

Class: 11  
Subject: Physics  
Topic: Oscillations  
No. of Questions:29

1. The mass and diameter of a planet are twice than that of the earth. What will be the period of oscillation of a pendulum on this planet if it is a second pendulum on earth?
- A. 2  
B.  $\sqrt{2}$   
C.  $\frac{1}{\sqrt{2}}$   
D.  $2\sqrt{2}$

Sol. d

$$T = 2\pi\sqrt{\frac{l}{g}}$$
$$T_1 = 2\pi\sqrt{\frac{l}{g_1}}$$
$$\frac{T_1}{T} = \sqrt{\frac{g}{g_1}}$$
$$g = \frac{GM}{R^2}$$
$$g_1 = \frac{G2M}{4R^2}$$
$$\frac{g}{g_1} = 2$$
$$\frac{T_1}{T} = \sqrt{2}$$
$$T_1 = T\sqrt{2} = 2\sqrt{2}$$

2. The length of a simple pendulum is increased by 44%. What is the percentage increase in its period?
- a. 20%
  - b. 30%
  - c. 40%
  - d. 50%

Sol. a

For a simple pendulum  $l \propto T^2$ . Where C is constant. Let  $l = 100$  cm then,  $100 = CT^2$ . When the length is increased by 44%. Let the time be increased by  $t$ , then,  $144 =$

$$C(T+t)^2. \text{ Dividing both the equations, } \frac{144}{100} = \frac{(T+t)^2}{T^2}.$$

Percentage increase

$$\frac{t}{T} * 100 = \frac{1}{5} * 100 = 20\%$$

3. A body executes simple harmonic motion. The potential energy (PE), kinetic energy (KE) and total energy (TE) are measured as function of displacement  $x$ . Which of the following statements is true?
- a. KE is maximum when  $x = 0$ .
  - b. TE is zero when  $x = 0$ .
  - c. KE is maximum when  $x$  is maximum.
  - d. PE is maximum when  $x = 0$ .

Sol. a

The KE is maximum at mean position, i.e at  $x = 0$ .

4. A particle of mass 0.1 kg is held between two springs of spring constant 8N/m and 2N/m. If this particle is displaced along the length of spring, what will be the time period of vibration of spring?

- a.  $\frac{\pi}{2\sqrt{5}}$  s  
 b.  $\frac{\pi}{20}$  s  
 c.  $\frac{\pi}{5}$  s  
 d.  $6\pi$  s

Sol. c

$$\begin{aligned}
 F_{\text{tot}} &= F_1 + F_2 \\
 F &= -K_1x - K_2x \\
 F &= -(K_1 + K_2) * x \\
 \text{Let } K_{\text{eff}} &= K_1 + K_2 \\
 T &= 2\pi \sqrt{\frac{m}{k_{\text{eff}}}} \\
 &= 2\pi \sqrt{\frac{0.1}{10}} \\
 &= 2\pi * \frac{1}{10} = \frac{\pi}{5} \text{ s}
 \end{aligned}$$

5. A point particle of mass 0.1 Kg is executing SHM of amplitude 0.1 m. When the particle passes through the mean position its KE is  $8 \times 10^{-3}$  J. What is the equation of motion of this particle if the initial phase of oscillation is  $45^\circ$ ?

- a.  $y = 0.1 \sin(4t + \frac{\pi}{4})$   
 b.  $y = 2 \sin(4t + \frac{\pi}{2})$   
 c.  $y = 0.3 \sin(2t + \frac{\pi}{3})$   
 d.  $y = 4 \sin(2t + \frac{\pi}{4})$

Sol. a

$$y = A \sin(\omega t + \phi)$$

$$A = 0.1 \text{ m}$$

$$\phi = \frac{\pi}{4}$$

$$\text{KE at mean position} = \text{Maximum KE} = \frac{1}{2} m \omega^2 A^2$$

$$8 * 10^{-3} = \frac{1}{2} * 0.1 \omega^2 * (0.1)^2$$

$$\omega = 4 \text{ sec}^{-1}$$

$$y = 0.1 \sin\left(4t + \frac{\pi}{4}\right)$$

6. A stretched wire of length 110 cm is divided into three segments whose frequencies are in the ratio of 1 : 2 : 3. Their lengths must be

- a. 20 cm; 30 cm; 60 cm
- b. 60 cm; 30 cm; 20 cm
- c. 60 cm; 20 cm; 30 cm
- d. 30 cm; 60 cm; 20 cm

