check the coordinates of C.

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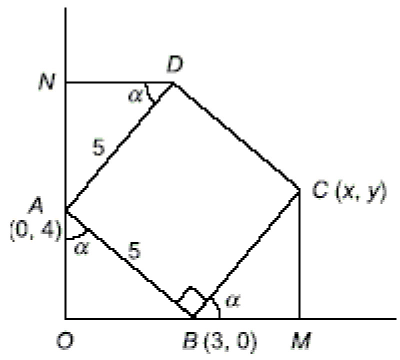

14. If the lines x + ay + a = 0, bx + y + b = 0 and cx + cy + 1 = 0 (a, b, c being distinct and $\neq$ 1) are concurrent, then the value of $\frac{a}{a-1}+\frac{b}{b-1}+\frac{c}{c-1}$ is

A. 1
B. 0
C. 1
D. None of these

Sol: C

$$\frac{x}{a} + y + 1 = 0, \qquad x + \frac{y}{b} + 1 = 0,$$

$$x + y + \frac{1}{c} = 0$$

$$x = \frac{c-1}{a-1}.\frac{a}{c}, \qquad y = \frac{c-1}{b-1}.\frac{b}{c}$$

$$x + y + \frac{1}{c} = 0 \Rightarrow \frac{a}{a-1}+\frac{b}{b-1}+\frac{1}{c-1} = 0$$

$$\Rightarrow \frac{a}{a-1}+\frac{b}{b-1}+\frac{c}{c-1} = 1$$

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15. The mid points of the sides AB and AC of a triangle ABC are (2, – 1) and (– 4, 7) respectively, then the length of BC is

A. 10
B. 20
C. 25
D. 30

Sol: B

Length of BC is twice the length of the line joining the mid-point of AB and AC.

16. If the vertices A and B of a triangle ABC are given by (2, 5) and (4, – 11) and C moves along the line L_1 : 9x + 7y + 4 = 0, the locus of the centroid of the triangle ABC is a straight line parallel to

A. AB
B. BC
C. CA
D. L_1

Sol: D

Let C (h, k), then Centroid is $\left(x = \frac{h+2+4}{3}, y = \frac{k+5-1}{3}\right)$ and $9h + 7k + 4 = 0 \Rightarrow 9(3x - 6) + 7(3y - 4) + 4 = 0$ is the locus of the centroid parallel to L_1.

17. In a right angled triangle ABC right angled at C; CA = a, CB = b. If the angular points A and B slide along x-axis and y-axis respectively then C lies on

A. $bx \pm ay = 0$
B. $ax \pm by = 0$
C. $\frac{x}{a} \pm \frac{y}{b} = 1$
D. $\frac{x}{b} + \frac{y}{a} = 1$

Sol: B

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Let $A(p, 0)$, $B(0, q)$ and $C(h, k)$, then
$(h - p)^2 + k^2 = a^2$, $h^2 + (k - q)^2 = b^2$
and $p^2 + q^2 = a^2 + b^2$
Eliminate p, q

18. If the circumcentre of a triangle lies at the point (a, a) and the centroid is the mid-point of the line joining the points (2a + 3, a + 4) and(a – 4, 2a – 3); then the orthocentre of the triangle lies on the line

A. y = x
B. (a – 1) x + (a + 1) y = 0
C. (a – 1) x – (a + 1) y = 0
D. (a + 1) x – (a – 1) y = 2a

Sol: D
Circumcentre, centroid and the orthocentre lie on the same line.

19. If A $(at^2, 2at)$, $B(a/t^2, – 2a/t)$ and S (a, 0) are three points, then $\frac{1}{SA} + \frac{1}{SB}$ is independent of

A. a
B. t
C. both a and t
D. none of these

Sol: B
$SA = a(1 + t^2)$, $SB = a(1 + t^2)/t^2$

20. The equation $ax^2 + 2hxy + ay^2 = 0$ represent a pair of coincident lines through the origin if

A. h = 2a
B. 2h = a
C. $h^2 = a$
D. $h^2 = a^2$

Sol: D
Lines are coincident if $(h)^2 = a . a$

21. Find the co-ordinates of the points which trisects the line segment PQ formed by joining the point P(4, 2, -6) and Q (10, -16,6)

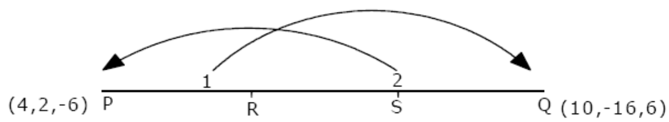
Sol:

Let R and S be the points of trisection of the segment PO. Then

$\therefore PR = RS = SQ$

$\Rightarrow 2PR = RQ$

$\Rightarrow \frac{PQ}{RQ} = \frac{1}{2}$



$\therefore$ R divides PQ in the ratio 1:2

$\therefore$ Co-ordinates of point $R\left[\frac{1(10)+2\times 4}{1+2}, \frac{1(-16)+2\times 2}{1+2}, \frac{1\times 6+2(-6)}{1+2}\right]$

