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Class: 11
Subject: Math's
Topic: Principle of Mathematical Induction
No. of Questions: 25

1. Let S (K) be $1 + 3 + 5 + \ldots + (2K - 1) = 3 + K^2$. Which of the following is true?

A. $S(K) \nRightarrow S(K+1)$
B. $S(K) \Rightarrow S(K+1)$
C. S (1) is correct.
D. Principle of mathematical induction can be used to prove the formula.

Sol. B

$S(1): 1 = 3 + 1^2$, which is not true.
Suppose S(K) is true, then
$1 + 3 + 5 + \ldots + (2K - 1) = 3 + K^2$
Adding (2K + 1) to both the sides, we get
$1 + 3 + 5 + \ldots + (2K - 1) + (2K + 1) = 3 + K^2 + 2K + 1 = 3 + (K + 1)^2$
which is S (K + 1).
Thus, $S(K) \Rightarrow S(K+1)$

2. If $a_n = \sqrt{7 + \sqrt{7 + \sqrt{7 + \ldots}}}$ having n radical signs, then by methods of mathematical induction which of the following is true?
A. $a_n > 7 \ \forall \ n \geq 1$
B. $a_n > 3 \ \forall \ n \geq 1$
C. $a_n < 4 \ \forall \ n \geq 1$
D. $a_n < 3 \ \forall \ n \geq 1$

Sol. C

We have $a_1 = \sqrt{7} < 4$. Assume $a_k < 4$ for some natural number k.
We have

$$a_{k+1} = \sqrt{7 + a_k} < \sqrt{7 + 4} < 4$$

$\therefore a_n < 4 \ \forall \ n \geq 1.$

3. For each $n \in N$, $x^{2n-1} + y^{2n-1}$ is divisible by

A. $x + y$
B. $(x + y)^2$
C. $x^3 + y^3$
D. none of these

Sol. A

$x^{2n-1} + y^{2n-1}$
$= (x + y)(x^{2n-3}y - x^{2n-3}y + x^{2n-4}y^2 - x^{2n-5}y^3 + ... + y^{2n-2})$
Thus $x^{2n-1} + y^{2n-1}$ is divisible by $x + y$ for all $n \in N$.

4. Let $x > -1$, then statement $P(n) : (1 + x)^n > 1 + nx$ is true for

A. all $n \in N$
B. all $n > 1$
C. all $n > 1$ provided $x \neq 0$
D. none of these

Sol. C

For n = 2,
$P(2): (1 + x)^2 = 1 + 2x + x^2 > 1 + 2x$ as $x \neq 0$.
Assume that
$P(k): (1 + x)^k > 1 + kx$ (1)
for some $k \in N$, $k > 1$.
As $x > -1$, multiplying both the sides of (1) by $1 + x$, we get
$(1 + x)^{k+1} > (1 + kx)(1 + x)$
$= 1 + (k + 1)x + kx^2 > 1 + (k + 1)x$
$[\because kx^2 > 0]$ Thus, $P(k + 1)$ is true.
By the principle of mathematical induction, $P(n)$ is true for all $n > 1$ provided $x \neq 0$.

5. Statement - 1: For each natural number n, $(n + 1)^7 - n^7 - 1$ is divisible by 7.
Statement - 2: For each natural number n, $n^7 - n$ is divisible by 7.

A. Statement – 1 is true, statement – 2 is true and statement – 2 is the correct explanation for statement – 1.
B. Statement – 1 is true, statement – 2 is true and statement – 2 is not the correct explanation for Statement – 1.
C. Statement – 1 is true, statement – 2 is false.
D. Statement – 1 is false, statement – 2 is true.

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Sol. A

For statement 2, write $n = 7r + k, k = -3, -2, -1, 0, 1, 2, 3$

Now, $n^7 - n = (7r + k)^7 - (7r + k)$

Using binomial theorem, we get

$n^7 - n - (k^7 - k)$ is divisible by 7.

