

Class: 11
Subject: Math's
Topic: Straight Lines
No. of Questions: 25

1. The point (2, 3) undergoes the following three transformation successively.
- (i) reflection about the line $y = x$
 - (ii) translation through a distance 2 units along the positive direction of y -axis
 - (iii) rotation through an angle of 45° about the origin in the anticlockwise direction

The final co-ordinates of the point are

- A. $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$
- B. $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$
- C. $\left(\frac{1}{\sqrt{2}}, -\frac{7}{\sqrt{2}}\right)$
- D. None of these

Sol. B

After the first transformation, we get
 $(2, 3) \Rightarrow (3, 2)$ reflection about the line $y = x$.
After second transformation, we get
 $(3, 2) \Rightarrow (3, 2 + 2) \Rightarrow (3, 4)$
After the last transformation, we get

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ \sqrt{2} \\ 7 \\ \sqrt{2} \end{bmatrix}$$

2. The lines $px + qy + r = 0$, $qx + ry + p = 0$, $rx + py + q = 0$ are concurrent if

- A. $p + q + r = 0$
- B. $p^2 + q^2 + r^2 + pq + qr + rp = 0$
- C. $p^2 + q^2 + r^2 + 3pqr = 0$
- D. all of the above

Sol. A

The given three lines will be concurrent if

$$\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

, which gives, $p + q + r = 0$

3. The straight lines $x + y = 0$, $3x + y - 4 = 0$ and $x + 3y - 4 = 0$ form which of the following triangles?

- A. Isoscles
- B. Equilateral
- C. Right-angled
- D. None of these

Sol. A

First two lines intersect at $(2, -2)$... (A)

The second line and the third line intersect at $(1, 1)$... (B)

The first line and the last line intersect at $(-2, 2)$... (C)

Distance of A and C from B is equal, whereas AC is not equal to them. Hence, it is an isosceles triangle.

4. The vertices of Δ are $(0, 0)$, $(3, 0)$ and $(0, 4)$. Its orthocentre is at

- A. $(0, 0)$
- B. $(1, 4/3)$
- C. $(3/2, 2)$
- D. None of these

Sol. A

Given vertices are of a right angled triangle, and the orthocenter of a right angled triangle lies on the intersection of perpendiculars. i.e. $(0, 0)$

5. One of the equations of lines passing through the point $(3, -2)$ and inclined at 60° to the line $\sqrt{3}x + y = 1$ is
- A. $y + 2 = 0$
 - B. $x + 2 = 0$
 - C. $x + y = 2$
 - D. $x - y = \sqrt{3}$

Sol. A

6. A straight line through the point $(2, 2)$ intersects the lines $\sqrt{3}x + y = 0$ and $\sqrt{3}x - y = 0$ at the points A and B. The equation to the line AB so that the triangle OAB is equilateral, is
- A. $x - 2 = 0$
 - B. $y - 2 = 0$
 - C. $x + y - 4 = 0$
 - D. none of these

Sol. B
Equation of the line passing from $(2, 2)$, and slope m (say), is

$$(y - 2) = m(x - 2); m = \frac{y - 2}{x - 2}$$

The angle between the line and given lines should be 60° , as they are required to form an equilateral triangle.

7. The incentre of a triangle with vertices $(1, \sqrt{3})$, $(0, 0)$ and $(2, 0)$ is

- A. $\left(1, \frac{\sqrt{3}}{2}\right)$
- B. $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$
- C. $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$
- D. None of these

Sol. D

Sides of the triangle are 2, 2 and 2

$$\text{Incentre: } \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right) = \left(1, \frac{1}{\sqrt{3}} \right)$$

8. If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product then c has the value
- A. 2
B. -1
C. 1
D. -2

Sol. A
 $x^2 + 2hxy + by^2 = 0$
 $m_1 + m_2 = -2h/b$ and
 $m_1 m_2 = a/b$
where, $m_1 m_2$ are slopes of lines
 $m_1 + m_2 = 4 m_1 m_2$
 $-2c/7 = -4/7$

9. A line passes through point (4, 1) and is perpendicular to line $3x + y = 3$, then its x - intercept is
- A. $\frac{1}{3}$
B. $\frac{2}{3}$
C. 1
D. $\frac{4}{3}$

