Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Class: XI
Subject: Maths
Topic: Limits and Derivatives
No. of Questions: 25

1. The notation ' $x \to 2^-$ ' denotes which of the following options?

A. x approaches to 2 from its left hand side.
B. x approaches to 2 from its right hand side.
C. x does not approach to 2.
D. none of these

Sol: A

2. Which of the following indeterminate form is the most fundamental one?

A. $\frac{\infty}{\infty}$
B. $0 \times \infty$
C. $\infty - \infty$
D. $\frac{0}{0}$

Sol: D

3. $\lim_{x \to a} \left\{ \frac{x^{45} - a^{45}}{x - a} \right\}$ is equal to

A. 45a
B. $45a^{44}$
C. $45a^{45}$
D. $44a^{45}$

Sol: B

Hint: $\left[\lim_{x\to a}\left(\frac{x^n - a^n}{x - a}\right) = na^{n-1}\right]$

4. $\lim_{x\to 0}\left(\frac{e^{4x} - 1}{x}\right)$ is equal to

A. 4x
B. 4
C. 3
D. $\frac{4}{x}$

Sol: B

$\left[\because \lim_{y\to 0}\left(\frac{e^y - 1}{y}\right) = 1\right]$

5. $\lim_{x\to 0}\left(\frac{4^x - 2^x}{x}\right)$ is equal to

A. log 4
B. log 4^x
C. log 2^x
D. log 2

Sol: D

Hint. $\lim_{x\to 0}\left(\frac{a^x - 1}{x}\right) = \log a$

6. $\lim_{x\to 0} \frac{\tan 3x}{\sin 5x}$ is equal to

A. $\frac{3}{5}$
B. $\frac{5}{3}$
C. 3
D. 1

Sol: A

7. $\lim_{x\to 0} \left(\frac{\sin x - \sin x \cos x}{x^3 \cos x}\right)$ is equal to

(A) $\frac{1}{2}$
(B) $\frac{2}{3}$
(C) $\frac{3}{2}$
(D) $\frac{1}{4}$

Sol: A

$$\lim_{x\to 0} \frac{\sin x(1-\cos x)}{x^3 \cos x} \Rightarrow \lim_{x\to 0} \left\{\frac{\sin x}{x} \cdot \frac{2\sin^2(x/2)}{(x/2)^2} \cdot \frac{1}{4} \cdot \frac{1}{\cos x}\right\}$$

$$\Rightarrow \frac{1}{2} \cdot \lim\left(\frac{\sin(x/2)}{(x/2)}\right)^2 \cdot \lim_{x\to 0} \frac{1}{\cos x}$$

$$= \left(\frac{1}{2} \times 1 \times 1^2 \times 1\right) = \frac{1}{2}$$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

8. What will be the value of 'k' if $\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \lim_{x \to k} \frac{x^4 - k^4}{x^3 - k^3}$?

A. $\frac{3}{2}$
B. $\frac{9}{4}$
C. $\frac{4}{3}$
D. 3

Sol: B

$\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \lim_{x \to 1} \frac{x^3 - 1^3}{x^3 - 1^3} = 3\,(1)^{3-1} = 3$

$\lim_{x \to a} \frac{x^4 - k^4}{x^3 - k^3} = \frac{x^4 - k^4}{(x - k)} \times \frac{(x - k)}{x^3 - k^3} = 4k^3 \div \frac{x^3 - k^3}{(x - k)} = 4k^3 \div 3k^2$

$= \frac{4k^3}{3k^2} = \frac{4}{3}k$

$\therefore\ 3 = \frac{4}{3}k$

$\frac{3 \times 3}{4} = k$

$k = \frac{9}{4}$

9. $\lim_{x \to 0} \frac{(a^x + b^x + c^x + d^x)}{4}$ is equal to

A. $(abcd)^{1/4}$
B. $\frac{abcd}{4}$
C. e^{abcd}
D. $\log(abc)^{\frac{1}{4}}$

Sol: A

$$\lim_{x\to 0}\left(\frac{a^x+b^x+c^x+d^x-4}{4}\right)^{\frac{1}{x}} = \lim_{x\to 0}\left\{\frac{1+(a^x-1)+(b^x-1)+(c^x-1)}{4}\right\}^{\frac{1}{4}} = \lim_{e^x\to 0}\frac{a^{x-1}}{4x} + \frac{b^{x-1}}{4x} + \frac{c^{x-1}}{4x} + \frac{d^x-1}{4x}$$

$$= e^{\frac{1}{4}}\left[\lim_{x\to 0}\frac{a^x-1}{x} + \lim_{x\to 0}\frac{b^x-1}{x} + \lim_{x\to 0}\frac{e^x-1}{x} + \lim_{d\to 0}\frac{d^x-1}{x}\right]$$

$$= e^{\frac{1}{4}}\{\log a + \log b + \log c + \log (D) = e^{\log(abcd)\frac{1}{4}} = (abcd)^{1/4}$$