Sol. b

$$l_1 + l_2 + l_3 = 110 \text{ cm}$$

$$n_1 l_1 = n_2 l_2 = n_3 l_3$$

$$n_1 : n_2 : n_3 = 1 : 2 : 3$$

$$\frac{n_1}{1} = \frac{1}{2} = \frac{l_2}{l_1}$$

$$\frac{n_2}{2} = \frac{1}{3} = \frac{l_3}{l_1}$$

$$\frac{n_3}{3} = \frac{1}{3} = \frac{l_1}{l_1}$$

$$l_1 + \frac{l_1}{2} + \frac{l_1}{3} = 110$$

$$l_2 = 30 \text{ cm}$$

7. A mass  $M$  is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period  $T$ . If the mass is increased by  $m$ , the time period becomes  $5T/3$ , then the ratio of  $m/M$  is
- A.  $3/5$
  - B.  $25/9$
  - C.  $16/9$
  - D.  $5/3$

Sol. c

$$\begin{aligned}\frac{T}{T'} &= \sqrt{\frac{M}{M+m}} \\ \frac{9}{25} &= \frac{M}{M+m} \\ \frac{m}{M} &= \frac{16}{9}\end{aligned}$$

8. If a child swinging on a swing in a sitting position stands up, then the time period of the swing will
- a. increase
  - b. decrease
  - c. remain the same
  - d. depend upon the height of the child

Sol. b

When child stands up centre of mass shifts upwards. The child swings in simple harmonic motion hence time-period is given by  $T = 2\pi \sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$  When child stands up, the length shortened decreases.

9. Time period of a simple pendulum will be double, if we
- decrease the length by 2 times
  - decrease the length by 4 times
  - increase the length by 2 times
  - increase the length by 4 times

Sol. d

$$T = 2\pi\sqrt{\frac{l}{g}}$$

If length is increased by the factor of 4, the time period will be doubled.

10. A spring has a certain mass suspended from it and its period for vertical oscillations is  $T_1$ . The spring is now cut into two equal halves and the same mass is suspended from one of the halves. The period of vertical oscillations is now  $T_2$ . The ratio  $T_2/T_1$  is equal to

- $\frac{1}{\sqrt{2}}$
- $\frac{1}{2}$
- $\frac{\sqrt{3}}{1}$
- $\frac{1}{\sqrt{3}}$

Sol. a

If the spring is cut into two equal parts, each part has a spring constant  $2k$ . Hence,

$$T_2 = 2\pi\sqrt{\frac{m}{2k}} T_1 = 2\pi\sqrt{\frac{m}{k}} \frac{T_2}{T_1} = \frac{1}{\sqrt{2}}$$

11. A linear simple harmonic oscillation has amplitude of 10 cm. What is its time period if the speed at the mean position is 31.4 cm/s?
- a. 8 s
  - b. 6 s
  - c. 4 s
  - d. 2 s

Sol. d

$$V_{\max} = a \omega = a \frac{2\pi}{T}$$
$$T = \frac{2\pi a}{V_{\max}} = \frac{2 \times 3.14 \times 10}{31.4}$$
$$= \frac{2 \times 10}{10} = 2 \text{ sec}$$

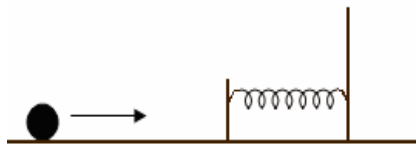
12. A particle executes SHM of time period 8 s and amplitude 4 cm. What is the speed of the particle 2 s after it passes through the mean position?
- a. 0 cm/s
  - b. 2 cm/s
  - c. 4 cm/s
  - d. 6 cm/s

Sol. a

$$V = a\omega \cos \omega t$$
$$= a \frac{2\pi}{T} \cos \frac{2\pi}{T} t$$
$$= 4 \times \frac{2\pi}{8} \cos \left( \frac{2\pi}{8} \times 2 \right)$$

$$\begin{aligned} &= \pi \cos \frac{\pi}{2} \\ &= 0 \end{aligned}$$

13. A 0.5 kg mass moving with a speed of 1.5 m/s on a horizontal smooth surface collides with a nearly weightless spring of force constant  $k = 50 \text{ N/m}$ . The maximum compression of the spring would be



