

Class: 11
Subject: Math's
Topic: Sequence and Series
No. of Questions: 25

1. If $x > 0$, and $\log_2 x + \log_2 (\sqrt{x}) + \log_2 (\sqrt[4]{x}) + \log_2 (\sqrt[8]{x}) + \log_2 (\sqrt[16]{x}) + \dots = 4$ then x equals
- A. 2
B. 3
C. 4
D. 5

Sol. C

We can write the given equation as

$$\log_2 \left(x^{1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots} \right) = 4$$

$$\Rightarrow \log_2(x^2) = 4 \quad \Rightarrow \quad x^2 = 2^4 \quad \Rightarrow \quad x = 4$$

2. If $(1 + 3 + 5 + \dots + p) + (1 + 3 + 5 + \dots + q) = (1 + 3 + 5 + \dots + r)$ where each set of parentheses contains the sum of consecutive odd integers as shown, the smallest possible value of $p + q + r$, (where $p > 6$) is
- A. 12
B. 21
C. 45
D. 54

Sol. B

We know that

$$1 + 3 + 5 + \dots + (2k - 1) = k^2$$

Thus, the given equation can be written as

$$\left(\frac{p+1}{2}\right)^2 + \left(\frac{q+1}{2}\right)^2 = \left(\frac{r+1}{2}\right)^2$$

$$\Rightarrow (p+1)^2 + (q+1)^2 = (r+1)^2$$

Therefore, $(p+1, q+1, r+1)$ forms a Pythagorean triplet.

As $p > 6$, $p+1 > 7$.

The first Pythagorean triplet containing a number > 7 is $(6, 8, 10)$.

\therefore We may take $p+1 = 8$, $q+1 = 6$, $r+1 = 10$

$$\Rightarrow p + q + r = 21.$$

3. Let a_1, a_2, \dots, a_{10} be in A.P. and h_1, h_2, \dots, h_{10} be in H.P. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then $a_4 h_7$ is

- A. 2
- B. 3
- C. 5
- D. 6

Sol. D

Let d be the common difference of the A.P.,

then

$$3 = a_{10} = 2 + 9d \quad \Rightarrow \quad d = 1/9$$

$$\therefore a_4 = 2 + 3d = 7/3$$

Next, let D be the common difference of the A.P.

$\frac{1}{h_1}, \frac{1}{h_2}, \dots, \frac{1}{h_{10}}$ Then

$$\frac{1}{3} = \frac{1}{h_{10}} = \frac{1}{2} + 9D \quad \Rightarrow \quad D = -\frac{1}{54}$$

$$\therefore \frac{1}{h_7} = \frac{1}{h_1} + 6D = \frac{7}{18} \quad \Rightarrow \quad h_7 = \frac{18}{7}$$

Hence, $a_4 h_7 = (7/3) (18/7) = 6$.

4. Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. If $a < b < c$ and $a + b + c = 3/2$, then the value of a is

- A. $\frac{1}{2\sqrt{2}}$
 B. $\frac{1}{2\sqrt{3}}$
 C. $\frac{1}{2} - \frac{1}{\sqrt{3}}$
 D. $\frac{1}{2} - \frac{1}{\sqrt{2}}$

Sol. D

$$\text{We have } a + c = 2b \Rightarrow 3b = a + b + c = 3/2 \\ \Rightarrow b = 1/2$$

$$\text{Thus, } a + c = 1 \text{ and } a^2 c^2 = b^4 = 1/16 \\ \Rightarrow a^2(1-a)^2 = 1/16 \Rightarrow a(1-a) = \pm 1/4 \\ \Rightarrow 4a^2 - 4a \pm 1 = 0 \Rightarrow (2a-1)^2 = 0 \text{ or } (2a-1)^2 = 2 \\ \Rightarrow a = \frac{1}{2} \text{ or } a = \frac{1}{2} \pm \frac{1}{\sqrt{2}}$$

$$\text{As } a < b < c \text{ and } b = \frac{1}{2}, \text{ we get } a = \frac{1}{2} - \frac{1}{\sqrt{2}}$$

