

Class: 11
Subject: Math's
Topic: Quadratic Equations
No. of Questions: 25

1. The number of solutions of $\sqrt{4-x} + \sqrt{x+9} = 5$ is
- A. 0
B. 1
C. 2
D. 3

Sol. C

$$\sqrt{4-x} + \sqrt{x+9} = 5 \quad (1)$$

Note that $4-x \geq 0$ and $x+9 \geq 0 \Rightarrow -9 \leq x \leq 4$.

We can write (1) as $\sqrt{x+9} = 5 - \sqrt{4-x}$

Squaring both the sides we get

$$\begin{aligned} x+9 &= 25 - 10\sqrt{4-x} + 4-x \\ \Rightarrow 10\sqrt{4-x} &= 20 - 2x \Rightarrow 5\sqrt{4-x} = 10 - x \end{aligned}$$

Squaring both the sides we get

$$\begin{aligned} 25(4-x) &= 100 - 20x + x^2 \Rightarrow x^2 + 5x = 0 \\ \Rightarrow x(x+5) &= 0 \Rightarrow x = 0 \text{ or } x = -5 \end{aligned}$$

Both $x = 0$ and $x = -5$ satisfy (1).

2. The value of a for which one root of the quadratic equation, $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as other, is
- A. $2/3$
B. $1/3$
C. $1/3$
D. $2/3$

Sol. D

$$(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0 \quad (1)$$

Let α and 2α be the roots of (1), then

$$(a^2 - 5a + 3)\alpha^2 + (3a - 1)\alpha + 2 = 0 \quad (2)$$

$$\text{and } (a^2 - 5a + 3)(4\alpha^2) + (3a - 1)(2\alpha) + 2 = 0 \quad (3)$$

Multiplying (2) by 4 and subtracting it from (3) we get

$$(3a - 1)(2\alpha) + 6 = 0$$

$$\text{Clearly } a \neq 1/3. \text{ Therefore, } \alpha = -3/(3a - 1)$$

Putting this value in (2) we get

$$(a^2 - 5a + 3)(9) - (3a - 1)^2(3) + 2(3a - 1)^2 = 0$$

$$\Rightarrow 9a^2 - 45a + 27 - (9a^2 - 6a + 1) = 0$$

$$\Rightarrow -39a + 26 = 0$$

$$\Rightarrow a = 2/3.$$

For $a = 2/3$, the equation becomes $x^2 + 9x + 18 = 0$, whose roots are $-3, -6$.

3. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ are such that $\min f(x) > \max g(x)$, then the relation between b and c , is

- A. no relation
- B. $0 < c < b/2$
- C. $|c| < \frac{|b|}{\sqrt{2}}$
- D. $|c| > \sqrt{2}|b|$

Sol. D

$$f(x) = (x + b)^2 + 2c^2 - b^2$$

$$\Rightarrow \min f(x) = 2c^2 - b^2$$

$$\text{Also } g(x) = -x^2 - 2cx + b^2 = b^2 + c^2 - (x + c)^2$$

$$\Rightarrow \max g(x) = b^2 + c^2$$

As $\min f(x) > \max g(x)$, we get $2c^2 - b^2 > b^2 + c^2$

$$\Rightarrow c^2 > 2b^2 \Rightarrow |c| > \sqrt{2}|b|$$

4. Let p and q be the roots of $x^2 - 2x + A = 0$ and let r and s be the roots of $x^2 - 18x + B = 0$. If $p < q < r < s$ are in an A.P. then the ordered pair (A, B) is equal to
- A. $(-3, 77)$
 - B. $(77, -3)$
 - C. $(-3, -77)$
 - D. None of these

Sol. A

We have $p + q = 2, pq = A$ (1)

and $r + s = 18, rs = B$ (2)

As p, q, r, s are in AP, we take

$$p = a - 3d, q = a - d, r = a + d, s = a + 3d.$$

As $p < q < r < s$, we have $d > 0$

Now, $2 = p + q = 2a - 4d$

and $18 = r + s = 2a + 4d$

Solving above equations, we get $a = 5$ and $d = 2$

$\therefore p = -1, q = 3, r = 7$ and $s = 11$

Thus, $A = pq = -3$ and $B = rs = 77$.

