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Class: 11
Subject: Math's
Topic: Trigo phase
No. of Questions: 21

1. A circular wire of radius 3 cm is cut and bent so as to lie along the circumference of a hoop whose radius is 48 cm. Find the angle in degrees which is subtended at the center of hoop.

Sol: Given that circular wire is of radius 3 cm, so when it is cut then its length = $2\pi \times 3 = 6\pi$ cm. Again, it is being placed along a circular hoop of radius 48 cm. Here, s = 6 π cm is the length of arc and r =48 cm is the radius of the circle. Therefore, the angle θ, in radian , subtended by the arc at the center of the circle is given by

$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{6\pi}{48} = \frac{\pi}{8} = 22.5°.$$

2. If A $= \cos^2\theta + \sin^4\theta$ for all values of θ, then prove that $\frac{3}{4} \le A \le 1$.

Sol:

We have $A = \cos^2\theta + \sin^4\theta = \cos^2\theta + \sin^2\theta \sin^2\theta \le \cos^2\theta + \sin^2\theta$

Therefore, $A \le 1$

Also, $A = \cos^2\theta + \sin^4\theta = (1 - \sin^2\theta) + \sin^4\theta$

$$= \left(\sin^2\theta - \frac{1}{2}\right)^2 + \left(1 - \frac{1}{4}\right) = \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4} \ge \frac{3}{4}$$

Hence, $\frac{3}{4} \le A \le 1$.

3. Find the value of $\sqrt{3}$ cosec 20°- sec 20°

Sol:

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We have

$$\sqrt{3}\ \text{cosec}\ 20° - \sec 20° = \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°}$$

$$= \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20° \cos 20°} = 4\left(\frac{\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°}{2\sin 20° \cos 20°}\right)$$

$$= 4\left(\frac{\sin 60° \cos 20° - \cos 60° \sin 20°}{\sin 40°}\right)$$

(Why?)

$$= 4\left(\frac{\sin(60° - 20°)}{\sin 40°}\right) = 4$$ (Why?)

4. If θ lies in the second quadrant, then show that

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = -2\sec\theta$$

Sol:

We have

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \frac{1-\sin\theta}{\sqrt{1-\sin^2\theta}} + \frac{1+\sin\theta}{\sqrt{1-\sin^2\theta}} = \frac{2}{\sqrt{\cos^2\theta}}$$

$$= \frac{2}{|\cos\theta|}$$ (Since $\sqrt{\alpha^2} = |\alpha|$ for every real number α)

Given that θ lies in the second quadrant so $|\cos\theta| = -\cos\theta$ (since $\cos\theta < 0$).

5. Find the value of tan 9° – tan 27° – tan 63° + tan 81°

Sol:

We have tan 9° – tan 27° – tan 63° + tan 81°

$$= \tan 9° + \tan 81° - \tan 27° - \tan 63°$$
$$= \tan 9° + \tan(90° - 9°) - \tan 27° - \tan(90° - 27°)$$
$$= \tan 9° + \cot 9° - (\tan 27° + \cot 27°) \quad (1)$$

Also $$\tan 9° + \cot 9° = \frac{1}{\sin 9° \cos 9°} = \frac{2}{\sin 18°}$$ (Why?) (2)

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Similarly, $\tan 27° + \cot 27° = \dfrac{1}{\sin 27° \cos 27°} = \dfrac{2}{\sin 54°} = \dfrac{2}{\cos 36°}$ (Why?) (3)

Using (2) and (3) in (1), we get

$$\tan 9° - \tan 27° - \tan 63° + \tan 81° = \frac{2}{\sin 18°} - \frac{2}{\cos 36°} = \frac{2 \times 4}{\sqrt{5}-1} - \frac{2 \times 4}{\sqrt{5}+1} = 4$$

6. Prove that $\dfrac{\sec 8\theta - 1}{\sec 4\theta - 1} = \dfrac{\tan 8\theta}{\tan 2\theta}$

Sol:

We have
$$\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{(1 - \cos 8\theta)\cos 4\theta}{\cos 8\theta\,(1 - \cos 4\theta)}$$

$$= \frac{2\sin^2 4\theta \cos 4\theta}{\cos 8\theta\; 2\sin^2 2\theta}$$

$$\frac{\ldots\theta \cos 4\theta)}{2 \cos 8\theta \sin^2 2\theta}$$

$$= \frac{\sin 4\theta \sin 8\theta}{2\cos 8\theta \sin^2 2\theta}$$

$$= \frac{2\sin 2\theta \cos 2\theta \sin 8\theta}{2\cos 8\theta \sin^2 2\theta}$$

$$= \frac{\tan 8\theta}{\tan 2\theta}$$

7. Solve the equation sin θ + sin 3θ + sin 5θ = 0

Sol:

