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Class: XI
Subject: Math's
Topic: Permutation & Combination
No. of Questions: 25

1. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P., then the value of n can be

A. 6
B. 7
C. 8
D. 9

Sol: B

As ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P.,

$$2\,({}^nC_5) = {}^nC_4 + {}^nC_6$$

$$\Rightarrow \quad 2 = \frac{{}^nC_4}{{}^nC_5} + \frac{{}^nC_6}{{}^nC_5}$$

$$= \frac{n!}{4!\,(n-4)!}\frac{5!(n-5)!}{n!} + \frac{n!}{6!\,(n-6)!}\frac{5!(n-5)!}{n!}$$

$$\Rightarrow \quad 2 = \frac{5}{n-4} + \frac{n-5}{6} \qquad \Rightarrow \qquad n^2 - 21n + 98 = 0$$

$$\Rightarrow \quad n = 7,\ 14.$$

2. 2. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then the value of r is equal to

A. 1
B. 2
C. 3
D. 4

Sol: C

We have

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{36}{84} \Rightarrow \frac{n!}{(r-1)!\,(n-r+1)!}\,\frac{r!\,(n-r)!}{n!} = \frac{3}{7}$$

$$\Rightarrow \qquad \frac{r}{n-r+1} = \frac{3}{7} \qquad \Rightarrow 10r = 3n + 3$$

Similarly, $\frac{^nC_r}{^nC_{r+1}} = \frac{84}{126} \Rightarrow \frac{r+1}{n-r} = \frac{2}{3}$

$\Rightarrow$ 5r + 3 = 2n $\Rightarrow$ 5r + 3 = 2n

Solving we obtain r = 3.

3. The number of words that can be formed by using the letters of the word MATHEMATICS that start as well as end with T is

A. 80720
B. 90720
C. 20860
D. 37528

Sol: B

The word MATHEMATICS contains 11 letters viz. M, M, A, A T, T, H, E, I, C, S. The number of words that begin with T and end with T is

$$\frac{9!}{2!\,2!} = 90720.$$

4. 4. m men and w women are to be seated in a row so that no two women sit together. If m > w, then the number of ways in which they can be seated is

A. $\frac{m!(m+1)!}{(m-w+1)!}$
B. $^mC_{m-w}(m-w)!$
C. $^{m+w}C_m\ (m-w)!$
D. none of these

Sol: A

We first arrange the m men. This can be done in m! ways. After m men have taken their seats, the women must choose w seats out of (m + 1) seats marked with X below.

X M X M X M X …X M X
1st 2nd 3rd mth

They can choose w seats in $^{m+1}C_w$ ways and take their seats in w! ways.

Thus, the required number of arrangements is

$$m!(^{m+1}C_w)\ (w!) = \frac{m!\,(m+1)!\,w!}{w!\,(m+1-w)!} = \frac{m!\,(m+1)!}{(m+1-w)!}$$

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5. The number of positive integers < 1,00,000 which contain exactly one 2, one 5 and one 7 in its decimal representation is

A. 2940
B. 7350
C. 2157
D. 1582

Sol: A

We may consider a 5-digit number as a number of the form

X X X X X

where X is a digit from 0 to 9. The digit 2 can occupy any of the five places, 3 can occupy any of the remaining 4 places and 7 in any of the 3-remaining places. The remaining 2 places can be filled up by 7 digits. Thus, there are (5) (4) (3) (7) (7) = 2940 positive integers in the desired category.

6. The sum S = $\frac{1}{9!}+\frac{1}{3!7!}+\frac{1}{5!5!}+\frac{1}{7!3!!}+\frac{1}{9!}$ equals

A. $\frac{2^9}{10!}$
B. $\frac{2^{10}}{8!}$
C. $\frac{2^{11}}{9!}$
D. $\frac{2^{10}}{7!}$

Sol: A

We can write S as follows

$$S=\frac{1}{10!}[{}^{10}C_1+{}^{10}C_3+{}^{10}C_5+{}^{10}C_7+{}^{10}C_9]$$

$$=\frac{1}{10!}(2^9).$$

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7. If $0 < r < s \leq n$ and ${}^nP_r = {}^nP_s$; then value of r + s is

A. 2n – 2
B. 2n – 1
C. 2
D. 1

Sol: B

$${}^nP_r = {}^nP_s \Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{(n-s)!}$$

$\Rightarrow$ (n – r) ! = (n – s)!

As r < s, n – r > n – s. But the only two different factorials which are equal are 0! and 1!. Thus n – r = 1 and n – s = 0

$\Rightarrow$ r = n – 1 ans s = n.