5. Suppose a, b, c are three non-zero real numbers. The equation $x^2 + (a + b + c)x + (a^2 + b^2 + c^2) = 0$ has
- A. two negative real roots
 - B. two positive real roots
 - C. two real roots with opposite signs
 - D. no real roots

Sol. D

$$x^2 + (a + b + c)x + (a^2 + b^2 + c^2) = 0 \quad (1)$$

Discriminant D of (1) is given by

$$\begin{aligned} D &= (a + b + c)^2 - 4(a^2 + b^2 + c^2) \\ &= -\{(b^2 + c^2 - 2bc) + (c^2 + a^2 - 2ca) + (a^2 + b^2 - 2ab) + (a^2 + b^2 + c^2)\} \\ &= -[(b - c)^2 + (c - a)^2 + (a - b)^2 + (a^2 + b^2 + c^2)] < 0 \end{aligned}$$

Thus, (1) cannot have real roots.

6. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$ in variable x has real roots, if p belongs to the interval
- A. $(0, 2\pi)$
 B. $(-\pi, 0)$
 C. $(-\pi/2, \pi/2)$
 D. $(0, \pi)$

Sol. D

$$(\cos p - 1)x^2 + (\cos p)x + \sin p = 0 \quad (1)$$

Discriminant of (1) is given by

$$\begin{aligned} D &= \cos^2 p - 4(\cos p - 1) \sin p \\ &= \cos^2 p + 4(1 - \cos p) \sin p \end{aligned}$$

Note that

$\cos^2 p \geq 0$, $1 - \cos p \geq 0$. Thus, $D \geq 0$ if $\sin p \geq 0$ i.e. if $p \in (0, \pi)$.

7. If a, b are two real numbers satisfying the relations $2a^2 - 3a - 1 = 0$ and $b^2 + 3b - 2 = 0$ and $ab \neq 1$, then the value of $\frac{ab + a + 1}{b}$ is

- A. 1
 B. 0
 C. 1
 D. 2

Sol. C

$$b^2 + 3b - 2 = 0 \Rightarrow 1 + \frac{3}{b} - \frac{2}{b^2} = 0$$
$$\Rightarrow \frac{2}{b^2} - \frac{3}{b} - 1 = 0$$

Thus, a and $1/b$ are roots of $2x^2 - 3x - 1 = 0$

$$\therefore a + \frac{1}{b} = \frac{3}{2} \text{ and } \frac{a}{b} = \frac{-1}{2}$$

$$\text{Now, } \frac{ab + a + 1}{b} = a + \frac{1}{b} + \frac{a}{b} = \frac{3}{2} - \frac{1}{2} = 1$$

8. Suppose a, b, c are the lengths of three sides of a $\triangle ABC$, $a > b > c$, $2b = a + c$ and b is a positive integer. If $a^2 + b^2 + c^2 = 84$, then the value of b is

- A. 7
- B. 6
- C. 5
- D. 4

Sol. C

We have

$$ac = \frac{1}{2} [(a + c)^2 - (a^2 + c^2)] = \frac{1}{2} [4b^2 - (84 - b^2)]$$
$$= 5b^2/2 - 42$$

Thus, a and c are the roots of the equation

$$x^2 - 2bx + (5b^2/2 - 42) = 0$$

As a and c are distinct real numbers the discriminant of the above must be positive, that is, $4b^2 - 4(5b^2/2 - 42) > 0$

$$\Rightarrow 6b^2 < 168 \text{ or } b^2 < 28.$$

$$\text{Also, } ac > 0 \Rightarrow 5b^2 > 84.$$

$$\therefore 84/5 < b^2 < 28.$$

As b is a positive integer, we get $b = 5$.

9. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are non-zero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

- A. $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$
 B. $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$
 C. $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$
 D. $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

Sol. B

$$q = \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

$$= -p^3 + 3\alpha\beta p$$

$$\Rightarrow \alpha\beta = \frac{p^3 + q}{3p}$$

We have

$$\begin{aligned} \frac{\alpha}{\beta} + \frac{\beta}{\alpha} &= \frac{\alpha^2 + \beta^2}{\alpha\beta} \\ &= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{(\alpha + \beta)^2}{\alpha\beta} - 2 \\ &= \frac{p^2}{(p^3 + q)/3p} - 2 \\ &= \frac{3p^3 - 2p^3 - 2q}{p^3 + q} = \frac{p^3 - 2q}{p^3 + q} \end{aligned}$$

$$\text{and } \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$$

Thus, required quadratic equation is

$$x^2 - \frac{(p^3 - 2q)}{p^3 + q} x + 1 = 0$$

$$\text{or } (p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$$

10. If a is the minimum root of the equation $x^2 - 3|x| - 2 = 0$, then the value of $-1/a$ is

- A. $(\sqrt{17} - 3)/4$
- B. $(\sqrt{17} + 3)/4$
- C. 2
- D. 3

Sol. A

$$\begin{aligned}x^2 - 3|x| - 2 = 0 &\text{ can be written as } |x|^2 - 3|x| - 2 = 0 \\ \Rightarrow |x| &= \frac{1}{2}(3 \pm \sqrt{17}) \\ \text{As } |x| \geq 0, &\text{ we get } |x| = \frac{1}{2}(3 + \sqrt{17}) \\ \Rightarrow x &= \pm \frac{1}{2}(3 + \sqrt{17}) \\ \therefore a &= -\frac{1}{2}(3 + \sqrt{17}) \Rightarrow -\frac{1}{a} = \frac{2}{\sqrt{17} + 3} = \frac{\sqrt{17} - 3}{4}\end{aligned}$$

11. Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in a G.P. then the integral values of p and q respectively, are

- A. 2, -32
- B. 2, 3
- C. 6, 3
- D. 6, -32

Sol. A

We have

$$\alpha + \beta = 1, \alpha\beta = p,$$

$$\gamma + \delta = 4, \gamma\delta = q$$

Let r be the common ratio of the GP $\alpha, \beta, \gamma, \delta$. Then

$$\alpha + \beta = 1 \Rightarrow \alpha + \alpha r = 1 \Rightarrow \alpha(1 + r) = 1$$

$$\gamma + \delta = 4 \Rightarrow \alpha r^2 + \alpha r^3 = 4$$

$$\Rightarrow \alpha r^2(1 + r) = 4$$

$$\text{Thus, } \frac{\alpha r^2(1 + r)}{\alpha(1 + r)} = 4 \Rightarrow r^2 = 4 \Rightarrow r = \pm 2$$

$$\text{When } r = 2,$$

$$\text{we get } \alpha(1 + r) = 1 \Rightarrow \alpha = 1/3$$

$$\text{In this case } p = \alpha\beta = \alpha(\alpha r) = \alpha^2 r$$

$$= \frac{1}{9}(2) = \frac{2}{9}$$

which is not an integer.

$$\text{Thus, } r = -2. \text{ In this case, } \alpha(1 + r) = 1 \Rightarrow \alpha = -1.$$

$$\therefore p = \alpha^2 r = (-1)^2(-2) = -2$$

$$\text{and } q = \gamma\delta = (\alpha r^2)(\alpha r^3) = \alpha^2 r^5 = -32$$

$$\text{Hence, } p = -2, q = -32.$$

12. If α and β ($\alpha < \beta$) are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then

- A. $0 < \alpha < \beta$
- B. $\alpha < 0 < \beta < |\alpha|$
- C. $\alpha < \beta < 0$
- D. $\alpha < 0 < |\alpha| < \beta$

Sol. B

$$\text{We have } \alpha + \beta = -b, \alpha\beta = c$$

As $c < 0, b > 0$, we get $\alpha < 0 < \beta$

$$\text{Also, } \beta = -b - \alpha < -\alpha = |\alpha|$$

$$\text{Thus, } \alpha < 0 < \beta < |\alpha|$$

13. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then

- A. $a < 2$
- B. $2 \leq a \leq 3$
- C. $3 < a \leq 4$
- D. $a > 4$

Sol. A

We can write the given equation as

$$(x - a)^2 = 3 - a \quad (1)$$

As x and a are real, $(x - a)^2 \geq 0 \Rightarrow 3 - a \geq 0$ or $a \leq 3$.

