

Class: 11
Subject: Math's
Topic: Principle of Mathematical Induction
No. of Questions: 25

- Let $S(K)$ be $1 + 3 + 5 + \dots + (2K - 1) = 3 + K^2$. Which of the following is true?
 - $S(K) \not\Rightarrow S(K + 1)$
 - $S(K) \Rightarrow S(K + 1)$
 - $S(1)$ is correct.
 - Principle of mathematical induction can be used to prove the formula.
- If $a_n = \sqrt{7 + \sqrt{7 + \sqrt{7 + \dots}}}$ having n radical signs, then by methods of mathematical induction which of the following is true?
 - $a_n > 7 \forall n \geq 1$
 - $a_n > 3 \forall n \geq 1$
 - $a_n < 4 \forall n \geq 1$
 - $a_n < 3 \forall n \geq 1$
- For each $n \in \mathbb{N}$, $x^{2n-1} + y^{2n-1}$ is divisible by
 - $x + y$
 - $(x + y)^2$
 - $x^3 + y^3$
 - none of these
- Let $x > -1$, then statement $P(n) : (1 + x)^n > 1 + nx$ is true for
 - all $n \in \mathbb{N}$
 - all $n > 1$
 - all $n > 1$ provided $x \neq 0$
 - none of these

5. **Statement - 1:** For each natural number n , $(n + 1)^7 - n^7 - 1$ is divisible by 7.
Statement - 2: For each natural number n , $n^7 - n$ is divisible by 7.
- A. Statement - 1 is true, statement - 2 is true and statement - 2 is the correct explanation for statement - 1.
B. Statement - 1 is true, statement - 2 is true and statement - 2 is not the correct explanation for Statement - 1.
C. Statement - 1 is true, statement - 2 is false.
D. Statement - 1 is false, statement - 2 is true.
6. Given $a, b, c \in \mathbb{N}$. If $a^n + b^n$ is divisible by c when n is odd but not when n is even, then the value of c is
- A. $a + b$
B. $a - b$
C. $a^3 + b^3$
D. $a^3 - b^3$
7. Let $S = \sqrt{1 + \frac{1}{1^2} + \frac{1}{2^2}} + \sqrt{1 + \frac{1}{2^2} + \frac{1}{3^2}} + \dots + \sqrt{1 + \frac{1}{1999^2} + \frac{1}{2000^2}}$, find $|2000(S-2000)|$.
8. A sequence is obtained by deleting all perfect squares from set of natural numbers. What is the remainder when the 2003^{rd} term of new sequence is divided by 2048?

9. **STATEMENT-1:** If $S_n = \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}\right)$, then

$$nS_n = n + \left(\frac{n-1}{1} + \frac{n-2}{2} + \dots + \frac{2}{n-2} + \frac{1}{n-1}\right)$$

STATEMENT-2: $S_n = n - \left(\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots + \frac{n-1}{n}\right)$

- A. Statement - 1 is true, Statement - 2 is true; Statement - 2 is the correct explanation for Statement - 1.
 B. Statement - 1 is true, Statement - 2 is true; Statement - 2 is NOT the correct explanation for Statement - 1.
 C. Statement - 1 is true, Statement - 2 is false.
 Statement - 1 is false, Statement - 2 is true.

10. For a positive integer, n , let $S(n) = \sum_{k=1}^{2^n-1} \frac{1}{k}$, then let us consider two statements:
 $A_1 : S(100) \leq 100$;
 $A_2 : S(200) > 100$, then

- A. A_1 is correct but A_2 is not correct
 B. A_2 is correct but A_1 is not correct
 C. A_1 and A_2 are both correct
 D. Neither A_1 nor A_2 is correct

11. For a positive integer n , let $S(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2^n - 1}$. Which of the following expressions holds true?