5. Let S_1, S_2, \dots be squares such that for each $n \geq 1$, the length of a side of S_n equals the length of a diagonal of S_{n+1} . If the length of a side of S_1 is 10 cm, then the smallest value of n for which Area (S_n) < 1 is

- A. 7
 B. 8
 C. 9
 D. 10

Sol. B

Let a_n denote the length of a side of S_n . We are given

Length of a side of $S_n =$ Length of a diagonal of S_{n+1}

$$\Rightarrow a_n = \sqrt{2} a_{n+1} \Rightarrow \frac{a_{n+1}}{a_n} = \frac{1}{\sqrt{2}}.$$

Thus, a_1, a_2, a_3, \dots is a G.P. with first term 10 and common ratio $1/\sqrt{2}$. Therefore,

$$a_n = 10(1/\sqrt{2})^{n-1}$$

Also, Area (S_n) = $a_n^2 < 1$

$$\Rightarrow [10(1/\sqrt{2})^{n-1}]^2 < 1 \Rightarrow 100 < 2^{n-1}$$

\Rightarrow smallest possible value of n is 8.

6. Let T_r be the r th term of an AP, for $r = 1, 2, \dots$. If for some positive integers m and n , we have

$$T_m = \frac{1}{n} \text{ and } T_n = \frac{1}{m}, \text{ the } T_{mn} \text{ equals}$$

- A. $\frac{1}{mn}$
 B. $\frac{1}{m} + \frac{1}{n}$
 C. 1
 D. 0

Sol. C

Let a be the first term and d be the common difference of the given A.P. Then according to the hypothesis,

$$T_m - T_n = \frac{1}{n} - \frac{1}{m}$$

$$\Rightarrow (m-n)d = \frac{m-n}{mn} \Rightarrow d = \frac{1}{mn}.$$

$$\Rightarrow T_{mn} - T_m = (mn-m) \frac{1}{mn} = 1 - \frac{1}{n} \Rightarrow T_{mn} = 1$$

7. Consider an infinite geometric series with the first term a and common ratio r . If its sum is 4 and the second term is $3/4$, then
- A. $a = 4/7, r = 3/7$
 - B. $a = 2, r = 3/8$
 - C. $a = 3/2, r = 1/2$
 - D. $a = 3, r = 1/4$

Sol. D

We have

$$\frac{a}{1-r} = 4 \text{ and } ar = \frac{3}{4} \Rightarrow r = \frac{3}{4a}$$

$$\text{Thus, } \frac{a}{1-3/4a} = 4 \Rightarrow a^2 - 4a + 3 = 0 \Rightarrow a = 1, 3$$

When $a = 1, r = 3/4$ and when $a = 3, r = 1/4$.

8. If the sum of first $2n$ terms of the A.P. 2, 5, 8, ... is equal to the sum of first n terms of the A.P. 57, 59, 61, ..., then n equals
- A. 10
 - B. 12
 - C. 11
 - D. 13

Sol. C

Sum to $2n$ terms of 2, 5, 8, ... is

$$\frac{2n}{2} [2(2) + (2n-1)(3)] = n(6n+1)$$

and sum to n terms of 57, 59, 61, ..., is

$$\frac{n}{2} [2(57) + (n-1)(2)] = n(56+n)$$

According to given condition

$$n(6n+1) = n(56+n) \Rightarrow 6n+1 = 56+n \Rightarrow n = 11$$

9. For a positive integer n , let $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{(2^n) - 1}$. Then
- A. $a(100) < 100$
 - B. $a(100) > 100$
 - C. $a(200) < 100$
 - D. none of these

Sol. A

We have

$$\begin{aligned}
 a(n) &= 1 + \left(\frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{4} + \dots + \frac{1}{7}\right) \\
 &+ \left(\frac{1}{8} + \dots + \frac{1}{15}\right) + \dots + \left(\frac{1}{2^{n-1}} + \frac{1}{2^{n-1}+1} + \dots + \frac{1}{(2^n)-1}\right) \\
 &< 1 + \frac{2}{2} + \frac{4}{4} + \frac{8}{8} + \dots + \frac{2^{n-1}}{2^{n-1}} = n
 \end{aligned}$$

Thus, $a(100) < 100$.