- a. 0.15 m
- b. 0.12 m
- c. 1.5 m
- d. 0.5 m

Sol. a

By the law of conservation of energy, kinetic energy of mass = energy stored in spring

$$\text{i.e., } \frac{1}{2} mv^2 = \frac{1}{2} kx^2$$

$$\therefore x^2 = \frac{mv^2}{k}$$

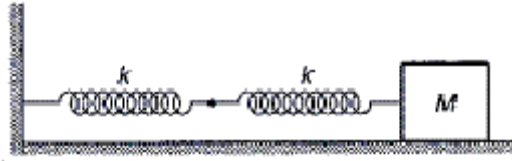
$$\Rightarrow x = \sqrt{\left\{ \frac{mv^2}{k} \right\}}$$

$$\Rightarrow x = \sqrt{\left( \frac{0.5 \times 1.5 \times 1.5}{50} \right)}$$

$$\text{So, } x = 0.15 \text{ m}$$



14. Two springs are connected to a block of mass  $M$ , placed on a frictionless surface, as shown in the figure below. If both the springs have a spring constant  $k$ , the frequency of the oscillation of block is



- a.  $\frac{1}{2\pi} \sqrt{\frac{k}{M}}$   
 b.  $\frac{1}{2\pi} \sqrt{\frac{k}{2M}}$   
 c.  $\frac{1}{2\pi} \sqrt{\frac{2k}{M}}$   
 d.  $\frac{1}{2\pi} \sqrt{\frac{M}{k}}$

Sol. b

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$F = -kl$$

Taking magnitude on both sides,

$$Mg = kl$$

$$l/g = M/k$$

Springs are in series  $k = 2kT = 2\pi \sqrt{\frac{M}{k}}$

Where  $k = 2k$

$$\nu = \frac{1}{2\pi} \sqrt{\frac{2k}{M}}$$

15. A ball of mass 2 kg moving with a velocity of 3 m/s, collides with a spring of natural length 2 m and force constant 144 N/m. What will be the length of the compressed spring?
- a. 2 m
  - b. 1.5 m
  - c. 1 m
  - d. 0.5 m

Sol. b

*The kinetic energy of spring-ball system is conserved.*

Let spring is compressed by a length  $x$ .

Kinetic energy of ball = Potential energy of spring

$$\text{i.e., } \frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

Given,  $m = 2$  kg,  $v = 3$  m/s,  $k = 144$  N/m

$$\therefore \frac{1}{2} \times 2 \times (3)^2 = \frac{1}{2} \times 144 \times x^2$$

$$\text{or } 9 = 72x^2$$

$$\therefore x = \sqrt{\frac{9}{72}} = \frac{1}{2\sqrt{2}} \text{ m}$$

Hence, length of compressed spring

$$= 2 - \frac{1}{2\sqrt{2}}$$

$$= \frac{4\sqrt{2} - 1}{2\sqrt{2}}$$

$$= 1.5 \text{ m}$$

16. A particle in SHM is described by the displacement equation  $x(t) = A \cos (\omega t + \theta)$ . If the initial ( $t = 0$ ) position of the particle is 1 cm and its initial velocity is  $\pi$  cm/s, what is its amplitude? (The angular frequency of the particle is  $\pi \text{ s}^{-1}$ )

- a. 1 cm
- b.  $\sqrt{2}$  cm
- c. 2 cm
- d. 2.5 cm

Sol. b

*Rate of change of displacement gives*

velocity.

Given,  $x = A \cos (\omega t + \theta)$

Velocity  $v = \frac{dx}{dt} = A \frac{d}{dt} \cos (\omega t + \theta)$

$$v = -A \omega \sin (\omega t + \theta)$$

Using  $\sin^2 \theta + \cos^2 \theta = 1$ , we have

$$v = -A \omega \sqrt{1 - \cos^2 (\omega t + \theta)}$$

$\therefore$  From equation  $x = A \cos (\omega t + \theta)$ , we have

$$v = -A \omega \sqrt{1 - x^2/A^2}$$

$$\Rightarrow v = -\omega \sqrt{A^2 - x^2}$$

Given,  $v = \pi$  cm/s,  $x = 1$  cm,  $\omega = \pi \text{ s}^{-1}$

$$\therefore \pi = -\pi \sqrt{A^2 - 1}$$

$$\Rightarrow 1 = A^2 - 1$$

$$\Rightarrow A = \sqrt{2} \text{ cm}$$

17. The angular amplitude of a simple pendulum is  $\theta_0$ . What will be the maximum tension in its string?

- a.  $mg(1 - \theta_0)$
- b.  $mg(1 + \theta_0)$
- c.  $mg(1 - \theta_0^2)$
- d.  $mg(1 + \theta_0^2)$

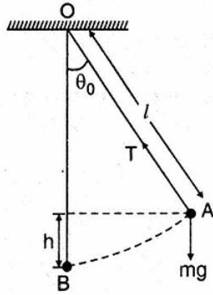
Sol. d

The simple pendulum at angular amplitude  $\theta_0$  is shown in the figure.

