

Class: 11
Subject: Math
Topic: Trigo
No. of Questions: 25

1. If for real values of x , $\cos x = x + 1/x$, then
- (A) $x > 90$
(B) $x < 90$
(C) $x = 90$
(D) no value of x is possible

Sol. D

Let $f(x) = \cos x$
 $f(x) \in [-1, 1]$
For all $x \in \mathbb{R}$,

Let $g(x) = x + \frac{1}{x}$

$g(x) \in (-\infty, -2] \cup [2, \infty)$

\therefore No value of x is possible where $f(x) = g(x)$

2. What is the value of $\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ$?
- (A) 1
(B) 2
(C) 3
(D) 4

Sol. D

$$\begin{aligned}\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ &= \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ} = \frac{\sqrt{30} \cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cos 20^\circ} = 4 \\ &= 4 \left(\frac{\frac{\sqrt{3}}{2} \cos 20^\circ - \frac{1}{2} \sin 20^\circ}{2 \sin 20^\circ \cos 20^\circ} \right) \\ &= 4 \left(\frac{\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ}{\sin 40^\circ} \right) \\ &= \left(\frac{\sin(60^\circ - 20^\circ)}{\sin 40^\circ} \right) = 4\end{aligned}$$

3. What is the value of $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$?

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Sol. D

$$\begin{aligned} \text{We have, } \tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ &= \tan 9^\circ + \tan 81^\circ - \tan 27^\circ - \tan 63^\circ \\ &= \tan 9^\circ + \tan (90^\circ - 9^\circ) - \tan 27^\circ - \tan (90^\circ - 27^\circ) \\ &= \tan 9^\circ + \cot 9^\circ - (\tan 27^\circ + \cot 27^\circ) \quad \dots\dots\dots(1) \end{aligned}$$

$$\text{Also, } \tan 9^\circ + \cot 9^\circ = \frac{1}{\sin 9^\circ \cos 9^\circ} = \frac{2}{\sin 18^\circ} \quad (\text{why?}) \quad \dots\dots\dots(2)$$

$$\text{Similarly, } \tan 27^\circ + \cot 27^\circ = \frac{1}{\sin 27^\circ \cos 27^\circ} = \frac{2}{\sin 54^\circ} = \frac{2}{\cos 36^\circ} \quad (\text{why?}) \quad \dots\dots\dots(3)$$

Using (2) and (3) in (1), we get

$$\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ = \frac{2}{\sin 18^\circ} - \frac{2}{\cos 36^\circ} = \frac{2 \times 4}{\sqrt{5} - 1} - \frac{2 \times 4}{\sqrt{5} + 1} = 4$$

4. What is the value of $\frac{\sec 8\theta - 1}{\sec 4\theta - 1}$?

- (A) $\frac{\tan 8\theta}{\tan 4\theta}$
- (B) $\frac{\tan 8\theta}{\tan 2\theta}$
- (C) $\frac{\tan (8\theta)}{\tan (\theta)}$
- (D) none of these

Sol. C

$$\begin{aligned} \frac{\sec 8\theta - 1}{\sec 4\theta - 1} &= \frac{(1 - \cos 8\theta) \cos 4\theta}{\cos 8\theta (1 - \cos 4\theta)} &&= \frac{2 \sin^2 4\theta \cos 4\theta}{\cos 8\theta 2 \sin^2 2\theta} \\ &= \frac{\sin 4\theta (2 \sin 4\theta \cos 4\theta)}{2 \cos 8\theta \sin^2 2\theta} \\ &= \frac{\sin 4\theta \sin 8\theta}{2 \cos 8\theta \sin^2 2\theta} \end{aligned}$$

$$\begin{aligned} & \frac{2 \sin 2\theta \cos 2\theta \sin 8\theta}{2 \cos 8\theta \sin^2 2\theta} \\ = & \frac{\tan 8\theta}{\tan 2\theta} \end{aligned}$$

$$\geq 2^{1 - \frac{1}{\sqrt{2}}}$$

5. The value of $x \in \mathbb{R}$, for which $2^{\sin x} + 2^{\cos x}$ does not hold true, is

- (A) $\frac{\pi}{4}$
(B) $\frac{3\pi}{4}$
(C) $\frac{\pi}{2}$
(D) None of these

Sol. B

Use options to eliminate the wrong answers.

6. $\tan x$ is periodic with period

- (A) $\frac{\pi}{2}$
(B) π
(C) 2π
(D) $\frac{3\pi}{2}$

Sol. B

As $\tan(\pi + x) = \tan x$ for all x , therefore, $\tan x$ is periodic with period π as π is the smallest positive number to satisfy the property.