Sol. C
Slope of the line perpendicular to the given line $3x + y = 3$ will be $1/3$, and it passes through (4, 1), its equation will be
 $(y - 1) = 1/3 (x - 4)$
Or $x - 3y = 1$
To find out the x intercept, put $y = 0$, we get
 $x = 1$

10. The value of λ for which the equation $x^2 - \lambda xy + 2y^2 + 3x - 5y + 2 = 0$ may represent a pair of straight lines is
- A. 2
 B. 3
 C. 4
 D. 1

Sol. B
 Comparing the equation with the standard second degree equation, i.e. $ax^2 + bx^2 + 2fx + 2gy + 2hxy + c = 0$, we get

$$a = 1, b = 1, c = 2, h = -\lambda/2, g = -5/2, f = 3/2$$

For straight lines:

$$\Delta = 0$$

$$\Delta =$$

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & -\frac{\lambda}{2} & \frac{-5}{2} \\ -\frac{\lambda}{2} & 1 & \frac{3}{2} \\ \frac{-5}{2} & \frac{3}{2} & 2 \end{vmatrix} = 0$$

So, we get

$$\lambda = 3$$

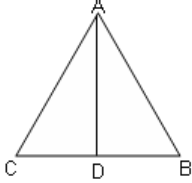
11. The equation of base of an equilateral triangle is $x + y = 2$ and the vertex opposite to this base is $(2, -1)$. Then the length of the side of triangle equals

- A. $\sqrt{\frac{1}{3}}$
 B. $\sqrt{3}$
 C. $\sqrt{\frac{2}{3}}$
 D. $\sqrt{\frac{3}{2}}$

Sol. C

Let the length of side be 'a'.

Distance of the opposite vertex from the given line is $AD = \frac{1}{\sqrt{2}}$



In triangle ADB, $AB^2 = AD^2 + BD^2$
 $a^2 = 1/2 + a^2/4$ (D is mid point on BC)

$$a = \sqrt{\frac{2}{3}}$$

12. If the distance between lines $5x + 12y - 1 = 0$ and $10x + 24y + k = 0$ is 2, what is the value of k?
- A. 54 only
 B. 50 only
 C. 54, 50
 D. 53 only

Sol. C
 On multiplying by 2 on both sides of the first equation, we get
 $10x + 24y - 2 = 0$
 Distance between the two lines:

$$\frac{|c - d|}{\sqrt{a^2 + b^2}} = 2$$

$$\frac{|-2 + k|}{\sqrt{100 + 576}} = 2$$

$$|k - 2| = 2 \sqrt{676}$$

Squaring both sides, we get:
 $k = -54, 50$

13. Foot of perpendicular drawn from (0, 5) in the line $3x - 4y - 5 = 0$ is
- A. (3, 2)
 B. (3, 1)
 C. (1, 3)
 D. (2, 3)

Sol. B

Distance of the point (0, 5) from the line is 5 units.

We can check from the options which point is 5 units away from the given point.

And only option 2, i.e. (3, 1) satisfies the given condition.

14. Let P (- 1, 0), Q (0, 0) and R (3, $3\sqrt{3}$) be three points. Then the equation of the bisector of the angle PQR is

A. $\frac{\sqrt{3}}{2}x + y = 0$

B. $x + \sqrt{3}y = 0$

C. $\sqrt{3}x + y = 0$

D. $x + \frac{\sqrt{3}}{2}y = 0$

Sol.

C

Line passing through PQ is $y = 0$

Line passing through QR is $y - \sqrt{3}x = 0$

Applying the formula of angle bisector between two lines, i.e.

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

We get

$$\sqrt{3}x + y = 0$$

15. If the lines $x + 2ay + a = 0$, $x + 3by + b = 0$ and $x + 4cy + c = 0$ are concurrent, then a, b, c are in

- A. A.P
 B. H.P
 C. G.P
 D. None of these

Sol.

B

Given three lines are concurrent if

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

. On solving it gives

$$b = \frac{2ac}{a + c}$$

, i.e a, b, c is in HP.

16. The equation of line parallel to x-axis and bisecting the join of (1, 4) and (-2, 6) is
- A. $y = 5$
 - B. $y = 3$
 - C. $y + 1 = 0$
 - D. $x = 5$

Sol. A
The mid point of the line segment joining the given point is $(1 - 2/2, 4 + 6/2)$, i.e. $(-1/2, 5)$. The line is bisecting the join of the points; it will pass through their mid point. Checking from options, we can say that option 1, i.e. $y = 5$, is the line.