10. $\lim_{x\to\frac{\pi}{2}}\frac{\cot x - \cos x}{(\pi - 2x)^3}$ is equal to

A. $\frac{1}{16}$

B. $\frac{1}{8}$

C. $\frac{2}{3}$

D. $\frac{1}{4}$

Sol: A

$$\lim_{h\to 0}\frac{\cot\left(\frac{\pi}{2}+h\right)-\cos\left(\frac{\pi}{2}+h\right)}{\left\{\pi-2\left(\frac{\pi}{2}+h\right)\right\}^3} = \lim_{h\to 0} - \frac{\tan h + \sin h}{-8h^3}$$

$$= \frac{1}{8}\lim_{h\to 0}\frac{\tan h}{h}\times\frac{1-\cos h}{h^2} = \frac{1}{8}\lim_{h\to 0}\frac{\tan h}{h}\times\lim_{h\to 0}\frac{2\sin^2\frac{h}{2}}{h^2}$$

$$\frac{1}{8}\times\frac{2}{4}\lim_{h\to 0}\frac{\sin^2\frac{h}{2}}{\left(\frac{h}{2}\right)^2} = \frac{1}{16}$$

11. If $y = \cos^2 x$, then $\frac{dy}{dx}$ is equal to

A. sin 2x
B. 2 sin x
C. $\sin^2 x$
D. - sin 2x

Sol: D

$y = \cos^2 x$
$y = (\cos x)^2 \Rightarrow -2\cos x \sin x \Rightarrow -\sin 2x$

12. If $y = \cos(x^2 + 1)$, then dy/dx is equal to

A. $\sin(x^2 + 1)$
B. $2x \sin(x^2 + 1)$
C. $\sin(x^2 + 1)$
D. sin 2x

Sol: B

$y = \cos(x^2 + 1)$, $\frac{dy}{dx} = -\sin(x^2 + 1) \cdot 2x$
$\Rightarrow -2x \sin(x^2 + 1)$

13. Find the derivative of $\tan x^2$.

A. $\sec x^2$
B. sec 2x
C. $2x \sec^2 x^2$
D. $2 \sec^2 x^2$

Sol: C

$y = \tan x \Rightarrow \frac{dy}{dx} = \sec^2 x^2 \cdot 2x \Rightarrow 2x \sec^2 x^2$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

14. Find the derivative of $e^{\sqrt{x}}$.

A. $e^{\sqrt{x}}$

B. $\frac{1}{2} e^{\sqrt{x}}$

C. $\frac{e^{\sqrt{x}}}{\sqrt{x}}$

D. $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$

Sol: D

$y = e^{\sqrt{x}}, \frac{dy}{dx} = e^{\sqrt{x}} . \frac{1}{2} x^{\frac{1}{2}-1} = e^{\sqrt{x}} . \frac{1}{2\sqrt{x}} = \frac{1}{2} \frac{e^{\sqrt{x}}}{\sqrt{x}}$

15. If $y = \sqrt{\sin 3x}$, then $\frac{dy}{dx}$ is equal to

A. $\sqrt{\cos 3x}$

B. $\frac{3}{2} \cos \sqrt{\sin 3x}$

C. $\frac{3 \cos 3x}{2\sqrt{\sin 3x}}$

D. $\sqrt{\sin 3x}$. cos x

Sol: C

$y = \sqrt{\sin 3x}\ \frac{dy}{dx} = \frac{1}{2} (\sin 3x)^{\frac{1}{2}-1} = \frac{1}{2} (\sin 3x)^{-\frac{1}{2}}$. 3 cos 3x

$= \frac{3 \cos 3x}{2\sqrt{\sin 3x}}$

16. Find the derivative of $x\,e^x$.

A. $(x + 1)\,e^x$
B. e^x
C. $x + e^x$
D. $x + 1$

Sol: A

Using product rule:
$xe^x + e^x \Rightarrow (x + 1)\,e^x$

17. If $y = \dfrac{x}{\sqrt{1 - x^2}}$, find $\dfrac{dy}{dx}$.

A. $\dfrac{1}{(1 - x^2)^{\frac{3}{2}}}$
B. $\dfrac{1}{(1 - x^2)^{\frac{1}{2}}}$
C. $\sqrt{1 - x^2}$
D. $(1 - x^2)^{\frac{3}{2}}$

Sol: A

$$y = \frac{x}{\sqrt{1 - x^2}} = \sqrt{1 - x^2}\,.\,\frac{d(x)}{dx} - x\,.\,\frac{d}{dx}\left(\sqrt{1 - x^2}\right)$$

$$\Rightarrow \frac{(1 - x^2) + x^2}{(1 - x^2)^{\frac{3}{2}}} = \frac{1}{(1 - x^2)^{\frac{3}{2}}}$$

