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Class: XI
Subject: Physics
Topic: Systems of Particles and Rotational motion
No. of Questions: 20

Q1. A shell fired from a gun at an angle to the horizontal explodes in mid-air. Then the centre of mass of the shell fragments will move

a. vertically down
b. horizontally
c. along the same parabolic path along which the 'intact' shell, was moving
d. along tangent to the parabolic path of the 'intact' shell, at the point of explosion

Soln: c

Since there no external forces, the centre of mass will continue to maintain its original motion. Hence the correct choice is (3)

Q2. A child is standing at one end of a long trolley moving with a speed v on a smooth horizontal track. If the child starts running towards the other end of the trolley with a speed u, the centre of mass of the system (trolley + child) will move with a speed

a. zero
b. (v + u)
c. (v – u)
d. v

Soln: d

The speed of the centre of mass of the system will remain unchanged (= v). The speed of the centre of mass of a system can change only if a net external force acts on it. The forces involved (such as the action and reaction and frictional forces), when the child runs on the trolley, are internal to the (trolley + child) system. Hence the correct choice is (4).

Q3. Choose the only incorrect statement from the following

a. The position of the centre of mass of a system of particles does not depend upon the internal forces between particles.
b. The centre of mass of a solid may lie outside the body of the solid.
c. A body tied to a string is whirled in a circle with a uniform speed. If the string is suddenly cut, the angular momentum of the body will change from its initial value.
d. The angular momentum of a comet revolving around a massive star, remains constant over the entire orbit.

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Soln: c

The only incorrect statement is (3). Since no external torque acts on the body even after the string is cut, the angular momentum will remain unchanged.

Q4. A carpet of mass M, made of an inextensible material, is rolled along its length in the form of a cylinder of radius R and kept on a rough floor. If the carpet is unrolled (without sliding) to a radius R/2, the decrease in potential energy will be

a. $\frac{1}{2}MgR$
b. $\frac{5}{8}MgR$
c. $\frac{3}{4}MgR$
d. $\frac{7}{8}MgR$

Soln: d

The centre of mass of the whole carpet is originally at a height R above the floor.
When the carpet unrolls itself and has a radius R/2, the centre of mass is at a height R/2 but the mass left over unrolled is

$\frac{M(R/2)^2}{R^2} = \frac{M}{4}$. Hence the decrease in P.E., is

$$MgR - \frac{M}{4}g.\frac{R}{2} = \frac{7}{8}MgR.$$

Hence the correct choice is (4).

Q5. A molecule consists of two atoms, each of mass m, separated by a distance a. The moment of inertia of the molecule about its centre of mass is

a. 2 ma^2
b. ma^2
c. $\frac{1}{2}ma^2$
d. $\frac{1}{4}ma^2$

Soln: c

Since the two atoms have the same mass, the centre of mass is at a distance of a/2 from each atom. Therefore, the moment of inertia of the molecule about its centre of mass is

$$I = m\left(\frac{a}{2}\right)^2 + m\left(\frac{a}{2}\right)^2 = \frac{ma^2}{2}$$

Hence, the correct choice is (3).

Q6. The moment of inertia of a solid sphere of mass M and radius R, about an axis through its centre is $\frac{2}{5}MR^2$. The moment of inertia about an axis tangential to the surface of the sphere will be

a. $\frac{4}{5}MR^2$
b. MR^2
c. $\frac{6}{5}MR^2$
d. $\frac{7}{5}MR^2$

Soln: d

Given

$$I_c = \frac{2}{5}MR^2$$

Using the parallel axes theorem, the moment of inertia about an axis tangential to the sphere will be

$$I = I_c + MR^2 = \frac{2}{5}MR^2 + MR^2$$

$$= \frac{7}{5}MR^2$$

Hence, the correct choice is (4).