$\Rightarrow$ r + s = 2n – 1.

8. The value of $E = \dfrac{(1+17)\left(1+\frac{17}{2}\right)\left(1+\frac{17}{3}\right)\ldots\left(1+\frac{17}{19}\right)}{(1+19)\left(1+\frac{19}{2}\right)\left(1+\frac{19}{3}\right)\ldots\left(1+\frac{19}{17}\right)}$ is

A. 1
B. ${}^{36}C_{17}$
C. 2/19
D. ${}^{36}C_1$

Sol: A

We have

$$(1+k)\left(1+\frac{k}{2}\right)\left(1+\frac{k}{3}\right)\ldots\left(1+\frac{k}{n}\right)$$

$$= \frac{(1+k)(2+k)(3+k)\ldots(n+k)}{(2)(3)\ldots(n)} = \frac{(n+k)!}{k!\,n!}$$

$= {}^{n+k}C_k$

Thus, both the numerator and the denominator of E equals

${}^{36}C_{17} = {}^{36}C_{19}$.

$\therefore$ E = 1.

9. If [y] denote the greatest integer $\le$ y, and $2\left[\frac{x}{8}\right]^2 + 3\left[\frac{x}{8}\right] = 20,$ then x lies in the smallest interval [a, b) where b – a is equal to

A. 6
B. 5
C. 4
D. 8

Sol: D

$$2\left[\frac{x}{8}\right]^2 + 3\left[\frac{x}{8}\right] - 20 = 0$$

$$\Rightarrow \left[\frac{x}{8}\right] = \frac{5}{2} \text{ or } -4$$

As $\left[\frac{x}{8}\right]$ is an integer, we take

$$\left[\frac{x}{8}\right] = -4 \quad \Rightarrow \quad -4 \le x/8 < -3$$

$\Rightarrow -32 \le x < -24$. Thus, $a = -32$, and $b = -24$. Therefore, $b - a = 8$

10. Let T_n denote the number of triangles which can be formed by using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals

A. 5
B. 7
C. 6
D. 4

Sol: B

The number of triangles that can be formed by using the vertices of a regular polygon is ${}^nC_3$. That is, $T_n = {}^nC_3$

Now, $T_{n+1} - T_n = 21$

$\Rightarrow {}^{n+1}C_3 - {}^nC_3 = 21$

$\Rightarrow {}^nC_2 + {}^nC_3 - {}^nC_3 = 21$ $[\because {}^{n+1}C_r = {}^nC_{r-1} + {}^nC_r]$

$\Rightarrow \frac{1}{2}n(n-1) = 21$

$\Rightarrow n = -6$ or 7.

As n is a positive integer, n = 7.

11. The number of rational numbers lying in the interval (2015, 2016) all whose digits after the decimal point are non-zero and are in decreasing order is

A. $\sum_{i=1}^{9} {}^9P_i$
B. $\sum_{i=1}^{10} {}^9P_i$
C. $2^9 - 1$
D. $2^{10} - 1$

Sol: C

A rational number of the desired category is of the form 2015. $x_1\, x_2 \ldots x_k$ where $1 \le k \le 9$ and $9 \ge x_1 > x_2 > \ldots > x_k \ge 1$. We can choose k digits out 9 in ${}^9C_k$ ways and arrange them in decreasing order in just one way. Thus, the desired number of rational numbers is ${}^9C_1 + {}^9C_2 + \ldots + {}^9C_9 = 2^9 - 1$.

12. 12. The number of positive integral solution of the equation $x_1\, x_2\, x_3\, x_4\, x_5 = 1050$ is

A. 1800
B. 1600
C. 1400
D. None of these

Sol: D

Using prime factorization of 1050, we can write the given equation as
$x_1\, x_2\, x_3\, x_4\, x_5 = 2 \times 3 \times 5^2 \times 7$
We can assign 2, 3 or 7 to any of 5 variables. We can assign entire 5^2 to just one variable in 5 ways or can assign $5^2 = 5 \times 5$ to two variables in ${}^5C_2$ ways. Thus, 5^2 can be assigned in
${}^5C_1 + {}^5C_2 = 5 + 10 = 15$ ways
Thus, the required number of solutions is $5 \times 5 \times 5 \times 15 = 1875$