We have $\sin\theta + \sin 3\theta + \sin 5\theta = 0$

or $(\sin\theta + \sin 5\theta) + \sin 3\theta = 0$

or $2\sin 3\theta \cos 2\theta + \sin 3\theta = 0$

or $\sin 3\theta\,(2\cos 2\theta + 1) = 0$

or $\sin 3\theta = 0$ or $\cos 2\theta = -\dfrac{1}{2}$

When $\sin 3\theta = 0$, then $3\theta = n\pi$ or $\theta = \dfrac{n\pi}{3}$

When $\cos 2\theta = -\frac{1}{2} = \cos\frac{2\pi}{3}$, then $2\theta = 2n\pi \pm \frac{2\pi}{3}$ or $\theta = n\pi \pm \frac{\pi}{3}$

which gives $\theta = (3n+1)\frac{\pi}{3}$ or $\theta = (3n-1)\frac{\pi}{3}$

All these values of θ are contained in $\theta = \frac{n\pi}{3}, n \in \mathbf{Z}$. Hence, the required solution set

is given by $\{\theta : \theta = \frac{n\pi}{3}, n \in \mathbf{Z}\}$

8. Solve $2\tan^2 x + \sec^2 x = 2$ for $0 \le x \le 2\pi$

Sol:

Here, $2\tan^2 x + \sec^2 x = 2$

which gives $\tan x = \pm\frac{1}{\sqrt{3}}$

If we take $\tan x = \frac{1}{\sqrt{3}}$, then $x = \frac{\pi}{6}$ or $\frac{7\pi}{6}$

Again, if we take $\tan x = \frac{-1}{\sqrt{3}}$, then $x = \frac{5\pi}{6}$ or $\frac{11\pi}{6}$

Therefore, the possible solutions of above equations are

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$ and $\frac{11\pi}{6}$ where $0 \le x \le 2\pi$

9. Find the value of $\left(1+cos\frac{\pi}{8}\right)\left(1+cos\frac{3\pi}{8}\right)\left(1+cos\frac{5\pi}{8}\right)\left(1+cos\frac{7\pi}{8}\right)$

Sol:

Write $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$

$=\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\left(\pi-\frac{3\pi}{8}\right)\right)\left(1+\cos\left(\pi-\frac{\pi}{8}\right)\right)$

$=\left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$

$$= \sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}$$

$$= \frac{1}{4}\left(1-\cos\frac{\pi}{4}\right)\left(1-\cos\frac{3\pi}{4}\right)$$

$$= \frac{1}{4}\left(1-\cos\frac{\pi}{4}\right)\left(1+\cos\frac{\pi}{4}\right)$$

$$= \frac{1}{4}\left(1-\cos^2\frac{\pi}{4}\right) = \frac{1}{4}\left(1-\frac{1}{2}\right) = \frac{1}{8}$$

10. If $x\cos\theta$ = y $\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right)$, then find the value of $xy+yz+zx$.

Sol:

Note that $xy + yz + zx = xyz \left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$.

If we put $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right) = k$ (say).

Then $x = \frac{k}{\cos\theta}, y = \frac{k}{\cos\left(\theta+\frac{2\pi}{3}\right)}$ and $z = \frac{k}{\cos\left(\theta+\frac{4\pi}{3}\right)}$

so that $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = \frac{1}{k}\left[\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)+\cos\left(\theta+\frac{4\pi}{3}\right)\right]$

$$= \frac{1}{k}[\cos\theta+\cos\theta\cos\frac{2\pi}{3}-\sin\theta\sin\frac{2\pi}{3}$$

$$+\cos\theta\cos\frac{4\pi}{3}-\sin\theta\sin\frac{4\pi}{3}]$$

$$= \frac{1}{k}[\cos\theta+\cos\theta\left(\frac{-1}{2}\right)-\frac{\sqrt{3}}{2}\sin\theta-\frac{1}{2}\cos\theta+\frac{\sqrt{3}}{2}\sin\theta] \text{ (Why?)}$$

$$= \frac{1}{k}\times 0 = 0$$

Hence, $xy + yz + zx = 0$

11. If α and β are the solutions of the equation a tan θ + b sec θ = c, then show that tan (α + β) = $\frac{2ac}{a^2-c^2}$.