10. If $x > 1, y > 1, z > 1$ are in G.P., then $\frac{1}{1 + \ln x}, \frac{1}{1 + \ln y}, \frac{1}{1 + \ln z}$ are in
- A. A.P.
 - B. G.P.
 - C. H.P.
 - D. None of these

Sol. C

$$\begin{aligned}
 &\text{As } x, y, z \text{ are in G.P.} \Rightarrow y^2 = xz \\
 \Rightarrow &\ln(y^2) = \ln(xz) \quad \Rightarrow \quad 2\ln(y) = \ln(x) + \ln(z) \\
 \Rightarrow &2(1 + \ln(y)) = (1 + \ln(x)) + (1 + \ln(z)) \\
 \Rightarrow &1 + \ln(x), 1 + \ln(y), 1 + \ln(z) \text{ are in A.P.} \\
 \Rightarrow &\frac{1}{1 + \ln(x)}, \frac{1}{1 + \ln(y)}, \frac{1}{1 + \ln(z)} \text{ are in H.P.}
 \end{aligned}$$

11. Let x be the arithmetic mean and y, z be the two geometric means between any two

positive numbers. Then value of $\frac{y^3 + z^3}{xyz}$ is

- A. 2
- B. 3
- C. $\frac{1}{2}$
- D. $\frac{3}{2}$

Sol. A

Let two positive numbers be a and b . Then $x = (a + b)/2$. Also, a, y, z, b are in G.P. If r is the common ratio of this G.P., then $b = ar^3 \Rightarrow r = (b/a)^{1/3}$.

We have

$$\frac{y^3 + z^3}{xyz} = \frac{a^3 r^3 + a^3 r^6}{x(ar)(ar^2)} = \frac{a(1+r^3)}{x} = \frac{a+b}{(a+b)/2} = 2$$

12. The sum to n terms of the series $\frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \frac{1+2+3}{1^3+2^3+3^3} + \dots$ is

- A. $\frac{n}{n+1}$
- B. $\frac{n}{2(n+1)}$
- C. $\frac{2n}{n+1}$
- D. $\frac{2}{n(n+1)}$

Sol. C

If t_n denotes the n th term of the series, then

$$t_n = \frac{1+2+3+\dots+n}{1^3+2^3+3^3+\dots+n^3} = \frac{\frac{1}{2}n(n+1)}{\frac{1}{4}n^2(n+1)^2}$$

$$= \frac{2}{n(n+1)} = 2\left[\frac{1}{n} - \frac{1}{n+1}\right]$$

$$\Rightarrow \sum_{k=1}^n t_k = 2\left\{\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \dots + \left(\frac{1}{n} - \frac{1}{n+1}\right)\right\}$$

$$= 2\left(1 - \frac{1}{n+1}\right) = \frac{2n}{n+1}$$

13. Let $\sum_{r=1}^n r^4 = f(n)$, then $\sum_{r=1}^n (2r-1)^4$ is equal to

- A. $f(2n) - 16f(n)$
- B. $f(2n) - 7f(n)$
- C. $f(2n-1) - 8f(n)$
- D. none of these

Sol. A

We have

$$\sum_{r=1}^n (2r-1)^4 = \sum_{r=1}^{2n} r^4 - \sum_{r=1}^n (2r)^4 = f(2n) - 16f(n)$$