Equation (1) implies

$$x - a = \pm \sqrt{3 - a} \Rightarrow x = a \pm \sqrt{3 - a}$$

Both the roots of (1) will be less than 3, if the larger of the two roots is less than 3, that is,

$$\begin{aligned} a + \sqrt{3 - a} < 3 &\Rightarrow \sqrt{3 - a} - (3 - a) < 0 \\ &\Rightarrow \sqrt{3 - a} (1 - \sqrt{3 - a}) < 0 \end{aligned}$$

$$\Rightarrow \sqrt{3 - a} > 0 \text{ and } 1 - \sqrt{3 - a} < 0$$

$$\Rightarrow a < 3 \text{ and } 1 < \sqrt{3 - a}$$

$$\Rightarrow a < 3 \text{ and } 1 < 3 - a$$

$$\Rightarrow a < 3 \text{ and } a < 2 \Rightarrow a < 2.$$

14. If $4^x - 3^{x-1/2} = 3^{x+1/2} - 2^{2x-1}$, then the value of x is equal to

- A. $5/2$
- B. 2
- C. $3/2$
- D. 1

Sol. C

$$\begin{aligned}4^x - 3^{x-1/2} &= 3^{x+1/2} - 2^{2x-1} \\ \Rightarrow 4^x + 2^{2x-1} &= 3^{x+1/2} + 3^{x-1/2} \\ \Rightarrow 4^x + \frac{4^x}{2} &= 3x\sqrt{3} + \frac{3^x}{\sqrt{3}} \\ \Rightarrow \left(\frac{3}{2}\right)4^x &= \left(\frac{4}{\sqrt{3}}\right)3^x \\ \Rightarrow \left(\frac{4}{3}\right)^x &= \frac{2}{3} \times \frac{4}{\sqrt{3}} = \frac{8}{3\sqrt{3}} = \left(\frac{2}{3^{1/2}}\right)^3 = \left(\frac{4}{3}\right)^{3/2} \\ \Rightarrow x &= 3/2\end{aligned}$$

15. The number of solutions of $\sqrt{x+1} - \sqrt{x-1} = 1$, ($x \in \mathbb{R}$) is

- A. 1
- B. 2
- C. 4
- D. Infinite

Sol. A

For the equation to make sense we must have
 $x+1 \geq 0$ and $x-1 \geq 0 \Rightarrow x \geq -1, x \geq 1$ i.e. $x \geq 1$.
We rewrite equation as

$$\sqrt{x+1} = 1 + \sqrt{x-1}$$

and square both the sides to obtain

$$x+1 = 1 + x-1 + 2\sqrt{x-1}$$

$$\Rightarrow \frac{1}{2} = \sqrt{x-1} \Rightarrow \frac{1}{4} = x-1 \text{ or } x = \frac{5}{4}$$

Also, $x = 5/4$ satisfies the given equation.

16. The equation $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$, ($x \in \mathbb{R}$) has
- A. no solution
 - B. one solution
 - C. two solutions
 - D. more than two solutions

Sol. A

The given equation is valid if $x + 1 \geq 0$, $x - 1 \geq 0$ and $4x - 1 \geq 0$ i.e. if $x \geq 1/4$.