$\Leftrightarrow$ $n^7 - n$ is divisible by 7 if and only if $k^7 - k$ is divisible by 7

For $k = \pm 3, \pm 2, \pm 1$. $k^7 - k$ is divisible by 7 if and only if $k^6 - 1$ is divisible by 7.

But $k^6 - 1 = 728, 63, 0$ for $k = \pm 3, \pm 2, \pm 1$.

$\therefore$ $k^6 - 1$ is divisible by 7 for $k = \pm 3, \pm 2, \pm 1$.

Thus, $n^7 - n$ is divisible by 7 for each $n \in \mathbf{N}$.

$\Rightarrow$ $(n + 1)^7 - (n + 1) - (n^7 - n)$ is divisible by 7

$\Rightarrow$ $(n + 1)^7 - n^7 - 1$ is divisible by 7.

Therefore, both the statements are true and statement - 2 is a correct reason for statement-1.

6. Given a, b, c $\in$ N. If $a^n + b^n$ is divisible by c when n is odd but not when n is even, then the value of c is

A. a + b
B. a – b
C. $a^3 + b^3$
D. $a^3 - b^3$

Sol. A

Use $a^{2m+1} + b^{2m+1}$

$= (a + b)(a^{2m} - a^{2m-1}b + a^{2m-2}b^2 - \ldots - ab^{2m-1} + b^{2m}]$

7. Let S = $\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+.....+\sqrt{1+\frac{1}{1999^2}+\frac{1}{2000^2}}$, find |2000(S-2000)|.

Sol.

$t_r = \sqrt{1+\frac{1}{r^2}+\frac{1}{(r+1)^2}}$

$= \sqrt{\frac{r^2+(r+1)^2+r^2(r+1)^2}{r^2(r+1)^2}}$

$= \sqrt{\frac{2r^2+2r+1+r^2(r^2+2r+1)}{r^2(r+1)^2}}$

$= \sqrt{\frac{r^4+2r^3+3r^2+2r+1}{r^2(r+1)^2}}$

$= \frac{r^2+r+1}{r(r+1)} = \frac{1}{r(r+1)}+1$

$= 1+\frac{1}{r}-\frac{1}{r+1}$,

S = 2000 – $\frac{1}{2000}$, $|2000(S-2000)| = 1$.

8. A sequence is obtained by deleting all perfect squares from set of natural numbers. What is the remainder when the
2003^{rd} term of new sequence is divided by 2048?

Sol.

Since $\left[\sqrt{2046}\right] = \left[\sqrt{2047}\right] = \left[\sqrt{2048}\right] = \left[\sqrt{2049}\right] = 45$

$\therefore 2003^{rd}$ term is 2003 + 45 = 2048

Hence remainder is 0

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9. STATEMENT–1: If $S_n = \left(1+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{n}\right)$, then

$nS_n = n+\left(\frac{n-1}{1}+\frac{n-2}{2}+.....+\frac{2}{n-2}+\frac{1}{n-1}\right)$

STATEMENT–2: $S_n = n-\left(\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+...+\frac{n-1}{n}\right)$

A. Statement – 1 is true, Statement – 2 is true; Statement – 2 is the correct explanation for Statement – 1.
B.Statement – 1 is true, Statement – 2 is true; Statement – 2 is NOT the correct explanation for Statement – 1.
C.Statement – 1 is true, Statement – 2 is false.
Statement – 1 is false, Statement – 2 is true.