$= R(6,-4,-2)$

Similarly $PS = 2SQ$

$\Rightarrow \frac{PS}{SQ} = \frac{2}{1}$

$\therefore$ S divider PQ in the ratio 2:1

$\therefore$ Co-ordinates of point $R\left[\frac{1(10)+2\times 4}{1+2}, \frac{1(-16)+2\times 2}{1+2}, \frac{1\times 6+2(-6)}{1+2}\right]$

$= R(6,-4,-2)$

Similarly $PS = 2SQ$

$\Rightarrow \frac{PS}{SQ} = \frac{2}{1}$

$\therefore$ S divider PQ in the ratio 2:1

$\therefore$ co-ordinates of point S $\left[\frac{2(10)+1(4)}{1+2}, \frac{2(-16)+1(2)}{1+2}, \frac{2(6)+1(-6)}{1+2}\right]$

$\therefore S(8,-10,2)$

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22. Show that the point P(1, 2, 3), Q (-1, -2, -1), R(2,3,2) and S (4, 7, 6) taken in order form the vertices of a parallelogram. Do these form a rectangle?

Sol:

Mid point of PR is $\left(\frac{1+2}{2},\frac{2+3}{2},\frac{3+2}{2}\right)$

i.e. $\left(\frac{3}{2},\frac{5}{2},\frac{5}{2}\right)$

also mid point of QS is $\left(\frac{-1+4}{2},\frac{-2+7}{2},\frac{-1+6}{2}\right)$

i.e. $\left(\frac{3}{2},\frac{5}{2},\frac{5}{2}\right)$

Then PR and QS have same mid points.

$\therefore$ PR and QS bisect each other. It is a Parallelogram.

Now $PR=\sqrt{(2-1)^2+(3-2)^2+(2-3)^2}=\sqrt{3}$ and

$QS=\sqrt{(4+1)^2+(7+2)^2+(6+1)^2}=\sqrt{155}$

$\therefore PR \neq QS$ diagonals an not equal

$\therefore PQRS$ are not rectangle.

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23. A point R with xco-ordinates 4 lies on the line segment joining the points P(2,-3, 4) and Q (8, 0, 10) find the co-ordinates of the point R

Sol:

Let the point. R divides the line segment joining the point P and Q in the ratio $\lambda:1$, Then co-ordinates of Point R

$$\left[\frac{8\lambda+2}{\lambda+1},\frac{-3}{\lambda+1},\frac{10\lambda+4}{\lambda+1}\right]$$

The x co-ordinates of point R is 4

$$\Rightarrow \frac{8\lambda+2}{\lambda+1}=4 \quad , \quad \lambda=\frac{1}{2}$$

$\therefore$ co-ordinates of point R

$$\left[4,\frac{-3}{\frac{1}{2}+1},\frac{10\times\frac{1}{2}+4}{\frac{1}{2}+1}\right] \qquad \text{i.e.}(4,-2,6)$$

24. If the points P(1, 0, -6), Q (-3, P,q) and R(-5, 9,6) are collinear, find the values of P and q

Sol:

Given points

$P(1,0,-6)$, $Q(-3,P,q)$ and $R(-5,9,6)$ are collinear

Let point Q divider PR in the ratio K:1

$\therefore$ co-ordinates of point $P\left(\frac{1-5K}{K+1},\frac{0+9K}{K+1},\frac{-6+6K}{K+1}\right)$

$Q(-3,P,q)$

$$\frac{1-5K}{K+1}=-3$$

$$1-5K=-3K-3$$

$$-2K=-4$$

$$K=\frac{-4}{-2}$$

$$K=2$$

$\therefore$ the value of P and q are 6 and 2.

25. Three consecutive vertices of a parallelogram ABCD are A (3, -1, 2), B (1, 2, -4) and C (-1, 1, 2) find forth vertex D

Sol:

Given vertices of 11gm ABCD

$A(3,-1,2),\quad B(1,2,-4),\quad C(-1,1,2)$

Suppose co-or dine of forth vertex $D(x,y,z)$

Mid point of $AC\left(\frac{3-1}{2},\frac{-1+1}{2},\frac{2+2}{2}\right)$

$=(1,0,2)$

Mid point of $BD\left(\frac{x+1}{2},\frac{y+2}{2},\frac{-4+z}{2}\right)$

Mid point of AC = mid point of BD

$\frac{x+1}{2}=1 \Rightarrow x=1$

$\frac{y+2}{2}=0 \Rightarrow y=-2$

$\frac{-4+z}{2}=2 \Rightarrow z=8$

Co-ordinates of point $D(1,-2,8)$