Maximum tension in the string is

$$T_{\max} = mg + \frac{mv^2}{l} \quad \dots(i)$$

When bob of the pendulum comes from A to B, it covers a vertical distance  $h$



$$\therefore \cos \theta_0 = \frac{l-h}{l}$$

$$\Rightarrow h = l(1 - \cos \theta_0) \quad \dots(ii)$$

Also during A to B, potential energy of bob converts into kinetic energy i.e.,  $mgh = \frac{1}{2}mv^2$

$$\therefore v = \sqrt{2gh} \quad \dots(iii)$$

Thus, using Eqs. (i), (ii) and (iii), we obtain

$$\begin{aligned} T_{\max} &= mg + \frac{2mg}{l} l(1 - \cos \theta_0) \\ &= mg + 2mg \left[ 1 - 1 + \frac{\theta_0^2}{2} \right] \\ &= mg (1 + \theta_0^2) \end{aligned}$$

18. The time period of a particle in simple harmonic motion is 8 seconds. At  $t = 0$  it is at the mean position. The ratio of the distances travelled by it in the 1<sup>st</sup> second and 2<sup>nd</sup> second is

- $\frac{1}{2}$
- $\frac{1}{\sqrt{2}}$
- $\frac{1}{\sqrt{2}-1}$
- $\frac{1}{\sqrt{3}}$

Sol. C

Time period of particle is given by

$$T = \frac{2\pi}{\omega} \quad (\because T = 8 \text{ s})$$

So,  $\frac{2\pi}{\omega} = 8$

$$\Rightarrow \omega = \frac{2\pi}{8} = \frac{\pi}{4}$$

Equation of SHM is

$$y = a \sin \omega t \quad \dots(i)$$

( $\because$  At mean position  $t = 0$ )

Hence,  $y = 0$

For  $t = 1 \text{ s}$  Eq. (i) becomes

$$y = a \sin \frac{\pi}{4} \times 1 = \frac{a}{\sqrt{2}} \quad \dots(ii)$$

For  $t = 2 \text{ s}$  Eq. (i) becomes

$$y = a \sin \frac{\pi}{4} \times 2 = a \sin \frac{\pi}{2} = a \quad \dots(iii)$$

Now, the distance covered in 2 s is given by

$$a - \frac{a}{\sqrt{2}} = a \left( \frac{\sqrt{2}-1}{\sqrt{2}} \right)$$

Again the ratio of the distance covered in first and second seconds is

$$= \frac{\frac{a}{\sqrt{2}}}{a \left( \frac{\sqrt{2}-1}{\sqrt{2}} \right)} = \frac{1}{(\sqrt{2}-1)}$$

19. A particle is executing two different simple harmonic motions, mutually perpendicular of different amplitudes and having phase difference of  $\pi/2$ . The path of the particle will be
- circular
  - straight
  - parabolic
  - elliptical

Sol. d

$$\text{Let } x = a \sin(\omega t + \delta) \quad \dots \text{ (i)}$$

$$y = b \sin \omega t \quad \dots \text{ (ii)}$$

are two perpendicular SHM waves.

The resultant can be obtained by eliminating  $\omega t$  from Eqs. (i) and (ii), we get

$$\sin \omega t = \frac{y}{b}$$

$$\therefore \cos \omega t = \sqrt{1 - \sin^2 \omega t} = \sqrt{1 - \frac{y^2}{b^2}}$$

$$\therefore \frac{x}{a} = \sin \omega t \cos \delta + \cos \omega t \sin \delta$$

$$\text{or } \frac{x}{a} = \frac{y}{b} \cos \delta + \sqrt{1 - \frac{y^2}{b^2}} \sin \delta$$

$$\Rightarrow \left( \frac{x}{a} - \frac{y}{b} \cos \delta \right)^2 = \sin^2 \delta \left( \sqrt{1 - \frac{y^2}{b^2}} \right)^2$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} \cos \delta = \sin^2 \delta$$

Given, phase difference between the waves is

$$\delta = \frac{\pi}{2}$$

So, resultant equation becomes,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

This represents the regular ellipse.