18. If $y = \sqrt{\frac{1 - \tan x}{1 + \tan x}}$, find $\frac{dy}{dx}$.

A. $\frac{-\sec^2 x}{(1 + \tan x)^{\frac{3}{2}}(1 - \tan x)^{\frac{1}{2}}}$

B. $\frac{\sec^2 x}{(1 + \tan x)^{\frac{3}{2}}(1 - \tan x)^{\frac{1}{2}}}$

C. $\frac{-\sec^2 x}{(1 + \tan x)^2}$

D. $\frac{-\sec^2 x}{(1 - \tan x)^{\frac{3}{2}}(1 + \tan x)^{\frac{1}{2}}}$

Sol: A

Putting $\frac{1 - \tan x}{1 + \tan x}$ = t, we get y = $\sqrt{t}$ and t = $\frac{1 - \tan x}{1 + \tan x}$ $\frac{dy}{dx} = \frac{1}{2} t^{\frac{-1}{2}} = \frac{1}{2\sqrt{t}}$

And $\frac{dt}{dx} = \frac{-2\sec^{2x}}{(1 + \sin x)^2}$

$\therefore \frac{dy}{dx} = \left(\frac{dy}{dt} \times \frac{dt}{dx}\right) = \frac{1}{2\sqrt{t}} \times \frac{-2\sec^2 x}{(1 + \tan x)^2}$

$= \frac{-\sec^2 x}{(1 + \tan x)^2} \times \frac{\sqrt{1 + \tan x}}{\sqrt{1 - \tan x}}$

$= \frac{-\sec^2 x}{(1 + \tan x)^{\frac{3}{2}}(1 - \tan x)^{\frac{1}{2}}}$

19. If $y = \cos^2 x^2$, then find $\frac{dy}{dx}$.

A. $2x \sin (2x^2)$
B. $\sin (2x^2)$
C. $x \sin (2x^2)$
D. $2x \cos (2x^2)$

Sol: A

$y = (\cos x^2)^2$, put $x^2 = t$ and $\cos x^2 = \cos t = u$, So, that $y = u^2 = u = \cos t$ and $t = x^2$

$\therefore \frac{dy}{du} = 2u$, $\frac{du}{dt} = -\sin t$ and $\frac{dt}{dx} = 2x$

So, $\frac{dy}{dx} = \left(\frac{dy}{du} \times \frac{du}{dt} \times \frac{dt}{dx}\right)$

$= -4ux \sin t = -4x \sin t \cos t$ $[\because u = \cos t]$

$= -4x \sin x^2 \cos x^2 = -2x \sin (2x^2)$ $[\because t = x^2]$

20. Find the derivative of $\left(\frac{\sin x + \tan x}{\sin x - \tan x}\right)$.

A. $\frac{-2}{(1 - \sin 2x)}$

B. $\frac{2}{1 - \sin 2x}$

C. $\frac{2}{1 + \sin x}$

D. $\frac{-2}{1 + \sin x}$

Sol: A

$$\frac{(\sin x - \cos x) \cdot \frac{d}{dx}(\sin x + \cos x) - (\sin x + \cos x) \cdot \frac{d}{dx}(\sin x - \cos x)}{(\sin x - \cos x)^2} =$$

$$\frac{(\sin x - \cos x)(\cos x - \sin x) - (\sin x + \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^2}$$

$$= \frac{-[(\sin x - \cos x)^2 + (\sin x + \cos x)^2]}{(\sin x - \cos x)^2}$$

$$= \frac{-2(\sin^2 x + \cos^2 x)}{(\sin^2 x + \cos^2 x - 2\sin x \cos x)} = \frac{-2}{(1 - \sin 2x)}$$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

21. Evaluate $\lim_{x\to\frac{\pi}{2}} \frac{1+\cos 2x}{(\pi-2x)^2}$

Sol:

$$\lim_{x\to\frac{\pi}{2}} \frac{1+\cos 2x}{(\pi-2x)^2}$$

$$let\ \pi-2x=y$$

$$2x=\pi-y$$

$$x\to\frac{\pi}{2},\ y\to 0$$

$$\lim_{y\to 0}\frac{1+\cos(\pi-y)}{y^2}=\lim_{y\to 0}\frac{1-\cos y}{y^2}$$

$$=\lim_{y\to 0}\frac{2\sin^2\frac{y}{2}}{4\times\frac{y^2}{4}}$$

$$=\lim_{y\to 0}\frac{1}{2}\times\frac{\sin^2\frac{y}{2}}{\left(\frac{y}{2}\right)^2}$$

$$=\frac{1}{2}\lim_{\frac{y}{2}\to 0}\left[\frac{\sin\frac{y}{2}}{\frac{y}{2}}\right]^2$$

$$=\frac{1}{2}\times 1^2=\frac{1}{2}$$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