14. The sum of first n terms of the series $1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + 5^2 + 2 \times 6^2 + \dots$ is $n(n+1)^2/2$ when n is even. When n is odd, the sum of the series is

- A. $n^2(3n+1)/4$
- B. $n^2(n+1)/2$
- C. $n^3(n-1)/2$
- D. none of these

Sol. B

Let $n = 2m$, then

$$\begin{aligned} S_{2m} &= 1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + \dots + \\ &\quad (2m-1)^2 + 2(2m)^2 \\ &= 2m(2m+1)^2/2 = m(2m+1)^2 \end{aligned}$$

When $n = 2m - 1$,

$$\begin{aligned} S_{2m-1} &= 1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + \dots + \\ &\quad (2m-1)^2 \\ &= S_{2m} - 2(2m)^2 = m(2m+1)^2 - 2(2m)^2 \\ &= m[4m^2 + 4m + 1 - 8m] = m(2m-1)^2 \\ &= n^2(n+1)/2 \end{aligned}$$

15. If $S_n = \sum_{r=1}^n t_r = \frac{1}{6} n(2n^2 + 9n + 13)$, then $\sum_{r=1}^n \sqrt{t_r}$ equals

- A. $\frac{1}{2}n(n+1)$
- B. $\frac{1}{2}n(n+2)$
- C. $\frac{1}{2}n(n+3)$
- D. $\frac{1}{2}n(n+5)$

Sol. C

We have $t_n = S_n - S_{n-1} \forall n \geq 2$

$$\begin{aligned} \therefore t_n &= \frac{1}{6} n(2n^2 + 9n + 13) - \frac{1}{6} (n-1) \{(2(n-1)^2 \\ &\quad + 9(n-1) + 13)\} \\ &= \frac{1}{6} [2(n^3 - (n-1)^3) + 9(n^2 - (n-1)^2) + 13(n \\ &\quad - n + 1)] \\ &= \frac{1}{6} [6n^2 - 6n + 2 + 9(2n - 1) + 13] \\ &= \frac{1}{6} (6n^2 + 12n + 6) \\ &= (n+1)^2 \end{aligned}$$

Also, $t_1 = S_1 = 4 = (1+1)^2$

$$\therefore \sum_{r=1}^n \sqrt{t_r} = \sum_{r=1}^n (r+1) = \frac{1}{2} (n+1)(n+2) - 1 = \frac{1}{2} n(n+3)$$

16. The sum to n terms of the series $\frac{1}{\sqrt{7} + \sqrt{10}} + \frac{1}{\sqrt{10} + \sqrt{13}} + \frac{1}{\sqrt{13} + \sqrt{16}} + \dots$ is

- A. $\frac{1}{3}(\sqrt{7+3n} - \sqrt{7})$
 B. $\frac{\sqrt{4+3n} - 2}{3}$
 C. $\frac{1}{3}(\sqrt{10+3n} - \sqrt{10})$
 D. None of these

Sol. A

We have

$$t_r = \frac{1}{\sqrt{4+3r} + \sqrt{7+3r}} = \frac{\sqrt{7+3r} - \sqrt{4+3r}}{3}$$

Thus, sum to n terms is equal to

$$\begin{aligned} \frac{1}{3} [(\sqrt{10} - \sqrt{7}) + (\sqrt{13} - \sqrt{10}) + \dots + (\sqrt{7+3n} - \sqrt{4+3n})] \\ = \frac{1}{3} (\sqrt{7+3n} - \sqrt{7}) \end{aligned}$$

17. If $\exp \{(\tan^2 x - \tan^4 x + \tan^6 x - \tan^8 x + \dots) \log_e 16\}$, $0 < x < \pi/4$, satisfies the quadratic equation $x^2 - 3x + 2 = 0$, then value of $\cos^2 x + \cos^4 x$ is
- A. 4/5
 B. 21/16
 C. 17/11
 D. 19/31

