

Class: 11
Subject: Math's
Topic: Trigo phase
No. of Questions: 21

1. A circular wire of radius 3 cm is cut and bent so as to lie along the circumference of a hoop whose radius is 48 cm. Find the angle in degrees which is subtended at the center of hoop.

Sol: Given that circular wire is of radius 3 cm, so when it is cut then its length = $2\pi \times 3 = 6\pi$ cm. Again, it is being placed along a circular hoop of radius 48 cm. Here, $s = 6\pi$ cm is the length of arc and $r = 48$ cm is the radius of the circle. Therefore, the angle θ , in radian, subtended by the arc at the center of the circle is given by

$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{6\pi}{48} = \frac{\pi}{8} = 22.5^\circ.$$

2. If $A = \cos^2 \theta + \sin^4 \theta$ for all values of θ , then prove that $\frac{3}{4} \leq A \leq 1$.

Sol:

We have $A = \cos^2 \theta + \sin^4 \theta = \cos^2 \theta + \sin^2 \theta \sin^2 \theta \leq \cos^2 \theta + \sin^2 \theta$

Therefore, $A \leq 1$

Also, $A = \cos^2 \theta + \sin^4 \theta = (1 - \sin^2 \theta) + \sin^4 \theta$

$$= \left(\sin^2 \theta - \frac{1}{2} \right)^2 + \left(1 - \frac{1}{4} \right) = \left(\sin^2 \theta - \frac{1}{2} \right)^2 + \frac{3}{4} \geq \frac{3}{4}$$

Hence, $\frac{3}{4} \leq A \leq 1$.

3. Find the value of $\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ$

Sol:

We have

$$\begin{aligned} \sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ &= \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ} \\ &= \frac{\sqrt{3} \cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cos 20^\circ} = 4 \left(\frac{\frac{\sqrt{3}}{2} \cos 20^\circ - \frac{1}{2} \sin 20^\circ}{2 \sin 20^\circ \cos 20^\circ} \right) \\ &= 4 \left(\frac{\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ}{\sin 40^\circ} \right) \quad (\text{Why?}) \\ &= 4 \left(\frac{\sin (60^\circ - 20^\circ)}{\sin 40^\circ} \right) = 4 \quad (\text{Why?}) \end{aligned}$$

4. If θ lies in the second quadrant, then show that

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = -2\sec\theta$$

Sol:

We have

$$\begin{aligned} \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} &= \frac{1-\sin\theta}{\sqrt{1-\sin^2\theta}} + \frac{1+\sin\theta}{\sqrt{1-\sin^2\theta}} = \frac{2}{\sqrt{\cos^2\theta}} \\ &= \frac{2}{|\cos\theta|} \quad (\text{Since } \sqrt{\alpha^2} = |\alpha| \text{ for every real number } \alpha) \end{aligned}$$

Given that θ lies in the second quadrant so $|\cos\theta| = -\cos\theta$ (since $\cos\theta < 0$).

5. Find the value of $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$

Sol:

$$\begin{aligned} \text{We have } \tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ \\ &= \tan 9^\circ + \tan 81^\circ - \tan 27^\circ - \tan 63^\circ \\ &= \tan 9^\circ + \tan (90^\circ - 9^\circ) - \tan 27^\circ - \tan (90^\circ - 27^\circ) \\ &= \tan 9^\circ + \cot 9^\circ - (\tan 27^\circ + \cot 27^\circ) \quad (1) \end{aligned}$$

$$\text{Also } \tan 9^\circ + \cot 9^\circ = \frac{1}{\sin 9^\circ \cos 9^\circ} = \frac{2}{\sin 18^\circ} \quad (\text{Why?}) \quad (2)$$

Similarly, $\tan 27^\circ + \cot 27^\circ = \frac{1}{\sin 27^\circ \cos 27^\circ} = \frac{2}{\sin 54^\circ} = \frac{2}{\cos 36^\circ}$ (Why?) (3)

Using (2) and (3) in (1), we get

$$\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ = \frac{2}{\sin 18^\circ} - \frac{2}{\cos 36^\circ} = \frac{2 \times 4}{\sqrt{5}-1} - \frac{2 \times 4}{\sqrt{5}+1} = 4$$

6. Prove that $\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$

Sol:

We have

$$\begin{aligned} \frac{\sec 8\theta - 1}{\sec 4\theta - 1} &= \frac{(1 - \cos 8\theta) \cos 4\theta}{\cos 8\theta (1 - \cos 4\theta)} \\ &= \frac{2 \sin^2 4\theta \cos 4\theta}{\cos 8\theta 2 \sin^2 2\theta} \cdot \frac{\cos 4\theta}{\cos 4\theta} \\ &= \frac{2 \cos 8\theta \sin^2 2\theta}{2 \cos 8\theta \sin^2 2\theta} \\ &= \frac{\sin 4\theta \sin 8\theta}{2 \cos 8\theta \sin^2 2\theta} \\ &= \frac{2 \sin 2\theta \cos 2\theta \sin 8\theta}{2 \cos 8\theta \sin^2 2\theta} \\ &= \frac{\tan 8\theta}{\tan 2\theta} \end{aligned}$$

7. Solve the equation $\sin \theta + \sin 3\theta + \sin 5\theta = 0$

Sol:

We have $\sin \theta + \sin 3\theta + \sin 5\theta = 0$
 or $(\sin \theta + \sin 5\theta) + \sin 3\theta = 0$
 or $2 \sin 3\theta \cos 2\theta + \sin 3\theta = 0$
 or $\sin 3\theta (2 \cos 2\theta + 1) = 0$
 or $\sin 3\theta = 0$ or $\cos 2\theta = -\frac{1}{2}$

When $\sin 3\theta = 0$, then $3\theta = n\pi$ or $\theta = \frac{n\pi}{3}$

When $\cos 2\theta = -\frac{1}{2} = \cos \frac{2\pi}{3}$, then $2\theta = 2m\pi \pm \frac{2\pi}{3}$ or $\theta = m\pi \pm \frac{\pi}{3}$

which gives $\theta = (3n+1)\frac{\pi}{3}$ or $\theta = (3n-1)\frac{\pi}{3}$

All these values of θ are contained in $\theta = \frac{n\pi}{3}, n \in \mathbf{Z}$. Hence, the required solution set

is given by $\{\theta : \theta = \frac{n\pi}{3}, n \in \mathbf{Z}\}$

8. Solve $2 \tan^2 x + \sec^2 x = 2$ for $0 \leq x \leq 2\pi$

Sol:

Here, $2 \tan^2 x + \sec^2 x = 2$

which gives $\tan x = \pm \frac{1}{\sqrt{3}}$

If we take $\tan x = \frac{1}{\sqrt{3}}$, then $x = \frac{\pi}{6}$ or $\frac{7\pi}{6}$

Again, if we take $\tan x = \frac{-1}{\sqrt{3}}$, then $x = \frac{5\pi}{6}$ or $\frac{11\pi}{6}$

Therefore, the possible solutions of above equations are

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$ and $\frac{11\pi}{6}$ where $0 \leq x \leq 2\pi$

9. Find the value of $\left(1 + \cos \frac{\pi}{8}\right) \left(1 + \cos \frac{3\pi}{8}\right) \left(1 + \cos \frac{5\pi}{8}\right) \left(1 + \cos \frac{7\pi}{8}\right)$

Sol:

$$\begin{aligned} & \text{Write } \left(1 + \cos \frac{\pi}{8}\right) \left(1 + \cos \frac{3\pi}{8}\right) \left(1 + \cos \frac{5\pi}{8}\right) \left(1 + \cos \frac{7\pi}{8}\right) \\ &= \left(1 + \cos \frac{\pi}{8}\right) \left(1 + \cos \frac{3\pi}{8}\right) \left(1 + \cos \left(\pi - \frac{3\pi}{8}\right)\right) \left(1 + \cos \left(\pi - \frac{\pi}{8}\right)\right) \\ &= \left(1 - \cos^2 \frac{\pi}{8}\right) \left(1 - \cos^2 \frac{3\pi}{8}\right) \end{aligned}$$

$$\begin{aligned}
 &= \sin^2 \frac{\pi}{8} \sin^2 \frac{3\pi}{8} \\
 &= \frac{1}{4} \left(1 - \cos \frac{\pi}{4}\right) \left(1 - \cos \frac{3\pi}{4}\right) \\
 &= \frac{1}{4} \left(1 - \cos \frac{\pi}{4}\right) \left(1 + \cos \frac{\pi}{4}\right) \\
 &= \frac{1}{4} \left(1 - \cos^2 \frac{\pi}{4}\right) = \frac{1}{4} \left(1 - \frac{1}{2}\right) = \frac{1}{8}
 \end{aligned}$$

10. If $x \cos \theta = y \cos \left(\theta + \frac{2\pi}{3}\right) = z \cos \left(\theta + \frac{4\pi}{3}\right)$, then find the value of $xy + yz + zx$.