17. The line formed by joining (-1, 1) and (5, 7) is divided by a line $x + y = 4$ in the ratio
- A. 1 : 2
 - B. 1 : 3
 - C. 3 : 4
 - D. 1 : 4

Sol. A
You might have got confused between the point of intersection and the ratio (1, 3) is the point of intersection and 1 : 2 is the ratio

18. If the points (1, 2) and (3, 4) were to be on the same side of line $3x - 5y + a = 0$, then
- A. $7 < a < 11$
 - B. $a = 7$
 - C. $a = 11$
 - D. $a < 7$ or $a > 11$

Sol. D

19. The extremities of diagonal of a parallelogram are the points (3, -4) and (-6, 5). If the third vertex is (-2, 1), then the fourth vertex will be
- A. (0, 1)
 - B. (1, 0)
 - C. (1, 1)
 - D. (-1, 0)

Sol. D

Let the fourth vertex be D (x, y) Given A(3, - 4), C(- 6, 5), B(- 2, 1)
Since diagonals of a parallelogram bisect each other, hence midpoints of AC and BD are the same.
 $(- 3/2, 1/2) = (- 2 + x /2, 1 + y /2)$
 $- 3 = - 2 + x, 1 = 1 + y$
 $x = - 1, y = 0$

20. Orthocentre of the triangle formed by the lines $x + y = 1$ and $xy = 0$ is

- A. (0, 0)
- B. (0, 1)
- C. (1, 0)
- D. (- 1, 1)

Sol. A

The given equation $xy = 0$ represents a pair of perpendicular lines intersecting at (0, 0). So the triangle formed is a right angled triangle. Whose orthocenter is at intersection of perpendiculars (0, 0)

21. If three points (h, 0) (a, b) and (0, k) lie on a line, show that $\frac{a}{h} + \frac{b}{k} = 1$

Sol:

Let $A(h, 0)$ $B(a, b)$ and $C(0, k)$

Slope of AB = slope of BC

$$\frac{b-0}{a-h} = \frac{k-b}{0-a}$$

$$\frac{b}{a-h} = \frac{h-b}{-a}$$

$$(a-h)(k-b) = -ab$$

$$ak - \cancel{ab} - hk + hb = -\cancel{ab}$$

$$ak + hb = hk$$

$$\frac{ak}{hk} + \frac{hb}{hk} = 1$$

$$\frac{a}{h} + \frac{b}{k} = 1$$

22. p (a, b) is the mid point of a line segment between axes. Show that equation of the line is

$$\frac{x}{a} + \frac{y}{b} = 2$$

Sol:

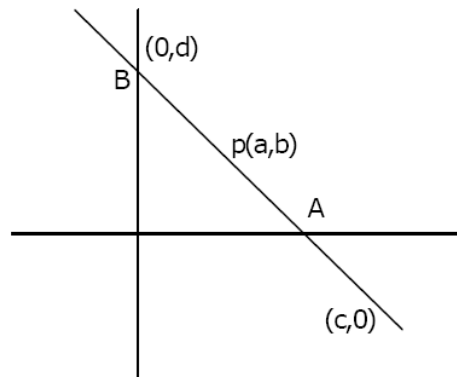
Req. eq. be

$$\frac{x}{c} + \frac{y}{d} = 1 \dots (i)$$

P is the mid point

$$\text{Coordinate of } p = \left(\frac{c}{2}, \frac{d}{2} \right)$$

$$(a, b) = \left(\frac{c}{2}, \frac{d}{2} \right)$$



$$\frac{a}{1} = \frac{c}{2}$$

$$c = 2a$$

$$\frac{b}{1} = \frac{d}{2}$$

$$d = 2b$$

Put the value of C and D in eq. (i)

$$\frac{x}{2a} + \frac{y}{2b} = 1$$

$$\frac{x}{a} + \frac{y}{b} = 2$$

23. The line \perp to the line segment joining the points (1,0) and (2,3) divides it in the ratio 1 : n find the equation of line.