Sol. B

We have

$$\begin{aligned} \tan^2 x - \tan^4 x + \tan^6 x - \tan^8 x + \dots \\ = \frac{\tan^2 x}{1 - (-\tan^2 x)} = \frac{\tan^2 x}{\sec^2 x} = \sin^2 x \end{aligned}$$

Therefore

$$\begin{aligned} y &= \exp \{(\tan^2 x - \tan^4 x + \tan^6 x - \tan^8 x + \dots) \log_e 16\} \\ &= \exp \{(\sin^2 x) \log_e 16\} \\ &= \exp \{\log_e (16^{\sin^2 x})\} = 16^{\sin^2 x} \end{aligned}$$

As y satisfies $x^2 - 3x + 2 = 0$, we get $y = 1$ or $y = 2$

$$\Rightarrow 16^{\sin^2 x} = 1 \text{ or } 16^{\sin^2 x} = 2$$

Since $0 < x < \pi/4$, $0 < \sin x < 1/\sqrt{2} \Rightarrow 0 < \sin^2 x < 1/2$

$\therefore 16^{\sin^2 x} = 1$ is not possible.

Thus, $16^{\sin^2 x} = 2 \Rightarrow \sin^2 x = 1/4$

Thus, $\cos^2 x + \cos^4 x = (1 - \sin^2 x) + (1 - \sin^2 x)^2 = 21/16$

18. If $\exp \{(\sin^2 x + \sin^4 x + \sin^6 x + \dots \text{ upto } \infty) \ln 2\}$ satisfies the equation $x^2 - 17x + 16 = 0$

then value of $\frac{2 \cos x}{\sin x + 2 \cos x}$ ($0 < x < \pi/2$) is

- A. $\frac{1}{2}$
- B. $\frac{3}{2}$
- C. $\frac{5}{2}$
- D. None of these

Sol. A

We have $\sin^2 x + \sin^4 x + \sin^6 x + \dots$

$$= \frac{\sin^2 x}{1 - \sin^2 x} = \tan^2 x$$

Therefore, $\alpha = \exp \{(\sin^2 x + \sin^4 x + \sin^6 x + \dots \text{ upto } \infty) \ln 2\}$
 $= \exp \{\tan^2 x \ln 2\} = \exp \{\ln 2^{\tan^2 x}\}$
 $= 2^{\tan^2 x}$

As α satisfies the equation $x^2 - 17x + 16 = 0$ we get

$$\alpha = 1 \text{ or } \alpha = 16$$

Since $0 < x < \pi/2$, $\tan^2 x > 0 \Rightarrow \alpha = 2^{\tan^2 x} > 1$. Therefore,

$$2^{\tan^2 x} = 16 = 2^4 \Rightarrow \tan^2 x = 4 \Rightarrow \tan x = 2$$

[$\because \tan x > 0$]

Thus, $\frac{2 \cos x}{\sin x + 2 \cos x} = \frac{2}{\tan x + 2} = \frac{2}{2 + 2} = \frac{1}{2}$

19. If three positive real numbers a, b, c are in A.P. such that $abc = 4$, then the minimum possible value of b is

- A. $2^{3/2}$
- B. $2^{2/3}$
- C. $2^{1/3}$
- D. $2^{5/2}$

Sol. B

Let d be the common difference of the A.P.,

then

$$4 = abc = (b - d)b(b + d) = b(b^2 - d^2)$$

$$\Rightarrow b^3 = 4 + bd^2 \geq 4 \quad [\because b > 0, d^2 \geq 0]$$

$$\Rightarrow b \geq 2^{2/3}$$

Thus, minimum possible value of b is $2^{2/3}$, that is the case when $d = 0$.