Sol:

Note that $xy + yz + zx = xyz \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$.

If we put $x \cos \theta = y \cos \left(\theta + \frac{2\pi}{3}\right) = z \cos \left(\theta + \frac{4\pi}{3}\right) = k$ (say).

Then $x = \frac{k}{\cos \theta}$, $y = \frac{k}{\cos \left(\theta + \frac{2\pi}{3}\right)}$ and $z = \frac{k}{\cos \left(\theta + \frac{4\pi}{3}\right)}$

so that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{k} \left[\cos \theta + \cos \left(\theta + \frac{2\pi}{3}\right) + \cos \left(\theta + \frac{4\pi}{3}\right) \right]$

$$\begin{aligned}
 &= \frac{1}{k} \left[\cos \theta + \cos \theta \cos \frac{2\pi}{3} - \sin \theta \sin \frac{2\pi}{3} \right. \\
 &\quad \left. + \cos \theta \cos \frac{4\pi}{3} - \sin \theta \sin \frac{4\pi}{3} \right] \\
 &= \frac{1}{k} \left[\cos \theta + \cos \theta \left(\frac{-1}{2}\right) - \frac{\sqrt{3}}{2} \sin \theta - \frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta \right] \text{ (Why?)} \\
 &= \frac{1}{k} \times 0 = 0
 \end{aligned}$$

Hence, $xy + yz + zx = 0$

11. If α and β are the solutions of the equation $a \tan \theta + b \sec \theta = c$, then show that $\tan (\alpha + \beta) = \frac{2ac}{a^2 - c^2}$.

Sol: Given that $a \tan \theta + b \sec \theta = c$ or $a \sin \theta + b = c \cos \theta$ Using the identities,

$$\sin \theta = \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} \text{ and } \cos \theta = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$$

We have,
$$\frac{a \left(2 \tan \frac{\theta}{2} \right)}{1 + \tan^2 \frac{\theta}{2}} + b = \frac{c \left(1 - \tan^2 \frac{\theta}{2} \right)}{1 + \tan^2 \frac{\theta}{2}}$$

or
$$(b + c) \tan^2 \frac{\theta}{2} + 2a \tan \frac{\theta}{2} + b - c = 0$$

Above equation is quadratic in $\tan \frac{\theta}{2}$ and hence $\tan \frac{\alpha}{2}$ and $\tan \frac{\beta}{2}$ are the roots of this equation (Why?). Therefore, $\tan \frac{\alpha}{2} + \tan \frac{\beta}{2} = \frac{-2a}{b+c}$ and $\tan \frac{\alpha}{2} \tan \frac{\beta}{2} = \frac{b-c}{b+c}$ (Why?)

Using the identity
$$\tan \left(\frac{\alpha}{2} + \frac{\beta}{2} \right) = \frac{\tan \frac{\alpha}{2} + \tan \frac{\beta}{2}}{1 - \tan \frac{\alpha}{2} \tan \frac{\beta}{2}}$$

We have,
$$\tan \left(\frac{\alpha}{2} + \frac{\beta}{2} \right) = \frac{\frac{-2a}{b+c}}{1 - \frac{b-c}{b+c}} = \frac{-2a}{2c} = \frac{-a}{c} \quad \dots (1)$$

Again, using another identity

$$\tan 2 \left(\frac{\alpha + \beta}{2} \right) = \frac{2 \tan \frac{\alpha + \beta}{2}}{1 - \tan^2 \frac{\alpha + \beta}{2}}$$

We have
$$\tan (\alpha + \beta) = \frac{2 \left(\frac{-a}{c} \right)}{1 - \frac{a^2}{c^2}} = \frac{2ac}{a^2 - c^2} \quad [\text{From (1)}]$$