Q7. When W joules of work is done on a flywheel, its frequency of rotation increases from ν_1 Hz to ν_2 Hz. The moment of inertia of the flywheel about its axis of rotation is given by

a. $\frac{W}{2\pi^2(v_2^2 - v_1^2)}$
b. $\frac{W}{2\pi^2(v_2^2 + v_1^2)}$
c. $\frac{W}{4\pi^2(v_2^2 - v_1^2)}$

d. $\dfrac{W}{4\pi^2(v_2^2 + v_1^2)}$

Soln: a

Work done = increase in kinetic energy or

$$W = \frac{1}{2} I\omega_2^2 - \frac{1}{2} I\omega_1^2$$

$$= \frac{I}{2}(\omega_2^2 - \omega_1^2)$$

$$= 2\pi^2 I(v_2^2 - v_1^2) \qquad (\because \omega = 2\pi v)$$

or $$I = \frac{W}{2\pi^2(v_2^2 - v_1^2)}$$

Hence the correct choice is (1).

Q8. If the earth were to suddenly contract to half its present size, without any change in its mass, the duration of the new day will be

a. 6 hours
b. 12 hours
c. 18 hours
d. 30 hours

Soln: a

Let M be the mass and R the initial radius of the earth. If ω is the angular velocity of the rotation of the earth, the duration T of the day is

$$T = \frac{2\pi}{\omega}$$

Let R' be the radius of the earth after contraction and ω' its angular velocity. From the conservation of angular momentum, we have

$$I\omega = I'\omega'$$

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where $I\left(=\frac{2}{5}MR^2\right)$ and $I'\left(=\frac{2}{5}MR'^2\right)$ are the moments of inertia of the earth before and after contraction, respectively.

$$\therefore \quad \frac{2}{5}MR^2\omega = \frac{2}{5}MR'^2\omega'$$

or $$\omega' = \frac{R^2\omega}{R'^2} = 4\omega \qquad (\because R' = R/2)$$

The duration T' of the new day will be (since $T = 2\pi/\omega$)

$$T' = \frac{2\pi}{\omega'} = \frac{2\pi}{4\omega} = \frac{T}{4}$$

$$T' = \frac{24 \text{ hours}}{4} = 6 \text{ hours}$$

Q9. A solid sphere rolls down from the top of an inclined plane. Its velocity on reaching the bottom of the plane is v. When the same sphere slides down from the top of the plane, its velocity on reaching the bottom is v'. The ratio v'/v is equal to

a. $\sqrt{\frac{3}{5}}$
b. 1
c. $\sqrt{\frac{7}{5}}$
d. $\frac{3}{\sqrt{5}}$

Soln: c

For rolling :

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$
$$= \frac{1}{2}Mv^2 + \frac{1}{2}\times\left(\frac{2}{5}MR^2\right)\times\frac{v^2}{R^2}$$
$$= \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 = \frac{7}{10}Mv^2$$
$$\left(\because I = \frac{2}{5}MR^2 \text{ and } \omega = \frac{v}{R}\right)$$

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For sliding :

$Mgh = \frac{1}{2} Mv'^2$. Therefore $\frac{1}{2} Mv'^2 = \frac{7}{10} Mv^2$

or $\frac{v'}{v} = \sqrt{\frac{7}{5}}$, which is choice (3).

Q 10. A circular disc rolls down an inclined plane without slipping. What fraction of its total energy is translational?

a. $\frac{1}{\sqrt{2}}$

b. $\frac{1}{2}$

c. $\frac{1}{3}$

d. $\frac{2}{3}$

Soln: d

Rotational kinetic energy is

$$(KE)_r = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$$

$$= \frac{1}{4} MR^2\omega^2 \qquad (\because I = \frac{1}{2} MR^2)$$

where M is the mass of the disc and R its radius.

Translational kinetic energy is

$$(KE)_t = \frac{1}{2} Mv^2 = \frac{1}{2} M(R\omega)^2 = \frac{1}{2} MR^2\omega^2$$

$$(\because v = R\omega)$$

Total energy, $KE = (KE)_r + (KE)_t = \frac{3}{4} MR^2\omega^2$

$$\therefore \quad \frac{(KE)_t}{KE} = \frac{2}{3}$$

Hence the correct choice is (4).