Sol: Given that atanθ + bsecθ = c or asinθ + b = c cos θ Using the identities,

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$$\sin\theta = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} \text{ and } \cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

We have, $$\frac{a\left(2\tan\frac{\theta}{2}\right)}{1+\tan^2\frac{\theta}{2}} + b = \frac{c\left(1-\tan^2\frac{\theta}{2}\right)}{1+\tan^2\frac{\theta}{2}}$$

or $$(b+c)\tan^2\frac{\theta}{2} + 2a\tan\frac{\theta}{2} + b - c = 0$$

Above equation is quadratic in $\tan\frac{\theta}{2}$ and hence $\tan\frac{\alpha}{2}$ and $\tan\frac{\beta}{2}$ are the roots of this equation (Why?). Therefore, $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} = \frac{-2a}{b+c}$ and $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{b-c}{b+c}$ (Why?)

Using the identity $$\tan\left(\frac{\alpha}{2}+\frac{\beta}{2}\right) = \frac{\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}}{1-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}}$$

We have, $$\tan\left(\frac{\alpha}{2}+\frac{\beta}{2}\right) = \frac{\frac{-2a}{b+c}}{1-\frac{b-c}{b+c}} = \frac{-2a}{2c} = \frac{-a}{c} \qquad \text{... (1)}$$

Again, using another identity

$$\tan 2\left(\frac{\alpha+\beta}{2}\right) = \frac{2\tan\frac{\alpha+\beta}{2}}{1-\tan^2\frac{\alpha+\beta}{2}},$$

We have $$\tan(\alpha+\beta) = \frac{2\left(-\frac{a}{c}\right)}{1-\frac{a^2}{c^2}} = \frac{2ac}{a^2-c^2} \quad \text{[From (1)]}$$

Alternatively, given that $a\tan\theta + b\sec\theta = c$

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$\Rightarrow \quad (a \tan\theta - c)^2 = b^2(1 + \tan^2\theta)$

$\Rightarrow \quad a^2 \tan^2\theta - 2ac \tan\theta + c^2 = b^2 + b^2 \tan^2\theta$

$\Rightarrow \quad (a^2 - b^2) \tan^2\theta - 2ac \tan\theta + c^2 - b^2 = 0$

Since α and β are the roots of the equation (1), so

$$\tan\alpha + \tan\beta = \frac{2ac}{a^2 - b^2} \text{ and } \tan\alpha \tan\beta = \frac{c^2 - b^2}{a^2 - b^2}$$

Therefore, $$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$$

$$= \frac{\frac{2ac}{a^2 - b^2}}{\frac{c^2 - b^2}{a^2 - b^2}} = \frac{2ac}{a^2 - c^2}$$

12. Show that $2 \sin^2 \beta + 4 \cos(\alpha + \beta) \sin \alpha \sin \beta + \cos 2(\alpha + \beta) = \cos 2\alpha$

Sol:

$$\text{LHS} = 2 \sin^2 \beta + 4 \cos(\alpha + \beta) \sin \alpha \sin \beta + \cos 2(\alpha + \beta)$$

$$= 2 \sin^2 \beta + 4 (\cos \alpha \cos \beta - \sin \alpha \sin \beta) \sin \alpha \sin \beta + (\cos 2\alpha \cos 2\beta - \sin 2\alpha \sin 2\beta)$$

$$= 2 \sin^2 \beta + 4 \sin \alpha \cos \alpha \sin \beta \cos \beta - 4 \sin^2 \alpha \sin^2 \beta + \cos 2\alpha \cos 2\beta - \sin 2\alpha \sin 2\beta$$

$$= 2 \sin^2 \beta + \sin 2\alpha \sin 2\beta - 4 \sin^2 \alpha \sin^2 \beta + \cos 2\alpha \cos 2\beta - \sin 2\alpha \sin 2\beta$$

$$= (1 - \cos 2\beta) - (2 \sin^2 \alpha)(2 \sin^2 \beta) + \cos 2\alpha \cos 2\beta \quad \text{(Why?)}$$

$$= (1 - \cos 2\beta) - (1 - \cos 2\alpha)(1 - \cos 2\beta) + \cos 2\alpha \cos 2\beta$$

$$= \cos 2\alpha \quad \text{(Why?)}$$

13. If angle θ is divided into two parts such that the tangent of one part is k times the tangent of other, and ϕ is their difference, then show that