Alternatively, given that $a \tan \theta + b \sec \theta = c$

$$\begin{aligned} \Rightarrow (a \tan \theta - c)^2 &= b^2(1 + \tan^2 \theta) \\ \Rightarrow a^2 \tan^2 \theta - 2ac \tan \theta + c^2 &= b^2 + b^2 \tan^2 \theta \\ \Rightarrow (a^2 - b^2) \tan^2 \theta - 2ac \tan \theta + c^2 - b^2 &= 0 \end{aligned}$$

Since α and β are the roots of the equation (1), so

$$\tan \alpha + \tan \beta = \frac{2ac}{a^2 - b^2} \quad \text{and} \quad \tan \alpha \tan \beta = \frac{c^2 - b^2}{a^2 - b^2}$$

Therefore,
$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\begin{aligned} &= \frac{\frac{2ac}{a^2 - b^2}}{\frac{c^2 - b^2}{a^2 - b^2}} = \frac{2ac}{c^2 - b^2} \end{aligned}$$

12. Show that $2 \sin^2 \beta + 4 \cos(\alpha + \beta) \sin \alpha \sin \beta + \cos 2(\alpha + \beta) = \cos 2\alpha$

Sol:

$$\begin{aligned} \text{LHS} &= 2 \sin^2 \beta + 4 \cos(\alpha + \beta) \sin \alpha \sin \beta + \cos 2(\alpha + \beta) \\ &= 2 \sin^2 \beta + 4(\cos \alpha \cos \beta - \sin \alpha \sin \beta) \sin \alpha \sin \beta \\ &\quad + (\cos 2\alpha \cos 2\beta - \sin 2\alpha \sin 2\beta) \\ &= 2 \sin^2 \beta + 4 \sin \alpha \cos \alpha \sin \beta \cos \beta - 4 \sin^2 \alpha \sin^2 \beta \\ &\quad + \cos 2\alpha \cos 2\beta - \sin 2\alpha \sin 2\beta \\ &= 2 \sin^2 \beta + \sin 2\alpha \sin 2\beta - 4 \sin^2 \alpha \sin^2 \beta + \cos 2\alpha \cos 2\beta - \sin 2\alpha \sin 2\beta \\ &= (1 - \cos 2\beta) - (2 \sin^2 \alpha)(2 \sin^2 \beta) + \cos 2\alpha \cos 2\beta \quad (\text{Why?}) \\ &= (1 - \cos 2\beta) - (1 - \cos 2\alpha)(1 - \cos 2\beta) + \cos 2\alpha \cos 2\beta \\ &= \cos 2\alpha \quad (\text{Why?}) \end{aligned}$$

13. If angle θ is divided into two parts such that the tangent of one part is k times the tangent of other, and ϕ is their difference, then show that

$$\sin \theta = \frac{k+1}{k-1} \sin \phi$$

Sol: Let $\theta = \alpha + \beta$. Then $\tan \alpha = k \tan \beta$

or
$$\frac{\tan \alpha}{\tan \beta} = \frac{k}{1}$$

Applying componendo and dividendo, we have

$$\frac{\tan \alpha + \tan \beta}{\tan \alpha - \tan \beta} = \frac{k+1}{k-1}$$

or
$$\frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\sin \alpha \cos \beta - \cos \alpha \sin \beta} = \frac{k+1}{k-1}$$

i.e.,
$$\frac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)} = \frac{k+1}{k-1} \quad (\text{Why?})$$

Given that $\alpha - \beta = \phi$ and $\alpha + \beta = \theta$. Therefore,

$$\frac{\sin \theta}{\sin \phi} = \frac{k+1}{k-1} \quad \text{or} \quad \sin \theta = \frac{k+1}{k-1} \sin \phi$$

14. If $\tan \theta = \frac{-4}{3}$, then $\sin \theta$ is

- (A) $\frac{-4}{5}$ but not $\frac{4}{5}$
- (B) $\frac{-4}{5}$ or $\frac{4}{5}$
- (C) $\frac{4}{5}$ but not $-\frac{4}{5}$
- (D) None of these

Sol: (B)

Since $\tan \theta = -\frac{4}{3}$ is negative, θ is negative, θ lies either in second quadrant or in fourth quadrant. Thus $\sin \theta = \frac{4}{5}$ if θ lies in the second quadrant or $\sin \theta = -\frac{4}{5}$, if θ lies in the fourth quadrant.