- A. $S_n \leq n$ $S_{2n} \leq n$
 B. $S_n < n$
 C. $S_{2n} \leq n$
 D. $S_{2n} > n$

12. The number $\frac{5^k + 3}{4}$ ($k \in \mathbb{N}$), when divided by 10, may leave remainder
- A. 2
B. 6
C. 7
D. 8
13. **Direction:** Let $f(n) = 3^{2n} + 3^n + 1$ for every positive integer n . Which of the following is true?
- A. $f(n+3) = 3^6 f(n) - 702 \cdot 3^n - 728$
B. $f(n+3) = 3^6 f(n) - 701 \cdot 3^n - 729$
C. $f(n+3) = 3^6 f(n) + 702 \cdot 3^n - 728$
D. none of these
14. **Direction:** Let $f(n) = 3^{2n} + 3^n + 1$ for every positive integer n . Which of the following is false?
- A. $f(100)$ is divisible by 13.
B. $f(1001)$ is divisible by 13.
C. $f(2007)$ is divisible by 13.
D. none of these
15. If $S_n = \sum_{r=1}^n t_r = \frac{1}{6}n(2n^2 + 9n + 13)$, then $\sum_{r=1}^n \sqrt{t_r}$ equals
- A. $\frac{1}{2}n(n+1)$
B. $\frac{1}{2}n(n+2)$
C. $\frac{1}{2}n(n+3)$
D. $\frac{1}{2}n(n+5)$

16. $1, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_k$ are divisions of number $N = 2^{n-1} (2^n - 1)$ where $2^n - 1$ is a prime number and $1 < \alpha_1 < \alpha_2 < \alpha_3 \dots < \alpha_k$, then value of $\left(1 + \frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \dots + \frac{1}{\alpha_k}\right)$ is

17. Answer the following questions based upon above passage.

Sum of the series

$$\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + \dots \text{to } 16 \text{ terms is}$$

- A. 346
B. 446
C. 546
D. 444
18. Answer the following questions based upon above passage.

$$\text{is } 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \dots \text{to } \infty$$

- A. $\frac{16}{35}$
B. $\frac{11}{8}$
C. $\frac{35}{16}$
D. $\frac{7}{16}$

19. Answer the following questions based upon above passage.

The sum of the series $\frac{1}{3 \times 7} + \frac{1}{7 \times 11} + \frac{1}{11 \times 15} + \dots$ to ∞ is

- A. $\frac{1}{3}$
- B. $\frac{1}{6}$
- C. $\frac{1}{9}$
- D. $\frac{1}{12}$

20. If n is a positive integer and p is a prime number, then $n^p - n$ is always divisible by

- A. $p - 1$
- B. p
- C. $p + 1$
- D. p^2

21. If $S_{(n)} = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}, (n \in \mathbb{N})$, then $S_{(1)} + S_{(2)} + \dots + S_{(n-1)}$ is equal to

- A. $nS_{(n)} - n$
- B. $nS_{(n)} - 1$
- C. $(n-1)S_{(n-1)} - n$
- D. $nS_{(n-1)} - n + 1$

22. **Statement - 1:** Let $d_1, d_2, d_3, \dots, d_k$ be all the factors of a fixed positive integer n including 1 and n .

If $d_1 + d_2 + d_3 + \dots + d_k = 72$, then the value of $\frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} + \dots + \frac{1}{d_k} = \frac{72}{n}$ because

Statement - 2: For a positive integer n , if d is a factor then $\frac{n}{d}$ is also a factor.

- A. Statement - 1 is true, Statement - 2 is true; Statement - 2 is the correct explanation for Statement - 1.
B. Statement - 1 is true, Statement - 2 is true; Statement - 2 is NOT the correct explanation for Statement - 1.
C. Statement - 1 is true, Statement - 2 is false.
D. Statement - 1 is false, Statement - 2 is true.
23. If 12 divides the seven digit number $ab313ab$, then the smallest values of $a + b$ is
- A. 2
B. 4
C. 6
D. 8
24. The sum of the cubes of three consecutive natural no. is divisible by 9.
25. Prove that $12^n + 25^{n-1}$ is divisible by 13