Sol:

Coordinate of C $\left(\frac{2+n}{1+n}, \frac{3}{1+n} \right)$

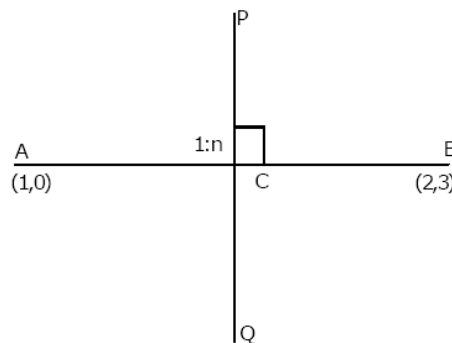
$$m_{AB} = 3$$

$$m_{PQ} = -\frac{1}{3}$$

Eq. of PQ is

$$\frac{y - \frac{3}{1+n}}{1} = -\frac{1}{3} \left(\frac{x - \frac{2+n}{1+n}}{1} \right)$$

$$(n+1)x + 3(n+1)y - (n+11) = 0$$



24. A line is such that its segment between that lines $5x-y+4=0$ and $3x+4y-4=0$ is bisected at the point $(1,5)$ obtain its equation.

Sol:

$$P(x_1, y_1) \text{ lies on } 5x - y + 4 = 0$$

$$\Rightarrow 5x_1 - y_1 + 4 = 0$$

$$\text{And } Q(x_2, y_2) \text{ lies on } 3x + 4y - 4 = 0$$

$$3x_2 + 4y_2 - 4 = 0$$

On solving

$$y_1 = 5x_1 + 4$$

$$y_2 = \frac{4 - 3x_2}{4}$$

Since R is the mid point of PQ

Since R is the mid point of PQ

$$\frac{x_1 + x_2}{2} = 1, \quad \frac{y_1 + y_2}{2} = 5$$

$$x_1 + x_2 = 2, \quad y_1 + y_2 = 10$$

On solving

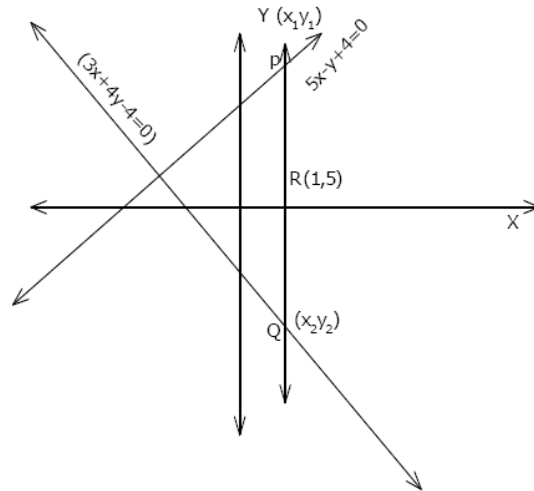
$$x_1 = \frac{26}{23}, \quad x_2 = \frac{20}{23}$$

$$\text{And } y_1 = \frac{222}{23}, \quad y_2 = \frac{8}{23}$$

Eq. of PQ

$$y - \frac{222}{23} = \frac{\frac{8}{23} - \frac{222}{23}}{\frac{20}{23} - \frac{26}{23}} \left(x - \frac{26}{23} \right)$$

$$107x - 3y - 92 = 0$$



25. Find the equation of the lines which pass through the point (4,5) and make equal angles with the lines $5x-12y+6=0$ and $3x-4y-7=0$

Sol:

The slopes of the given lines are $\frac{5}{12}$ and $\frac{3}{4}$

Let m be the slope of a required line

$$\text{ATQ } \left| \frac{m - \frac{5}{12}}{1 + m \cdot \frac{5}{12}} \right| = \left| \frac{m - \frac{3}{4}}{1 + m \cdot \frac{3}{4}} \right|$$

$$\Rightarrow \left| \frac{12m - 5}{12 + 5m} \right| = \left| \frac{4m - 3}{4 + 3m} \right|$$

$$\frac{12m - 5}{12 + 5m} = \frac{4m - 3}{4 + 3m}$$

$$16m^2 = -16$$

$$m^2 = -1$$

Neglect

$$\frac{12m - 5}{12 + 5m} = -\frac{4m - 3}{4 + 3m}$$

$$m = \frac{4}{7}, \frac{-7}{4}$$

Req. eq. are

$$y - 5 = \frac{4}{7}(x - 4)$$

$$4x - 7y + 19 = 0$$

$$y - 5 = \frac{-7}{4}(x - 4)$$

$$7x + 4y - 48 = 0$$