$\sin\theta = \frac{k+1}{k-1} \sin\phi$

Sol: Let $\theta = \alpha + \beta$. Then $\tan \alpha = k \tan \beta$

or $$\frac{\tan\alpha}{\tan\beta} = \frac{k}{1}$$

Applying componendo and dividendo, we have

$$\frac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta} = \frac{k+1}{k-1}$$

or $$\frac{\sin\alpha \cos\beta + \cos\alpha \sin\beta}{\sin\alpha \cos\beta - \cos\alpha \sin\beta} = \frac{k+1}{k-1}$$

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i.e., $$\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{k+1}{k-1} \quad \text{(Why?)}$$

Given that $\alpha - \beta = \phi$ and $\alpha + \beta = \theta$. Therefore,

$$\frac{\sin\theta}{\sin\phi} = \frac{k+1}{k-1} \quad \text{or} \quad \sin\theta = \frac{k+1}{k-1}\sin\phi$$

14. If $\tan\theta = \frac{-4}{3}$, *then* $\sin\theta$ *is*
(A) $\frac{-4}{5}$ but not $\frac{4}{5}$
(B) $\frac{-4}{5}$ or $\frac{4}{5}$
(C) $\frac{4}{5}$ *but not* $-\frac{4}{5}$
(D) None of these

Sol: (B)

Since $\tan\theta = -\frac{4}{3}$ is negative, θ is negative, θ lies either in second quadrant or in fourth quadrant. Thus $\sin\theta = \frac{4}{5}$ if θ lies in the second quadrant or $\sin\theta = -\frac{4}{5}$, if θ lies in the fourth quadrant.

15. If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 - bx + c = 0$, then a, b and c satisfy the relation.
(A) $a^2 + b^2 + 2ac = 0$
(B) $a^2 - b^2 + 2ac = 0$
(C) $a^2 + c^2 + 2ab = 0$
(D) $a^2 - b^2 - 2ac = 0$

Sol: (B)

Given that $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 - bx + c = 0$, so $\sin\theta + \cos\theta = \frac{b}{a}$ and $\sin\theta\cos\theta = \frac{c}{a}$

Using the identity $(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$, we have

$$\frac{b^2}{a^2} = 1 + \frac{2c}{a} \quad \text{or } a^2 - b^2 + 2ac = 0$$

16. The greatest value of sin x cos x is
(A) 1
(B) 2
(C) $\sqrt{2}$
(D) $\frac{1}{2}$

Sol: (D)

$$\sin x\ \cos x = \frac{1}{2}\sin 2x \le \frac{1}{2}, \text{ since } |\sin 2x| \le 1.$$

17. The value of sin 20° sin 40° sin 60° sin 80° is

(A) $\frac{-3}{16}$

(B) $\frac{5}{16}$

(C) $\frac{3}{16}$

(D) $\frac{1}{16}$

Sol: (C)

Indeed sin 20° sin 40° sin 60° sin 80°.

$$= \frac{\sqrt{3}}{2}\sin 20^\circ \sin(60^\circ - 20^\circ)\sin(60^\circ + 20^\circ) \text{ (since } \sin 60^\circ = \frac{\sqrt{3}}{2})$$

$$= \frac{\sqrt{3}}{2}\sin 20^\circ [\sin^2 60^\circ - \sin^2 20^\circ]$$

$$= \frac{\sqrt{3}}{2}\sin 20^\circ [\frac{3}{4} - \sin^2 20^\circ]$$

$$= \frac{\sqrt{3}}{2}\times\frac{1}{4}[3\sin 20^\circ - 4\sin^3 20^\circ]$$

$$= \frac{\sqrt{3}}{2}\times\frac{1}{4}(\sin 60^\circ)$$

$$= \frac{\sqrt{3}}{2}\times\frac{1}{4}\times\frac{\sqrt{3}}{2} = \frac{3}{16}$$

18. The value of $cos\ \frac{\pi}{5}\cos\frac{2\pi}{5}\ cos\frac{4\pi}{5} cos\frac{8\pi}{5}\ is$

(A) $\frac{1}{16}$

(B) 0

(C) $\frac{-1}{8}$

(D) $\frac{-1}{16}$

Sol: (D)