7. The period of $\tan(x + 3x + 5x + 7x)$ is

- (A) π
- (B) $\frac{\pi}{12}$
- (C) $\frac{\pi}{16}$
- (D) $\frac{\pi}{2}$

Sol. C

The equation is $\tan 16x$.

Let $f(x) = \tan 16x$ and $f(x + T) = \tan (16x + 16T)$

$f(x + T) = f(x)$ if function is periodic. Hence, $16x + 16T = n\pi + 16x$ and $T = \frac{\pi}{16}$ (if $n = 1$)

So, its period is $\frac{\pi}{16}$.

8. Which of the following is correct?

- (A) $\tan 1 = \tan 2$
- (B) $\tan 1 = \frac{2}{3} \tan 2$
- (C) $\tan 1 > \tan 2$
- (D) $\tan 1 < \sin 2$

Sol. C

As $\tan \theta$ increases from 0 to ∞ , θ varies from 0 to $\frac{\pi}{2}$ and $1c = \frac{180}{\pi}$ degrees

9. The magnitude of a radian is

- (A) 180°
- (B) $58^\circ 59'$
- (C) $57^\circ 17' 44.8''$
- (D) 60°

Sol. C

$$1 \text{ radian} = \frac{180}{\pi} \text{ degree} = \frac{180}{3.14} \\ = 57^\circ 17' 44.8''$$

10. For the trigonometric functions, which of the following statements is true?

- (A) $|\cos x|$ is not a periodic function
- (B) period of $|\cos x|$ is the same as that of $\cos x$
- (C) $|\cos x|$ and $\tan x$ both have period π
- (D) $\tan x$ has period π , but $|\cos x|$ does not have period π

Sol. C

$|\cos x|$, $\tan x$ both have period π .

11. The greatest value of $\sin x \cos x$ is

- (A) 1
- (B) 2
- (C) $1/2$
- (D) 1

Sol. C

$\sin 2x = 2 \sin x \cos x$
 \therefore Option (3) is correct.

12. If $\sin 2\theta = \cos 4\theta$, then θ is equal to

- (A) 12°
- (B) 15°
- (C) 18°
- (D) 21°

Sol. B

According to the question, $\sin 2\theta = \cos 4\theta$ It can be solved from the options. Take $\theta = 15^\circ$
, $\sin 30^\circ = \cos 60^\circ$ which are exactly equal.

13. If θ is an acute angle and $\sin \theta = \frac{p-6}{8-p}$, then p must satisfy

- (A) $6 \leq p < 8$
- (B) $6 \leq p < 7$
- (C) $3 \leq p \leq 4$
- (D) $4 \leq p < 7$

Sol. B

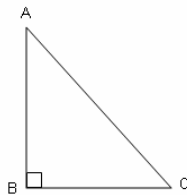
θ is an acute angle, $\theta < 90^\circ$
 \therefore Only 2nd value satisfies the value of p.

14. In a right angled triangle ABC, if $\angle B = 90^\circ$ and $AB : AC = \sqrt{3} : 2$, find the measure of $\angle A$.

- (A) 75°
- (B) 60°
- (C) 45°
- (D) 30°

Sol. D

In $\triangle ABC$,



$$\frac{AB}{AC} = \frac{\sqrt{3}}{2} \dots(i)$$

$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{AC} \quad [\because \text{For angle A, opposite side BC is perpendicular, so AB is the base}]$$

$$\Rightarrow \cos A = \frac{\sqrt{3}}{2} \text{ by equation (i)}$$

$$\text{Also, } \cos 30^\circ = \frac{\sqrt{3}}{2}, \text{ thus } \angle A = 30^\circ$$

15. If $u = \sin^6 x + \cos^6 x$, then maximum and minimum values of u are

- (A) $\pm 1/4$
- (B) ± 1
- (C) 1 and $1/4$
- (D) $1/4$ and -1

Sol. C

As $\sin x$ and $\cos x$ have even powers. \therefore The value can't be negative.
Hence, (3) option is correct.