Sol. A

Statement II

$S_n = n-\left(\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+....+\frac{n+1}{n}\right)$

$= 1+1+1+......+1-\left(\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+.....+\frac{n-1}{n}\right)$

$= \frac{1}{2}+\frac{1}{3}+\frac{1}{4}+......+\frac{1}{n-2}+\frac{1}{n-1}$

$nS_n = n+n\left[1+\frac{1}{2}+......+\frac{1}{n-1}\right]-(n-1)$

$nS_n = n+n\left[1+\frac{1}{2}+......+\frac{1}{n-1}\right]-(n-1)$

$= n+n\left[1+\frac{1}{2}+....+\frac{1}{n-1}\right]-n+1$

$= 1+1+\frac{1}{2}+....+\frac{1}{n-1}-1+\frac{1}{n}$

$S_n = 1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}$

10. For a positive integer, n, let $S(n) = \sum_{k=1}^{2^n-1}\frac{1}{k}$, then let us consider two statements:
$A_1 : S(100) \leq 100$;
$A_2 : S(200) > 100$, then

A. A_1 is correct but A_2 is not correct
B.A_2 is correct but A_1 is not correct
C.A_1 and A_2 are both correct
D. Neither A_1 nor A_2 is correct

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Sol. C

$$S_{(n)} > 1 + \frac{1}{2} + \left(\frac{1}{4}+\frac{1}{4}\right) + \left(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}\right) + \ldots + \left(\frac{1}{2^{n-1}} + \ldots + \frac{1}{2^n}\right) - \frac{1}{2^n} \qquad S_{(n)} > 1 + \frac{n}{2} - \frac{1}{2^n}$$

$$S_{(200)} > 1 + 100 - \frac{1}{2^{100}} > 100$$

Also

$$S_n = 1 + \left(\frac{1}{2}+\frac{1}{3}\right) + \left(\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\right) + \ldots + \left(\frac{1}{2^n-1}\right)$$

$$\leq 1 + 2\left(\frac{1}{2}\right) + 4\left(\frac{1}{4}\right) + \ldots = n$$

$S(100) \leq 100$.

11. For a positive integer n, $let\ S(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{2^n - 1}$. Which of the following expressions holds true?

A. $S_n \leq n \ S_{2n} \leq n$
B. $S_n < n$
C. $S_{2n} \leq n$
D. $S_{2n} > n$

Sol. A,D

$$S(n) = 1 + \left(\frac{1}{2}+\frac{1}{3}\right) + \left(\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\right) + \ldots + \left(\frac{1}{2^{n-1}} + \frac{1}{2^{n-1}+1} + \ldots + \frac{1}{2^n - 1}\right)$$

$$\leq 1 + \left(\frac{1}{2}+\frac{1}{2}\right) + \left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right) + \ldots + \left(\frac{1}{2^{n-1}} + \frac{1}{2^{n-1}} + \ldots + \frac{1}{2^{n-1}}\right)$$

$= 1 + 1 + 1 + \ldots + 1$ (n terms) $= n$

Also

$$S(n) \geq 1 + \frac{1}{2} + \left(\frac{1}{3}+\frac{1}{4}\right) + \left(\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}\right) + \ldots + \left(\frac{1}{2^{n-2}+1} + \frac{1}{2^{n-2}+2} + \ldots + \frac{1}{2^{n-1}}\right)$$

$$> 1 + \frac{1}{2} + \frac{1}{2} + \ldots + \frac{1}{2} = 1 + \left(\frac{n-1}{2}\right) = \frac{n+1}{2}$$

$$\therefore S(2n) > \frac{2n+1}{2} = n + \frac{1}{2} > n$$

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12. The number $\frac{5^k+3}{4}(k \in N)$, when divided by 10, may leave remainder

A. 2
B. 6
C. 7
D. 8

Sol. A,C

$$\frac{5^k+3}{4} = \frac{5^k-5+8}{4} = \frac{5(5^{k-1}-1)}{5-1} + 2 = 5+5^2+5^3+\ldots(k-1)\text{ terms} + 2$$

So if k – 1 = even, the last digit is 2
If k –1 = odd, the last digit is 5 + 2 = 7

13. Direction: Let f(n) = $3^{2n} + 3^n + 1$ for every positive integer n. Which of the following is true?

A. $f(n+3) = 3^6 f(n) - 702.3^n - 728$
B. $f(n+3) = 3^6 f(n) - 701.3^n - 729$
C. $f(n+3) = 3^6 f(n) + 702.3^n - 728$
D. none of these

Sol. A

$$f(n) = 3^{2n} + 3^n + 1$$
$$f(n+3) = 3^{2n+6} + 3^{n+3} + 1$$
$$f(n+3) - 3^6 f(x) = -702.3^n - 728. \Rightarrow$$

So, (1) is correct.