20. A particle executes linear simple harmonic motion with amplitude of 2 cm. When the particle is at 1 cm from the mean position, the magnitude of its velocity is equal to that of its acceleration. Then its time period in seconds is

- $\frac{1}{2\pi\sqrt{3}}$
- $2\pi\sqrt{3}$
- $\frac{2\pi}{\sqrt{3}}$
- $\frac{\sqrt{3}}{2\pi}$

Sol. c

$$\begin{aligned} \text{Velocity} &= \text{acceleration} \\ \omega \sqrt{a^2 - y^2} &= \omega^2 y \\ \sqrt{(2)^2 - (1)^2} &= \omega \quad (1) \\ \Rightarrow \omega &= \sqrt{3} \\ T &= \frac{2\pi}{\omega} \\ \Rightarrow T &= \frac{2\pi}{\sqrt{3}} \end{aligned}$$

21. Which of the following condition is not sufficient for simple harmonic motion and why?

- (i) acceleration and displacement
- (ii) restoring force and displacement

Sol: Condition (i) is not sufficient, because direction of acceleration is not mentioned. In SHM, the acceleration is always in a direction opposite to that of the displacement.

How is the time period effected, if the amptutude of a simple pendulum is

No effect on time period when amptitude of pendulum is increased or decreased.

22. Does the direction of acceleration at various points during the oscillation of a simple pendulum remain towards mean position?

Sol: No, the resultant of tension in the string and weight of bob is not always towards the mean position.

23. No, the resultant of tension in the string and weight of bob is not always towards the mean position.

What is the phase relationship between displacement, velocity and accelection in SHM?

Sol: In SHM, - The velocity leads the displacement by a phase  $\pi/2$  radians and acceleration leads the velocity by a phase  $\pi/2$  radians.

24. How will the time period of a simple pendulum change when its length is doubled?

Sol:  $\sqrt{2}$  times, as  $T \propto \sqrt{\ell}$

25. In a forced oscillation of a particle, the amplitude is maximum for a frequency  $\omega_1$  of the force, while the energy is maximum for a frequency  $\omega_2$  of the force. What is the relation between  $\omega_1$  and  $\omega_2$ ?

Sol: Both amplitude and energy of the particle can be maximum only in the case of resonance, for resonance to occur  $\omega_1 = \omega_2$ .

26. Time period of a particle in S.H.M depends on the force constant  $K$  and mass  $m$  of the particle ( $T = \frac{1}{2\pi} \sqrt{\frac{m}{k}}$ ). A simple pendulum for small angular displacement independent of the mass of the pendulum of a pendulum independent of the mass of the pendulum?

Sol: Restoring force in case of simple pendulum is given by

$$F = \frac{mg}{l} y \Rightarrow K = mg / l$$

So force constant itself proportional to  $m$  as the value  $k$  is substituted in the formula,  $m$  is cancelled out.

27. The displacement of a particle in S.H.M may be given by  $y = a \sin(\omega t + \Phi)$  show that if the time  $t$  is increased by  $2\pi/\omega$ , the value of  $y$  remains the same.

Sol:

The displacement at any time  $t$  is

$$y = a \sin(\omega t + \phi)$$

$\therefore$  displacement at any time  $(t + 2\pi/\omega)$  will be

$$y = a \sin [\omega(t + 2\pi/\omega) + \phi] = [\sin(\omega t + \phi) + 2\pi]$$

$$\Rightarrow y = a \sin(\omega t + \phi) [\because \sin(2\pi + \phi) = \sin\phi]$$

Hence, the displacement at time  $t$  and  $(t + 2\pi/\omega)$  are same.



28. The time period of a body executing S.H.M is 1s. After how much time will its displacement be  $\frac{1}{\sqrt{2}}$  of its amplitude.

Sol:

$$\text{Soln : } y = r \sin \omega t = r \sin \frac{2\pi}{T} t$$

$$\text{Here } y = \frac{1}{3}r \text{ and } T = 1\text{s}$$

$$\therefore \frac{1}{\sqrt{2}}r = r \sin \frac{2\pi}{T} t \Rightarrow 2\pi t = \pi/4$$

$$\Rightarrow t = \frac{1}{8}\text{s}$$

29. A particle executes S.H.M of amplitude 25 cm and time period 3s. what is the minimum time required for the particle to move between two points 12.5 cm on either side of the mean position?

Sol:

$$\text{Given, } r = 25 \text{ cm; } T = 3\text{s ; } y = 12.5 \text{ cm}$$

$$\text{The displacement } y = r \sin \frac{2\pi}{T} t$$

$$12.5 = 25 \sin \frac{2\pi}{3} t \text{ or } \frac{2\pi}{3} t = \frac{\pi}{6} \text{ or } t = 0.25 \text{ s.}$$

$$\text{The minimum time taken by the particle } 2t = 0.5 \text{ s}$$