15. If $\sin \theta$ and $\cos \theta$ are the roots of the equation $ax^2 - bx + c = 0$, then a, b and c satisfy the relation.

- (A) $a^2 + b^2 + 2ac = 0$
- (B) $a^2 - b^2 + 2ac = 0$
- (C) $a^2 + c^2 + 2ab = 0$
- (D) $a^2 - b^2 - 2ac = 0$

Sol: (B)

Given that $\sin \theta$ and $\cos \theta$ are the roots of the

equation $ax^2 - bx + c = 0$, so $\sin \theta + \cos \theta = \frac{b}{a}$ and $\sin \theta \cos \theta = \frac{c}{a}$

Using the identity $(\sin \theta + \cos \theta)^2 = \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta$, we have

$$\frac{b^2}{a^2} = 1 + \frac{2c}{a} \quad \text{or} \quad a^2 - b^2 + 2ac = 0$$

16. The greatest value of $\sin x \cos x$ is

- (A) 1
- (B) 2
- (C) $\sqrt{2}$
- (D) $\frac{1}{2}$

Sol: (D)

$$\sin x \cos x = \frac{1}{2} \sin 2x \leq \frac{1}{2}, \text{ since } |\sin 2x| \leq 1.$$

17. The value of $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$ is

- (A) $\frac{-3}{16}$
- (B) $\frac{5}{16}$
- (C) $\frac{3}{16}$
- (D) $\frac{1}{16}$

Sol: (C)

Indeed $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$.

$$\begin{aligned} &= \frac{\sqrt{3}}{2} \sin 20^\circ \sin (60^\circ - 20^\circ) \sin (60^\circ + 20^\circ) \text{ (since } \sin 60^\circ = \frac{\sqrt{3}}{2} \text{)} \\ &= \frac{\sqrt{3}}{2} \sin 20^\circ [\sin^2 60^\circ - \sin^2 20^\circ] \\ &= \frac{\sqrt{3}}{2} \sin 20^\circ \left[\frac{3}{4} - \sin^2 20^\circ \right] \\ &= \frac{\sqrt{3}}{2} \times \frac{1}{4} [3\sin 20^\circ - 4\sin^3 20^\circ] \\ &= \frac{\sqrt{3}}{2} \times \frac{1}{4} (\sin 60^\circ) \\ &= \frac{\sqrt{3}}{2} \times \frac{1}{4} \times \frac{\sqrt{3}}{2} = \frac{3}{16} \end{aligned}$$

18. The value of $\cos \frac{\pi}{5} \cos \frac{2\pi}{5} \cos \frac{4\pi}{5} \cos \frac{8\pi}{5}$ is

- (A) $\frac{1}{16}$
- (B) 0
- (C) $\frac{-1}{8}$
- (D) $\frac{-1}{16}$

Sol: (D)

$$\begin{aligned}
 & \cos \frac{\pi}{5} \cos \frac{2\pi}{5} \cos \frac{4\pi}{5} \cos \frac{8\pi}{5} \\
 &= \frac{1}{2 \sin \frac{\pi}{5}} 2 \sin \frac{\pi}{5} \cos \frac{\pi}{5} \cos \frac{2\pi}{5} \cos \frac{4\pi}{5} \cos \frac{8\pi}{5} \\
 &= \frac{1}{2 \sin \frac{\pi}{5}} \sin \frac{2\pi}{5} \cos \frac{2\pi}{5} \cos \frac{4\pi}{5} \cos \frac{8\pi}{5} \\
 &= \frac{1}{4 \sin \frac{\pi}{5}} \sin \frac{4\pi}{5} \cos \frac{4\pi}{5} \cos \frac{8\pi}{5} \\
 &= \frac{1}{8 \sin \frac{\pi}{5}} \sin \frac{8\pi}{5} \cos \frac{8\pi}{5} \\
 &= \frac{\sin \frac{16\pi}{5}}{16 \sin \frac{\pi}{5}} = \frac{\sin \left(3\pi + \frac{\pi}{5} \right)}{16 \sin \frac{\pi}{5}} \\
 &= \frac{-\sin \frac{\pi}{5}}{16 \sin \frac{\pi}{5}} \\
 &= -\frac{1}{16}
 \end{aligned}$$