14. Direction: Let f(n) = $3^{2n} + 3^n + 1$ for every positive integer n. Which of the following is false?

A. f(100) is divisible by 13.
B.f(1001) is divisible by 13.
C.f(2007) is divisible by 13.
D. none of these

Sol. C

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Indeed 702 and 728 are divisible by 13.
The answer in the above question shows that if
(i) f(n) is divisible by 13 then f(n+1) is also divisible by 13.
(ii) f(n) is not divisible by 13 then f(n+1) is not divisible by 13
Now f(1) and f(2) are divisible by 13 but f(3) is not divisible by 13
$\Rightarrow f(100), f(1001)$ are divisible by 13 but f(2007) is not divisible by 13.
(3) is correct.

15. If $S_n = \sum_{r=1}^{n} t_r = \frac{1}{6}n(2n^2+9n+13)$, then $\sum_{r=1}^{n}\sqrt{t_r}$ equals

A. $\frac{1}{2}n(n+1)$
B. $\frac{1}{2}n(n+2)$
C. $\frac{1}{2}n(n+3)$
D. $\frac{1}{2}n(n+5)$

Sol. C

We have $t_n = S_n - S_{n-1} \quad \forall\ n \geq 2$

$$\therefore \quad t_n = \frac{1}{6}\left[2\left(n^3-(n-1)^3\right)+9\left(n^2-(n-1)^2\right)+13(n-n+1)\right]$$

$$= \frac{1}{6}\left[6n^2-6n+2+9(2n-1)+13\right]$$

$$= \frac{1}{6}\left(6n^2+12n+6\right) = (n+1)^2$$

$$\therefore \quad \sum_{r=1}^{n}\sqrt{t_r} = \sum_{r=1}^{n}(r+1) = \frac{1}{2}(n+1)(n+2)-1 = \frac{1}{2}n(n+3)$$

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16. $1, \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_k$ are divisions of number N = $2^{n-1}(2^n - 1)$ where $2^n - 1$ is a prime number and $1 < \alpha_1 < \alpha_2 < \alpha_3 \ldots < \alpha_k$, then value of $\left(1+\frac{1}{\alpha_1}+\frac{1}{\alpha_2}+\ldots+\frac{1}{\alpha_k}\right)$ is

Sol.

Divisors of N = $2^{n-1}(2^n - 1)$ are

$1, 2, 2^2, \ldots, 2^{n-1}, 2^n - 1, 2(2^n - 1), 2^2(2^n - 1), \ldots, 2^{n-1}(2^n - 1)$

$$1 + \frac{1}{\alpha_1}+\frac{1}{\alpha_2}+\ldots+\frac{1}{\alpha_k} = 1+\frac{1}{2}+\ldots+\frac{1}{2^{n-1}}+\frac{1}{2^n-1}+\frac{1}{2(2^n-1)}+\ldots+\frac{1}{2^{n-1}(2^n-1)}$$

$$= \left(1+\frac{1}{2}+\ldots+\frac{1}{2^{n-1}}\right)+\frac{1}{2^n-1}\left(1+\frac{1}{2}+\ldots+\frac{1}{2^{n-1}}\right)$$

$$= \left(1+\frac{1}{2}+\ldots+\frac{1}{2^{n-1}}\right)\left(1+\frac{1}{2^n-1}\right)$$

$$= \frac{1.\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}}\left(\frac{2^n}{2^n-1}\right) = \frac{2^n}{2^{n-1}} = 2$$

Passage type

$$\sum n = \frac{n(n+1)}{2}$$

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}, \sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

If $T_n = n(n+1)$, then

$$S_n = [n(n+1)]\frac{(n+2)}{3}$$

if $T_n = n(n+1)(n+2)$, then

$$S_n = [n(n+1)(n+2)]\frac{(n+3)}{4}.$$

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17. Answer the following questions based upon above passage.