13. The number of positive integers n such that 2^n divides n! is

A. exactly 1
B. exactly 2
C. infinite
D. none of these

Sol: D

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The exponent of 2 in $n!$ is given by

$$E = \left[\frac{n}{2}\right] + \left[\frac{n}{2^2}\right] + \left[\frac{n}{2^3}\right] + \ldots$$

where $[x]$ denotes greatest integer $\le x$, As $[x] \le x \;\forall\; x$,

$\left[\frac{n}{2^m}\right] = 0$ after finite number of terms. Thus we get

$$E < \frac{n}{2} + \frac{n}{2^2} + \frac{n}{2^3} + \ldots = \frac{n/2}{1 - 1/2} = n$$

Thus, there is no positive integer for which 2^n divides $n!$

14. The value of $^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is

A. $^{56}C_3$
B. $^{56}C_4$
C. $^{55}C_4$
D. $^{55}C_3$

Sol: B

$^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$

$= {}^{50}C_4 + ({}^{55}C_3 + {}^{54}C_3 + \ldots + {}^{50}C_3)$

$= ({}^{50}C_4 + {}^{50}C_3) + ({}^{51}C_3 + \ldots + {}^{55}C_3)$

$= ({}^{51}C_4 + {}^{51}C_3) + ({}^{52}C_3 + \ldots + {}^{55}C_3)$

$= \ldots\ldots$

$= {}^{56}C_4$

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15. The sum of the factors of 9! which are odd and are of the form 3m + 2, where m is a natural number is

A. 40
B. 45
C. 51
D. 54

Sol: A

We have $9! = 2^7 \times 3^4 \times 5 \times 7$

Odd factors of the form 3m + 2 are neither multiples of 2 nor multiples of 3. So the factors may be 1, 5, 7, 35 of which just 5 and 35 are of the form 3m + 2. Their sum is 40.

16. The number of ordered pairs (m, n), m, n $\in$ {1, 2, …, 100} such that $7^m + 7^n$ is divisible by 5 is

A. 1250
B. 2000
C. 2500
D. 5000

Sol: C

Note that 7^r $(r \in N)$ ends in 7, 9, 3 or 1 (corresponding to $r = 1, 2, 3$ and 4 respectively.)
Thus, $7^m + 7^n$ cannot end in 5 for any values of $m, n \in N$.
In other words, for $7^m + 7^n$ to be divisible by 5, it should end in 0.
For $7^m + 7^n$ to end in 0, the forms of m and n should be as follows:

	m	n
1	$4r$	$4s + 2$
2	$4r + 1$	$4s + 3$
3	$4r + 2$	$4s$
4	$4r + 3$	$4s + 1$

Thus, for a given value of m there are just 25 values of n for which $7^m + 7^n$ ends in 0. [For instance, if $m = 4r$, then $n = 2, 6, 10, \ldots, 98$]
$\therefore$ there are $100 \times 25 = 2500$ ordered pairs (m, n) for which $7^m + 7^n$ is divisible by 5.

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17. The number of ways of arranging p numbers out of 1, 2, 3, …, q so that maximum is q – 2 and minimum is 2 (repetition of number is allowed) such that maximum and minimum both occur exactly once, (p > 5, q > 3) is

A. ${}^{p-3}C_{q-2}$
B. ${}^{p}C_2\,(q-3)^{q-1}$
C. ${}^{p}C_2 \times {}^{q}C_3$
D. $p(p-1)(q-5)^{p-2}$

Sol: D
First we take one of the numbers as 2 and one another as q – 2. We can arrange these two numbers in p(p – 1) ways. We have to choose remaining p– 2 numbers from the numbers 3, 4, …, q – 4, q – 3. This can be done in $(q-5)^{p-2}$ ways.
Thus, the total number of ways of arranging the numbers in desired way is
$p(p-1)(q-5)^{p-2}$.

18. The number of integers x, y, z, w such that x + y + z + w = 20 and x, y, z, w $\geq$ – 1, is

A. ${}^{24}C_3$
B. ${}^{25}C_3$
C. ${}^{26}C_3$
D. ${}^{27}C_3$

Sol: D
Put x = a – 1, y = b – 1, z = c – 1, w = d – 1, then a, b, c, d $\geq$ 0 and (a – 1) + (b – 1) + (c – 1) + (d – 1) = 20
$\Rightarrow$ a + b + c + d = 24
The number of non-negative integral solutions of this equation is
$${}^{24+4-1}C_{4-1} = {}^{27}C_3$$

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19. There are three piles of identical yellow, black and green balls and each pile contains at least 20 balls. The number of ways of selecting 20 balls if the number of black balls to be selected is twice the number of yellow balls, is

A. 6
B. 7
C. 8
D. 9

Sol: B

Let the number of yellow balls be x, that of black be 2x and that of green be y. Then
$x + 2x + y = 20$ or $3x + y = 20$
$\Rightarrow$ $y = 20 - 3x$.
As $0 \leq y \leq 20$, we get $0 \leq 20 - 3x \leq 20$
$\Rightarrow$ $0 \leq 3x \leq 20$ or $0 \leq x \leq 6$
$\therefore$ The number of ways of selecting the balls is 7.