19. If $3 \tan (\theta - 15^\circ) = \tan (\theta + 15^\circ)$, $0^\circ < \theta < 90^\circ$, then $\theta =$ _____

Sol:

Given that $3 \tan (\theta - 15^\circ) = \tan (\theta + 15^\circ)$ which can be rewritten as

$$\frac{\tan(\theta+15^\circ)}{\tan(\theta-15^\circ)} = \frac{3}{1}$$

Applying componendo and Dividendo; we get $\frac{\tan(\theta+15^\circ) + \tan(\theta-15^\circ)}{\tan(\theta+15^\circ) - \tan(\theta-15^\circ)} = 2$

$$\Rightarrow \frac{\sin(\theta+15^\circ)\cos(\theta-15^\circ) + \sin(\theta-15^\circ)\cos(\theta+15^\circ)}{\sin(\theta+15^\circ)\cos(\theta-15^\circ) - \sin(\theta-15^\circ)\cos(\theta+15^\circ)} = 2$$

$$\Rightarrow \frac{\sin 2\theta}{\sin 30^\circ} = 2 \quad \text{i.e.,} \quad \sin 2\theta = 1$$

$$\text{giving } \theta = \frac{\pi}{4}$$

20. "The inequality $2^{\sin\theta} + 2^{\cos\theta} \geq 2^{1-\frac{1}{\sqrt{2}}}$ holds for all real values of θ "

Sol:

True. Since $2^{\sin\theta}$ and $2^{\cos\theta}$ are positive real numbers, so A.M. (Arithmetic Mean) of these two numbers is greater or equal to their GM. (Geometric Mean) and hence

$$\begin{aligned} \frac{2^{\sin\theta} + 2^{\cos\theta}}{2} &\geq \sqrt{2^{\sin\theta} \times 2^{\cos\theta}} = \sqrt{2^{\sin\theta + \cos\theta}} \\ &\geq 2^{\frac{\sin\theta + \cos\theta}{2}} = 2^{\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta\right)} \\ &\geq 2^{\frac{1}{\sqrt{2}}\sin\left(\frac{\pi}{4} + \theta\right)} \end{aligned}$$

Since, $-1 \leq \sin\left(\frac{\pi}{4} + \theta\right) \leq 1$, we have

$$\frac{2^{\sin\theta} + 2^{\cos\theta}}{2} \geq 2^{\frac{-1}{\sqrt{2}}} \Rightarrow 2^{\sin\theta} + 2^{\cos\theta} \geq 2^{1-\frac{1}{\sqrt{2}}}$$

21. Match each item given under the column C_1 to its correct answer given under column C_2

C_1	C_2
(a) $\frac{1-\cos x}{\sin x}$	(i) $\cot^2 \frac{x}{2}$
(b) $\frac{1+\cos x}{1-\cos x}$	(ii) $\cot \frac{x}{2}$
(c) $\frac{1+\cos x}{\sin x}$	(iii) $ \cos x + \sin x $
(d) $\sqrt{1+\sin 2x}$	(iv) $\tan \frac{x}{2}$

Sol:

$$(a) \frac{1-\cos x}{\sin x} = \frac{2\sin^2 \frac{x}{2}}{2\sin \frac{x}{2} \cos \frac{x}{2}} = \tan \frac{x}{2}.$$

Hence (a) matches with (iv) denoted by (a) \leftrightarrow (iv)

$$(b) \frac{1+\cos x}{1-\cos x} = \frac{2\cos^2 \frac{x}{2}}{2\sin^2 \frac{x}{2}} = \cot^2 \frac{x}{2}. \text{ Hence (b) matches with (i) i.e., (b) } \leftrightarrow \text{ (i)}$$

$$(c) \frac{1+\cos x}{\sin x} = \frac{2\cos^2 \frac{x}{2}}{2\sin \frac{x}{2} \cos \frac{x}{2}} = \cot \frac{x}{2}.$$

Hence (c) matches with (ii) i.e., (c) \leftrightarrow (ii)

$$(d) \sqrt{1+\sin 2x} = \sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x} \\ = \sqrt{(\sin x + \cos x)^2} \\ = |(\sin x + \cos x)|. \text{ Hence (d) matches with (iii), i.e., (d) } \leftrightarrow \text{ (iii)}$$