16. If $A = \frac{\pi}{8}$, then $(\cos 3A \cos 2A + \sin 2A \sin 3A) \sin A$ is equal to

- (A) $\frac{1}{\sqrt{2}}$
- (B) $\sqrt{2}$
- (C) $\frac{1}{2\sqrt{2}}$
- (D) None of these

Sol. C

As $A = \frac{\pi}{8}$ then $\cos 3A \cos 2A + \sin 2A \sin 3A$ can be written as

$\cos(3A - 2A) = \cos A$ So, equation becomes $\cos A \sin A$. So, $\cos A \sin A = \frac{1}{2} \sin 2A$ where

$$A = \frac{\pi}{8}$$

$$\text{So, it becomes } \frac{1}{2} \sin \frac{\pi}{4} = \frac{1}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

17. If $\cot^2 \theta = \cot(\theta - \alpha) \cot(\theta - \beta)$, then $\cot 2\theta$ is equal to

- (A) $\cot \alpha + \cot \beta$
- (B) $1/2 (\cot \alpha + \cot \beta)$
- (C) $1/2 (\tan \alpha + \tan \beta)$
- (D) None of these

Sol. B

It can be solved using the options. Let $\theta = 30^\circ$, $\alpha = 60^\circ$, $\beta = 60^\circ$

Hence, $\cot 2\theta = \cot 60^\circ = 1/\sqrt{3}$ Now go with the options.

18. The value of $\sin 36^\circ \cdot \sin 72^\circ \cdot \sin 108^\circ \cdot \sin 144^\circ$ is

- (A) 1/16
- (B) 3/16
- (C) 5/16
- (D) None of these

Sol. C

$$\sin 36^\circ = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$$

$$\cos 36^\circ = \frac{\sqrt{5} + 1}{4}$$

$$\sin 18^\circ = \frac{\sqrt{5} - 1}{4}$$

19. $\tan \frac{2\pi}{5} - \tan \frac{\pi}{15} - \sqrt{3} \tan \frac{2\pi}{5} \tan \frac{\pi}{15}$ is equal to

- (A) $-\sqrt{3}$
- (B) $1 - \sqrt{3}$
- (C) 1
- (D) $\sqrt{3}$

Sol. D

$$\frac{\tan \frac{6\pi}{15} - \tan \frac{\pi}{15}}{1 + \tan \frac{6\pi}{15} \tan \frac{\pi}{15}} = \tan \frac{\pi}{3}$$

We have

$$\Rightarrow \tan \frac{6\pi}{15} - \tan \frac{\pi}{15} = \sqrt{3} + \sqrt{3} \tan \frac{6\pi}{15} \tan \frac{\pi}{15}$$

$$\Rightarrow \tan \frac{6\pi}{15} - \tan \frac{\pi}{15} - \sqrt{3} \tan \frac{6\pi}{15} \tan \frac{\pi}{15} = \sqrt{3}$$

20. If $\tan^{\theta_1} = k \tan^{\theta_2}$, then $\frac{\cos(\theta_1 - \theta_2)}{\cos(\theta_1 + \theta_2)}$ equals

- (A) $\frac{1+k}{1-k}$
(B) $\frac{1-k}{1+k}$
(C) $\frac{k+1}{k-1}$
(D) $\frac{k-1}{k+1}$

Sol. A

Use formula $\cos(A - B)$ and $\cos(A + B)$ and $\frac{\sin \theta_1}{\cos \theta_1} = k \frac{\sin \theta_2}{\cos \theta_2}$.

21. If $\cot x = -\frac{5}{12}$, x lies in second quadrant find the values of other five trigonometric functions.

Sol:

$$\cot x = -\frac{5}{12}$$

$$\tan x = -\frac{12}{5}$$

$$\sec^2 x = 1 + \tan^2 x$$

$$\sec x = \pm \frac{13}{5}$$

$$\sec x = -\frac{13}{5} \quad [\because x \text{ lies in II}^{\text{nd}} \text{ quad.}]$$

$$\cos x = -\frac{5}{13}$$

$$\sin x = \tan x \cdot \cos x$$

$$= \frac{-12}{5} \times \left(\frac{-5}{13}\right) = \frac{12}{13}$$

$$\operatorname{Cosec} x = \frac{13}{12}$$

22. Prove that $\frac{\sin 5x - 2 \sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$

Sol :

$$\begin{aligned} \text{L. H. S} &= \frac{\sin 5x + \sin x - 2 \sin 3x}{\cos 5x - \cos x} \\ &= \frac{2 \sin 3x \cdot \cos 2x - 2 \sin 3x}{-2 \sin 3x \cdot \sin 2x} \\ &= \frac{\cancel{2 \sin 3x} (\cos 2x - 1)}{-\cancel{2 \sin 3x} \cdot \sin 2x} \\ &= \frac{-(1 - \cos 2x)}{-\sin 2x} \\ &= \frac{\cancel{2 \sin^2 x}}{\cancel{2 \sin x} \cdot \cos x} \\ &= \frac{\sin x}{\cos x} = \tan x \end{aligned}$$