Squaring both the sides we get

$$\begin{aligned} x + 1 + x - 1 - 2\sqrt{(x+1)(x-1)} &= 4x - 1 \\ \Rightarrow 1 - 2x &= 2\sqrt{(x+1)(x-1)} \end{aligned}$$

Squaring again, we get

$$\begin{aligned} 1 - 4x + 4x^2 &= 4(x^2 - 1) \\ \Rightarrow 4x &= 5 \text{ or } x = 5/4. \end{aligned}$$

Putting this value of x in the given equation, we get

$$\begin{aligned} \sqrt{\frac{5}{4} + 1} - \sqrt{\frac{5}{4} - 1} &= \sqrt{4\left(\frac{5}{4}\right) - 1} \\ \Rightarrow \frac{3}{2} - \frac{1}{2} &= 2 \text{ or } 1 = 2 \end{aligned}$$

which is not true.

Thus, the given equation has no solution.

17. If c, d are roots of $x^2 - 10ax - 11b = 0$ and a, b are roots of $x^2 - 10cx - 11d = 0$, then the value of $a + b + c + d$ is
- A. 1210
 - B. 1
 - C. 2530
 - D. 11

Sol. A

$$c + d = 10a \quad (1)$$

$$a + b = 10c \quad (2)$$

Subtracting (1) from (2) we get

$$(a - c) + (b - d) = 10(c - a)$$

$$\Rightarrow b - d = 11(c - a) \quad (3)$$

As c is a root of $x^2 - 10ax - 11b = 0$,
we get

$$c^2 - 10ac - 11b = 0 \quad (3)$$

Similarly, $a^2 - 10ac - 11d = 0 \quad (4)$

Subtracting (4) from (3), we get

$$c^2 - a^2 = 11(b - d)$$

$$\Rightarrow (c - a)(c + a) = 11(b - d)$$

$$\Rightarrow c + a = 121$$

$$\begin{aligned} \therefore a + b + c + d &= 10(a + c) \quad [\text{from (1) and (2)}] \\ &= 10(121) = 1210 \end{aligned}$$

18. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$, is equal to the sum of the squares of their reciprocals, then

- A. ab^2, ca^2, bc^2 are in A.P.
- B. ab^2, bc^2, ca^2 are in A.P.
- C. ab^2, bc^2, ca^2 are in G.P.
- D. none of these

Sol. A

Let α, β be the roots of $ax^2 + bx + c = 0$, then

$$\alpha + \beta = -b/a \text{ and } \alpha\beta = c/a.$$

$$\text{We are given } \alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2} \Rightarrow \alpha^2\beta^2(\alpha + \beta) = \alpha^2 + \beta^2$$

$$\Rightarrow (\alpha\beta)^2(\alpha + \beta) = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\Rightarrow \frac{c^2}{a^2} \left(\frac{-b}{a} \right) = \frac{b^2}{a^2} - \frac{2c}{a}$$

$$\Rightarrow -bc^2 = ab^2 - 2a^2c$$

$$\Rightarrow 2a^2c = ab^2 + bc^2$$

$$\Rightarrow ab^2, ca^2, bc^2 \text{ are in A.P.}$$

19. If a and b are the non-zero distinct roots of $x^2 + ax + b = 0$, then the least value of $x^2 + ax + b$ is

- A. $2/3$
- B. $9/4$
- C. $9/4$
- D. 1

Sol. C

$$\text{We have } a + b = -a, ab = b$$

$$\text{As } b \neq 0, \text{ we get } a = 1$$

$$\therefore 1 + b = -1 \Rightarrow b = -2$$

$$\text{Thus, } x^2 + ax + b = x^2 + x - 2$$

$$= \left(x + \frac{1}{2} \right)^2 - \frac{9}{4} \geq -\frac{9}{4}$$

\therefore least value of $x^2 + ax + b$ is $-9/4$ which is attained at $x = -1/2$.