20. The exponent of 7 in ${}^{100}C_{50}$ is

A. 0
B. 2
C. 4
D. None of these

Sol: A

We have ${}^{100}C_{50} = \frac{100!}{50!50!}$.

The exponent of 7 in 50 ! is $\left[\frac{50}{7}\right] + \left[\frac{50}{7^2}\right] = 7 + 1 = 8$, and the exponent of 7 in 100! is $\left[\frac{100}{7}\right] + \left[\frac{100}{7^2}\right]$
$= 14 + 2 = 16$
Thus, exponent of 7 in ${}^{100}C_{50}$ is 16 – 2 = (8) = 0.

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21. How many words with or without meaning can be formed using all the letters of the word EQUATION' at a time so that vowels and consonants occur together

Sol: In the word 'EQUATION' there are 5 vowels [A.E.I.O.U.] and 3 consonants [Q.T.N]

Total no. of letters = 8

Arrangement of 5 vowels = $\lfloor 5$

Arrangements of 3 consonants = $\lfloor 3$

Arrangements of vowels and consonants = $\lfloor 2$

$\therefore$ Total number of words $= \lfloor 5 \times \lfloor 3 \times \lfloor 2$

$= 5.4.3.2.1 \times 3.2.1 \times 2.1 = 1440$

22. From a class of 25 students 10 are to be chosen for an excursion party. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can excursion party be chosen?

Sol:

Total no. of students = 25

No. of students to be selected = 10

I case :

3 students all of them will join the excursion party.

Then remaining 7 students will be selected out of (25-3 = 22) in ${}^{22}C_7$ ways

II case :

All 3 students will not join the party then 10 students will be selected in ${}^{22}C_{10}$ ways

Total no. of selection $= {}^{22}C_7 + {}^{22}C_{10}$

$= \frac{\lfloor 22}{\lfloor 15 \lfloor 7} + \frac{\lfloor 22}{\lfloor 12 \lfloor 10} = 817190$

23. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

Sol:

No. of red balls = 6

No. of white balls = 5

No. of blue balls = 5

No. of selecting each colour balls = 3

$\therefore$ required no. of selection = ${}^6C_3 \times {}^5C_3 \times {}^5C_3$

$$= \frac{\lfloor 6}{\lfloor 3 \lfloor 3} \times \frac{\lfloor 5}{\lfloor 2 \lfloor 3} \times \frac{\lfloor 5}{\lfloor 2 \lfloor 3}$$

$$= \frac{6.5.4.\lfloor 3}{\lfloor 3.3.2.1} \times \frac{5.4^2.\lfloor 3}{2.1.\lfloor 3} \times \frac{5.{}^2 4.\lfloor 3}{2.1.\lfloor 3}$$

$$= 5 \times 4 \times 5 \times 2 \times 5 \times 2$$

$$= 20 \times 10 \times 10 = 2000$$

24. Find the number of 3 digit even number that can be made using the digits 1, 2, 3, 4, 5, 6, 7, if no digit is repeated ?

Sol: For making 3 digit even numbers unit place of digit can be filled in 3 ways. Ten's place of Digit can be filled in 5 ways. Hundred place of digit can be filled in 4 ways $\therefore$ required Numbers of 3 digit number $= 3 \times 4 \times 4 = 60$

25. Prove that the product r of consecutive positive integer is divisible by $\lfloor r$

Sol:

Suppose & consecutive positive integers are $(n+1),\ (n+2),\ ......(n+r)$

Then product $=(n+1).(n+2).(n+3)......(n+r)$

$=\dfrac{\lfloor n\ \ (n+1).(n+2).(n+3)......(n+r)}{\lfloor n}$

$=\dfrac{1.2.3......n.(n+1)(n+2)(n+3)......(n+r)}{\lfloor n}$

$=\dfrac{\lfloor n+r}{\lfloor n}=\dfrac{\lfloor n+r}{\lfloor r\ \lfloor n+r-r}\quad \lfloor r$

$=\left({}^{n+r}C_r\right)\ \lfloor r$ which is divisible by $\lfloor r$