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$$\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5}$$

$$=\frac{1}{2\sin\frac{\pi}{5}}2\sin\frac{\pi}{5}\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5}$$

$$=\frac{1}{2\sin\frac{\pi}{5}}\sin\frac{2\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5}$$

$$=\frac{1}{4\sin\frac{\pi}{5}}\sin\frac{4\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5}$$

$$=\frac{1}{8\sin\frac{\pi}{5}}\sin\frac{8\pi}{5}\cos\frac{8\pi}{5}$$

$$=\frac{\sin\frac{16\pi}{5}}{16\sin\frac{\pi}{5}}=\frac{\sin\left(3\pi+\frac{\pi}{5}\right)}{16\sin\frac{\pi}{5}}$$

$$=\frac{-\sin\frac{\pi}{5}}{16\sin\frac{\pi}{5}}$$

$$=-\frac{1}{16}$$

19. If 3 tan (θ – 15°) = tan (θ + 15°), 0° < θ < 90°, then θ = ________

Sol:

Given that 3 tan (θ – 15°) = tan (θ + 15°) which can be rewritten as

$$\frac{\tan(\theta+15^\circ)}{\tan(\theta-15^\circ)}=\frac{3}{1}.$$

Applying componendo and Dividendo; we get $\frac{\tan(\theta+15^\circ)+\tan(\theta-15^\circ)}{\tan(\theta+15^\circ)-\tan(\theta-15^\circ)}=2$

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$$\Rightarrow \frac{\sin(\theta+15^\circ)\cos(\theta-15^\circ)+\sin(\theta-15^\circ)\cos(\theta+15^\circ)}{\sin(\theta+15^\circ)\cos(\theta-15^\circ)-\sin(\theta-15^\circ)\cos(\theta+15^\circ)} = 2$$

$$\Rightarrow \frac{\sin 2\theta}{\sin 30^\circ} = 2 \quad \text{i.e.,} \quad \sin 2\theta = 1$$

giving $\theta = \frac{\pi}{4}$

20. "The inequality $2^{sin\theta} + 2^{cos\theta} \geq 2^{1-\frac{1}{\sqrt{2}}}$ holds for all real values of θ"

Sol:

True. Since $2^{\sin\theta}$ and $2^{\cos\theta}$ are positive real numbers, so A.M. (Arithmetic Mean) of these two numbers is greater or equal to their G.M. (Geometric Mean) and hence

$$\frac{2^{\sin\theta}+2^{\cos\theta}}{2} \geq \sqrt{2^{\sin\theta}\times 2^{\cos\theta}} = \sqrt{2^{\sin\theta+\cos\theta}}$$

$$\geq 2^{\frac{\sin\theta+\cos\theta}{2}} = 2^{\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\cos\theta\right)}$$

$$\geq 2^{\frac{1}{\sqrt{2}}\sin\left(\frac{\pi}{4}+\theta\right)}$$

Since, $-1 \leq \sin\left(\frac{\pi}{4}+\theta\right) \leq 1$, we have

$$\frac{2^{\sin\theta}+2^{\cos\theta}}{2} \geq 2^{\frac{-1}{\sqrt{2}}} \Rightarrow 2^{\sin\theta}+2^{\cos\theta} \geq 2^{1-\frac{1}{\sqrt{2}}}$$

21. Match each item given under the column C_1 to its correct answer given under column C_2

	C_1		C_2
(a)	$\frac{1-\cos x}{\sin x}$	(i)	$\cot^2\frac{x}{2}$
(b)	$\frac{1+\cos x}{1-\cos x}$	(ii)	$\cot\frac{x}{2}$
(c)	$\frac{1+\cos x}{\sin x}$	(iii)	$\lvert\cos x+\sin x\rvert$
(d)	$\sqrt{1+\sin 2x}$	(iv)	$\tan\frac{x}{2}$

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(a) $\frac{1-\cos x}{\sin x} = \frac{2\sin^2 \frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \tan\frac{x}{2}.$

Hence (a) matches with (iv) denoted by (a) $\leftrightarrow$ (iv)

(b) $\frac{1+cosx}{1-cosx} = \frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}} = \cot^2\frac{x}{2.}$. Hence (b) matches with (i) i.e., (b) $\leftrightarrow$ (i)

(c) $\frac{1+\cos x}{\sin x} = \frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \cot\frac{x}{2}.$

Hence (c) matches with (ii) i.e., (c) $\leftrightarrow$ (ii)

(d) $\sqrt{1+\sin 2x} = \sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x}$

$= \sqrt{(\sin x+\cos x)^2}$

$= \left|(\sin x+\cos x)\right|$. Hence (d) matches with (iii), i.e., (d) $\leftrightarrow$ (iii)