Sum of the series

$$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots\text{to 16 terms is}$$

A. 346
B. 446
C. 546
D. 444

Sol. B

$$T_n=\frac{\sum n^3}{\frac{n}{2}\left[2.1+(n-1).2\right]}$$

$$=\frac{1}{4}.\frac{n^2(n+1)^2}{n^2}=\frac{1}{4}\left(n^2+2n+1\right)$$

$$S_n=\frac{1}{4}\left[\sum n^2+2\sum n+\sum 1\right]$$

$$=\frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6}+2.\frac{(n+1)n}{2}+n\right]$$

Putting n = 16, we get

$S_{16} = 446$

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18. Answer the following questions based upon above passage.

is $1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+....\text{to } \infty$

A. $\frac{16}{35}$

B. $\frac{11}{8}$

C. $\frac{35}{16}$

D. $\frac{7}{16}$

Sol. C

$$S = 1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+....\infty$$

Then $\frac{1}{5}S = \frac{1}{5}+\frac{4}{5^2}+\frac{7}{5^3}+.....$

$$S\left(1-\frac{1}{5}\right) = 1+3\left[\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+....\infty\right]$$

$$\frac{4}{5}S = 1+3\left[\frac{1/5}{1-1/5}\right] = 1+\frac{3}{4} = \frac{7}{4}$$

$\therefore$ $S = \frac{35}{16}$

Note: Your many use the formula

i.e. $S_\infty = \frac{ab}{1-r}+\frac{dbr}{(1-r)^2}$

where a = 1, d= 3, b = 1, r = 1/5

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19. Answer the following questions based upon above passage.

The sum of the series $\frac{1}{3\times 7}+\frac{1}{7\times 11}+\frac{1}{11\times 15}+\ldots\text{to }\infty$ is

A. $\frac{1}{3}$

B. $\frac{1}{6}$

C. $\frac{1}{9}$

D. $\frac{1}{12}$

Sol. D

$$S=\frac{1}{4}\left[\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{11}\right)+\left(\frac{1}{11}-\frac{1}{15}\right)+\ldots\infty\right]$$

$$S_{\infty}=\frac{1}{4}\cdot\frac{1}{3}=\frac{1}{12}$$

20. If n is a positive integer and *p* is a prime number, then $n^p - n$ is always divisible by

A. p – 1
B. p
C. p + 1
D. p^2

Sol. B

$f(n)=n^p-n$ and hence divisible by p

$f(n+1)-f(n)=(n+1)^p-(n+1)-n^p+n=(n+1)^p-n^p-1={}^pC_1.n^{p-1}+{}^pC_2.n^{p-2}+\ldots+{}^pC_{p-1}.n$ is divisible by p,

since ${}^pC_r$ is divisible by p, for r=1,2,3,.....p-1

Hence by induction ,f(n) is divisible by p

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21. If $S_{(n)} = 1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n},(n \in N)$, then $S_{(1)}+S_{(2)}+.......+S_{(n-1)}$ is equal to

A. $nS_{(n)} - n$

B. $nS_{(n)} - 1$

C. $(n-1)S_{(n-1)} - n$

D. $nS_{(n-1)} - n + 1$

Sol. A

$S_{(1)}+S_{(2)}+.....+S_{(n-1)}$

$S_{(1)}$: 1

$S_{(2)}$: $1+\frac{1}{2}$

$S_{(3)}$: $1+\frac{1}{2}+\frac{1}{3}$

…………

…………..