22. Differentiate the function y = $\frac{(x+2)(3x-1)}{(2x+5)}$ with respect to x

Sol:

$$y=\frac{(x+2)(3x-1)}{(2x+5)}$$

$$\frac{dy}{dx}=\frac{d}{dx}\frac{(x+2)(3x-1)}{(2x+5)}$$

$$=\frac{(2x+5)\frac{d}{dx}(x+2)(3x-1)-(x+2)(3x-1)\frac{d}{dx}(2x+5)}{(2x+5)^2}$$

$$=\frac{(2x+5)\left[(x+2)\frac{d}{dx}(3x-1)+(3x-1)\frac{d}{dx}(x+2)\right]-(x+2)(3x-1)[2+0]}{(2x+5)^2}$$

$$=\frac{(2x+5)[(x+2)\times 3+(3x-1)\times 1]-2\left[3x^2+6x-x-2\right]}{(2x+5)^2}$$

$$=\frac{(2x+5)[3x+6+3x-1]-6x^2-12x+2x+4}{(2x+5)^2}$$

$$=\frac{12x^2+30x+\cancel{10x}+25-6x^2-\cancel{10x}+4}{(2x+5)^2}$$

$$=\frac{6x^2+30x+29}{(2x+5)^2}$$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

23. Find $\lim_{x\to 5} |x|-5$

Sol:

$$L.H.S. \lim_{x\to 5^-} f(x)$$

$$x = 5 - h$$

$$x \to 5, h \to 0$$

$$\lim_{h\to 0} f(5-h)$$

$$\lim_{h\to 0} |5-h|-5$$

$$= 0$$

$$R.H.S. \lim_{x\to 5^+} f(x)$$

$$put \ x = 5 + h$$

$$x \to 5, \ h \to 0$$

$$\lim_{h\to 0} f(5+h) = \lim_{h\to 0} |5+h|-5$$

$$= 0$$

$$R.H.S. = R.H.S.$$

$$\therefore \lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x)$$

$$\therefore \lim_{x\to 5} f(x) \text{ e} xist$$

$$\therefore \lim_{x\to 5} f(x) = 0$$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

24. *Find* $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$ where $f(x)=\begin{cases} 2x+3; x\le 0 \\ 3(x+1); (x>0) \end{cases}$

Sol:

$$given\ f(x)=\begin{cases} 2x+3, x\le 0 \\ 3(x+1), x>0 \end{cases}$$

$$for\ x=0$$

$$L.H.S. \lim_{x\to 0^-} f(x)=\lim_{x\to 0} 2x+3=3$$

$$R.H.S. \lim_{x\to 0^+} f(x)=\lim_{x\to 0} 3(x+1)=3$$

$$L.H.S.=R.H.S.$$

$$\therefore \lim_{x\to 0} f(x)\ exist$$

$$\therefore \lim_{x\to 0} f(x)=3$$

$$for\ x=1$$

$$L.H.S.$$

$$\therefore \lim_{x\to 1^-} f(x)=\lim_{x\to 1} 3(x+1)$$

$$=3(1+1)=6$$

$$R.H.S.$$

$$\lim_{x\to 1^+} f(x)=\lim_{x\to 1} 3(x+1)$$

$$=3(1+1)=6$$

$$L.H.S.=R.H.S.$$

$$\therefore \lim_{x\to 1} f(x)\ exist$$

$$\lim_{x\to 1} f(x)=6$$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

25. *Find derivative of sec x by first principle*

Sol:

$$let\ f(x)=\sec x$$

$$f(x+h)=\sec(x+h)$$

$$f^1(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\sec(x+h)-\sec x}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{\cos(s+h)}-\frac{1}{\cos x}}{h}$$

$$=\lim_{h\to 0}\frac{\cos x-\cos(x+h)}{\cos(x+h)\cos xh}$$

$$=\lim_{h\to 0}\frac{-2\sin\left[\frac{2x+h}{2}\right]\sin\left[\frac{-h}{2}\right]}{\cos(x+h)\cos x.h}$$

$$=\lim_{h\to 0}\frac{2\sin\left[\frac{2x+h}{2}\right]\sin\frac{h}{2}}{\cos(x+h)\cos xh}\qquad\left[\sin(-\theta=-\sin\theta)\right]$$

$$=\lim_{h\to 0}\frac{2\sin\frac{h}{2}}{2\frac{h}{2}}\times\frac{\lim_{h\to 0}\sin\frac{(2x+h)}{2}}{\lim_{h\to 0}\cos(x+h)\cos x}$$

$$=1\times\frac{\sin\left(\frac{2x+0}{2}\right)}{\cos(x+0)\cos x}=\frac{\sin x}{\cos x\ \cos x}$$

$$=\tan x\ \sec x$$