20. If $a < b < c < d$, then the equation $3(x - a)(x - c) + 5(x - b)(x - d) = 0$ has

- A. real and distinct roots
- B. real and equal roots
- C. purely imaginary roots
- D. none of these

Sol. A

Let $f(x) = 3(x - a)(x - c) + 5(x - b)(x - d)$

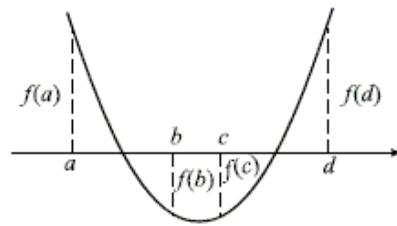
Since f is a polynomial, f is continuous on \mathbf{R} . Also, since $a < b < c < d$,

$$f(a) = 5(a - b)(a - d) > 0$$

$$f(b) = 3(b - a)(b - c) < 0$$

$$f(c) = 5(c - b)(c - d) < 0$$

$$f(d) = 3(d - a)(d - c) > 0$$



As f is continuous on \mathbf{R} , $y = f(x)$ crosses x -axis at least once between a and b and once between c and d .

Thus, f has two distinct real roots, one lying between a and b and one lying between c and d .

21. $\sqrt{x+3-4\sqrt{x-1}} + \sqrt{x+8-6\sqrt{x-1}} = 1$. Solve for x .

(A) $\{5,10\}$ (B) $[1,\infty)$

(C) $[5,10]$ (D) none of these

Put $x - 1 = t^2$ in the given equation, we get

$$\sqrt{t^2 + 1 + 3 - 4t} + \sqrt{t^2 + 1 + 8 - 6t} = 1$$

$$|t - 2| + |t - 3| = 1$$

$$t \in [2, 3]$$

$$x \in [5, 10]$$

22. If the equation $x^2 + 5bx + 8c = 0$, does not have two distinct real roots, then minimum value of $5b + 8c$ is

- (A) 1 (B) 2
 (C) -2 (D) -1

Sol. Since the equation $x^2 + 5bx + 8c = 0$ does not have two distinct real roots and coefficient of x^2 is positive, hence $x^2 + 5bx + 8c \geq 0$

$\Rightarrow 5b + 8c \geq -1$. Hence the minimum value is -1 .

23. Let a, b, c be real numbers with $a \neq 0$ and let α, β be the roots of the equation $ax^2 + bx + c = 0$. Then the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β are given by

- (A) α, β (B) $\frac{c\alpha}{a}, \frac{c\beta}{a}$
 (C) $a\alpha, c\beta$ (D) $c\alpha, a\beta$

Sol. Given equation $a^3x^2 + abcx + c^3 = 0$

written as $a\left(\frac{ax}{c}\right)^2 + b\left(\frac{ax}{c}\right) + c = 0$.

Clearly roots of this equation are $\frac{ax}{c} = \alpha, \beta$

$\Rightarrow x = \frac{c\alpha}{a}, \frac{c\beta}{a}$

24. If a, b, c be the sides of $\triangle ABC$ and equations $ax^2 + bx + c = 0$ and $5x^2 + 12x + 13 = 0$ have a common root, then $\angle C$ is

- (A) 60° (B) 90°
 (C) 120° (D) 45°

Solution: (B) Since $5x^2 + 12x + 13 = 0$ has imaginary roots as $D = 144 - 4 \times 5 \times 13 < 0$

So, both roots of $ax^2 + bx + c = 0$ and $5x^2 + 12x + 13 = 0$ will be common

$$\therefore \frac{a}{5} = \frac{b}{12} = \frac{c}{13} \Rightarrow a^2 + b^2 = c^2 \Rightarrow \angle C = 90^\circ$$

25. If $f(x) = (x - a_1)^2 + (x - a_2)^2 + \dots + (x - a_n)^2$ find x where $f(x)$ is minimum.

- (A) ∞ (B) $\frac{a_1 + a_2 + \dots + a_n}{n}$
(C) $\frac{a_1 - a_2 - \dots - a_n}{n}$ (D) $-\infty$

Solution:

(B) minimum value of $f(x) = nx^2 - 2[a_1 + a_2 + \dots + a_n]x + a_1^2 + a_2^2 \dots + a_n^2$

exist at $x = -\frac{B}{2A} = \frac{a_1 + a_2 + \dots + a_n}{n}$.