23. Prove that $\sin x + \sin 3x + \sin 5x + \sin 7x = 4 \cos x \cdot \cos 2x \cdot \sin 4x$

Sol:

$$\begin{aligned} \text{L. H. S.} &= \sin x + \sin 3x + \sin 5x + \sin 7x \\ &= \sin x + \sin 7x + \sin 3x + \sin 5x \\ &= 2 \sin \left(\frac{x+7x}{2} \right) \cdot \cos \left(\frac{x-7x}{2} \right) + 2 \sin \left(\frac{3x+5x}{2} \right) \cos \left(\frac{3x-5x}{2} \right) \\ &= 2 \sin 4x \cdot \cos 3x + 2 \sin 4x \cdot \cos x \\ &= 2 \sin 4x [\cos 3x + \cos x] \\ &= 2 \sin 4x \left[2 \cos \left(\frac{3x+x}{2} \right) \cdot \cos \left(\frac{3x-x}{2} \right) \right] \\ &= 2 \sin 4x [2 \cos 2x \cdot \cos x] \\ &= 4 \cos x \cdot \cos 2x \cdot \sin 4x \end{aligned}$$

24. Prove that $\cos^2 x + \cos^2 \left(x + \frac{\pi}{3}\right) + \cos^2 \left(x - \frac{\pi}{3}\right) = \frac{3}{2}$

Sol:

$$\begin{aligned}
 \text{L. H. S} &= \frac{1 + \cos 2x}{2} + \frac{1 + \cos \left(2x + \frac{2\pi}{3}\right)}{2} + \frac{1 + \cos \left(2x - \frac{2\pi}{3}\right)}{2} \\
 &= \frac{1}{2} \left[1 + 1 + \cos 2x + \cos \left(2x + \frac{2\pi}{3}\right) + \cos \left(2x - \frac{2\pi}{3}\right) \right] \\
 &= \frac{1}{2} \left[3 + \cos 2x + \cos \left(2x + \frac{2\pi}{3}\right) + \cos \left(2x - \frac{2\pi}{3}\right) \right] \\
 &= \frac{1}{2} \left[3 + \cos 2x + 2 \cos \left(\frac{2x + \frac{2\pi}{3} + 2x - \frac{2\pi}{3}}{2} \right) \cdot \cos \left(\frac{2x + \frac{2\pi}{3} - 2x + \frac{2\pi}{3}}{3} \right) \right] \\
 &= \frac{1}{2} \left[3 + \cos 2x + 2 \cos 2x \cdot \cos \frac{2\pi}{3} \right] \\
 &= \frac{1}{2} \left[3 + \cos 2x + 2 \cos 2x \cdot \cos \frac{2\pi}{3} \right] \\
 &= \frac{1}{2} \left[3 + \cos 2x + 2 \cos 2x \cdot \cos \left(\pi - \frac{\pi}{3} \right) \right] \\
 &= \frac{1}{2} \left[3 + \cos 2x + 2 \cos 2x \cdot \left(-\frac{1}{2} \right) \right] \\
 &= \frac{3}{2}
 \end{aligned}$$

25. Prove that $\cos 2x \cdot \cos \frac{x}{2} - \cos 3x \cdot \cos \frac{9x}{2} = \sin 5x \sin \frac{5x}{2}$

Sol:

$$\begin{aligned} \text{L. H. S} &= \frac{1}{2} \left[2 \cos 2x \cos \frac{x}{2} - 2 \cos 3x \cdot \cos \frac{9x}{2} \right] \\ &= \frac{1}{2} \left[\cos \left(2x + \frac{x}{2} \right) + \cos \left(2x - \frac{x}{2} \right) - \cos \left(\frac{9x}{2} + 3x \right) - \cos \left(\frac{9x}{2} - 3x \right) \right] \\ &= \frac{1}{2} \left[\cos \frac{5x}{2} + \cancel{\cos \frac{3x}{2}} - \cos \frac{15x}{2} - \cancel{\cos \frac{3x}{2}} \right] \\ &= \frac{1}{2} \left[\cos \frac{5x}{2} - \cos \frac{15x}{2} \right] \\ &= \frac{1}{2} \left[-2 \sin \left(\frac{\frac{5x}{2} + \frac{15x}{2}}{2} \right) \cdot \sin \left(\frac{\frac{5x}{2} - \frac{15x}{2}}{2} \right) \right] \\ &= -\sin 5x \cdot \sin \left(\frac{-5x}{2} \right) \\ &= \sin 5x \cdot \sin \frac{5x}{2} \end{aligned}$$