$S_{(n-1)}$: $1+\frac{1}{2}+\frac{1}{3}+......\frac{1}{n-1}$

Adding vertically :

$$= (n-1)+\frac{(n-2)}{2}+\frac{(n-3)}{3}+......\left(\frac{n-(n-1)}{(n-1)}\right)$$

$$= n\left[1+\frac{1}{2}+\frac{1}{3}+....\frac{1}{n-1}\right]-[1+1+1+.....1] = nS_{(n-1)}-(n-1) = nS_n - n$$

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22. Statement – 1: Let $d_1, d_2, d_3 \ldots d_k$ be all the factors of a fixed positive integer n including 1 and n.

If $d_1 + d_2 + d_3 + \ldots\ldots + d_k = 72$, then the value of $\frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} + \ldots\ldots + \frac{1}{d_k} = \frac{72}{n}$ because

Statement – 2: For a positive integer n, if d is a factor then $\frac{n}{d}$ is also a factor.

A. Statement – 1 is true, Statement – 2 is true; Statement – 2 is the correct explanation for Statement – 1.
B. Statement – 1 is true, Statement – 2 is true; Statement – 2 is NOT the correct explanation for Statement – 1.
C. Statement – 1 is true, Statement – 2 is false.
D. Statement – 1 is false, Statement – 2 is true.

Sol. A

If d is a factor of n, then $\frac{n}{d}$ is also a factor of n is correct.

$\therefore \frac{n}{d_1} + \frac{n}{d_2} + \frac{n}{d_3} + \ldots + \frac{n}{d_k} = 72$

$\therefore \frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} + \ldots + \frac{1}{d_k} = \frac{72}{n}$

23. If 12 divides the seven digit number ab313ab, then the smallest values of a + b is

A. 2
B. 4
C. 6
D. 8

Sol. B

Let $N = ab313ab$ is div. by12 then it is div. by 3.

$a + b + 3 + 1 + 3 + a + b$ is div. by 3

$2(a+b) + 1 = 3K \Rightarrow a + b = \frac{1}{2}(3K - 1)$

$a + b = 1, 4, 7, 10, \ldots$

But N is div. by 4 also. So ab is div. by 4. If $a + b = 4$

$\Rightarrow a = 4, b = 0$

$N = 4031340$

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24. The sum of the cubes of three consecutive natural no. is divisible by 9.

Sol:

P(n) $\left[k^3+(k+1)^3+(k+2)^3\right]$ is divisible by 9

For n = 1

P (1) : 1 + 8 + 9 = 18

which is divisible by 9

Let p (k) be true

$p(k):\left[k^3+(k+1)^3+(k+2)^3\right]$ is divisible by 9

$\Rightarrow k^3+(k+1)^3+(k+2)^3=9\lambda (i)$

we want to prove that p (k+1) is true

p(k+1) : $(k+1)^3+(k+2)^3+(k+3)^3$

L.H.S = $(k+1)^3+(k+2)^3+(k+3)^3$

$=(k+1)^3+(k+2)^3+k^3+9k^2+27k+27$

$=\underbrace{k^3+(k+1)^3+(k+2)^3}+9(k^2+3k+3)$

$=9\lambda+9(k^2+3k+3)$ (from i)

$=9\left[\lambda+(k^2+3k+3)\right]$ which is $\div$ by 9.

25. Prove that $12^n + 25^{n-1}$ is divisible by 13

Sol:

P(n) : $12^n + 25^{n-1}$ is divisible by 13

For n = 1

P(1) : $12 + (25)^0 = 13$

which is divisible by 13

Let p (k) be true

P(k) : $12^k + 25^{k-1}$ is divisible by 13

$\Rightarrow 12^k + 25^{k-1} = 13\lambda (i)$

we want to prove that result is true for n = k+1

$$12^{(k+1)} + 25^{k+1-1} = 12^k.12^1 + 25^k$$

$$= \left(13\lambda - 25^{k-1}\right).12 + 25^k \,(\text{from } i)$$

$$= 13\times 12\lambda - 25^{k-1}.12 + 25^k$$

$$= 13\times 12\lambda + 25^{k-1}\left(-12+25\right)$$

$$= 13\left(12\lambda + 25^{k-1}\right)$$

which is divisible by 13.