20. If the ratio of sums to n terms of two A.P's is $(5n + 7) : (3n + 2)$, then the ratio of their 17th term is

- A. 172 : 99
- B. 172 : 101
- C. 175 : 99
- D. 175 : 101

Sol. B

Let two A.P.'s be

$$a, a + d, a + 2d, a + 3d, \dots$$

and $A, A + D, A + 2D, A + 3D, \dots$

According to the given condition,

$$\frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}[2A + (n-1)D]} = \frac{5n+7}{3n+2} \Rightarrow \frac{a + \frac{n-1}{2}d}{A + \frac{n-1}{2}D} = \frac{5n+7}{3n+2}$$

Putting $\frac{n-1}{2} = 16$ or $n = 33$, we get

$$\frac{a+16d}{A+16D} = \frac{5(33)+7}{3(33)+2} = \frac{172}{101}$$

21. If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M. between a and b . then find the value of n .

Sol:

$$\frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \frac{a+b}{2}$$

$$2a^n + 2b^n = (a+b)(a^{n-1} + b^{n-1})$$

$$2a^n + 2b^n = a^n + ab^{n-1} + b.a^{n-1} + b^n$$

$$a^n + b^n = a.b^{n-1} + b.a^{n-1}$$

$$a^n - b.a^{n-1} = a.b^{n-1} - b^n$$

$$a^{n-1}(a-b) = b^{n-1}(a-b)$$

$$\left(\frac{a}{b}\right)^{n-1} = \frac{a-b}{a-b}$$

$$\left(\frac{a}{b}\right)^{n-1} = \left(\frac{a}{b}\right)^0 \left(\because \left(\frac{a}{b}\right)^0 = 1\right)$$

$$n-1 = 0$$

$$n = 1$$

22. If A.M. and G.M of two positive no. a and b are 10 and 8 respectively find the no.

Sol:

$$A.M = \frac{a+b}{2} = 10$$

$$G.M = \sqrt{ab} = 8$$

$$a+b = 20$$

$$ab = 64$$

$$(a-b)^2 = (a+b)^2 - 4ab$$

$$(a-b)^2 = 400 - 256$$

$$a-b = \sqrt{144}$$

$$a-b = \pm 12$$

$$a = 4, b = 16$$

$$\text{or } a = 16, b = 4$$

23. Find the sum to n terms of the series 5+11+19+29+41+

Sol:

$$\begin{aligned} a_n &= 5 + \frac{(n-1)[12 + (n-2) \times 2]}{2} \\ &= 5 + (n-1)(n+4) \\ &= n^2 + 3n + 1 \\ S_n &= \sum_{k=1}^n k^2 + 3 \sum_{k=1}^n k + n \\ &= \frac{n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + n \\ &= \frac{n(n+2)(n+4)}{3} \end{aligned}$$

24. Find the sum of integers from 1 to 100 that are divisible by 2 or 5

Sol:

Divisible by 2

2, 4, 6, ----- 100

$a = 2, d = 2, a_n = 100$

$100 = 2 + (n-1) \times 2$

$n = 50$

$$S_{50} = \frac{50}{2}[2+100] = 2550$$

divisible by 5

$a = 5, d = 5, a_n = 100$

$5 + (n-1) \times 5 = 100$

$n = 20$

$$S_{20} = 1050$$

divisible by both 2 or 5

10, 20, 30, ----- 100

$a = 10, d = 10, a_n = 100$

$100 = 10 + (n-1) \times 10$

$n = 10$

$$S_{10} = \frac{10}{2}[10+100] \\ = 550$$

$$\text{A T Q Sum} = 2550 + 1050 - 550 \\ = 3050$$

25. The sum of the first four terms of an A.P. is 56. The sum of the last four term is 112. If its first term is 11, then find the no. of terms.

Sol:

$$\text{A T Q } a = 11$$

$$(a) + (a + d) + (a + 2d) + (a + 3d) = 56$$

$$2a + 3d = 28 \quad (1)$$

$$a_n + a_{n-1} + a_{n-2} + a_{n-3} = 112$$

$$2nd - 5d = 14$$

$$D = 2$$

$$N = 11$$

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