Q11. A circular disc is rolling down an inclined plane without slipping. If the angle of inclination is 30°, the acceleration of the disc down the inclined plane is

a. g

b. $\frac{g}{2}$

c. $\frac{g}{3}$

d. $\frac{\sqrt{2}}{3}$ g

Soln: c

Downward force $F = Mg \sin\theta$. The effective mass of the rolling disc is $M_{\text{eff}} = M + \frac{I_{\text{CM}}}{R^2}$, where I_{CM} is the moment of inertia about its centre of mass, which is

$$I_{\text{CM}} = \frac{1}{2}MR^2$$

$$\therefore \quad M_{\text{eff}} = M + \frac{1}{2}\frac{MR^2}{R^2} = \frac{3M}{2}$$

$$\therefore \quad \text{Acceleration } a = \frac{F}{M_{\text{eff}}} = \frac{Mg\sin\theta}{3M/2} = \frac{2}{3}g\sin 30° = \frac{g}{3}$$

Hence the correct choice is (3).

Q12. Two circular loops A and B are made of the same wire and their radii are in the ratio 1 : n. Their moments of inertia about the axis passing through the centre and perpendicular to their plane are in the ratio 1 : m. The relation between m and n is

a. m = n
b. m = n^2
c. m = n^3
d. m = n^4

Soln: c

Here $M_A = 2\pi\mu R$ and $M_B = 2\pi n\mu R$. Their moments of inertia are

$$I_A = M_A R_A^2 = 2\pi\mu R \times R^2 = 2\pi\mu R^3$$

and $$I_B = M_B R_B^2 = 2\pi n\mu R \times (nR)^2 = 2\pi n^3 \mu R^3$$

$$\therefore \quad \frac{I_B}{I_A} = n^3; \text{ but } \frac{I_B}{I_A} = m \text{ (given)}$$

Thus, $m = n^3$. Hence the correct choice is (3).

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Q13. Three particles, each of mass m, are placed at the corners of an equilateral triangle of side a, as shown in the figure. The position vector of the centre of mass is

a. $\frac{a}{2}(i + j/\sqrt{3})$
b. $\frac{a}{2}(3\,i + j)$
c. $\frac{a}{2}(3\,i + \sqrt{3}\,j)$
d. $\frac{a}{2}(3\,i + j/\sqrt{3})$

Soln: a

Refer to Fig. The (x, y) co-ordinates of the masses at O, A and B respectively are

$$(x_1 = 0, y_1 = 0), (x_2 = a, y_2 = 0), \left(x_3 = \frac{a}{2}, y_3 = \frac{a\sqrt{3}}{2}\right)$$

Therefore, the (x, y) co-ordinates of the centre of mass are

$$x_{CM} = \frac{m \times 0 + m \times a + m \times a/2}{m + m + m} = \frac{a}{2}$$

$$y_{CM} = \frac{m \times 0 + m \times a + m \times a\sqrt{3}/2}{m + m + m}$$

$$= \frac{a}{2\sqrt{3}}$$

$\therefore$ Position vector of centre of mass is $\frac{a}{2}\left(\mathbf{i} + \frac{\mathbf{j}}{\sqrt{3}}\right)$.
Hence the correct choice is (1).

Q14. A circular disc of mass m and radius r is rolling on a horizontal surface with a constant speed v. Its kinetic energy is

a. $\frac{1}{4}mv^2$
b. $\frac{1}{2}mv^2$
c. $\frac{3}{4}mv^2$
d. mv^2

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Soln: c

The kinetic energy of a rolling disc consists of two parts: translational energy = $\frac{1}{2}mv^2$ and rotational energy = $\frac{1}{2}I\omega^2$.

$$\therefore \quad KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2} \times \left(\frac{1}{2}mr^2\right) \times \left(\frac{v}{r}\right)^2$$

$$(\because I = \frac{1}{2}mr^2)$$

$$= \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$$

Hence the correct choice is (3).

Q15. A cylinder, released from the top of an inclined plane, rolls without sliding and reaches the bottom with speed v_r. Another identical cylinder, released from the top of the same inclined plane, slides without rolling and reaches the bottom with speed v_s. Then

a. $v_r > v_s$
b. $v_r < v_s$
c. $v_r = v_s$
d. $v_r = v_s = 0$

Soln: b

When the cylinder rolls without sliding, the acceleration down the plane is

$$a_r = \frac{2}{3} g \sin\theta$$

When the cylinder slides without rolling, the acceleration is

$$a_s = g \sin \theta$$

where θ is the inclination of the plane.
If h is the height of the inclined plane, their speeds on reaching the bottom are given by

$$v_r = \sqrt{2a_r h} \text{ and } v_s = \sqrt{2a_s h}$$

Since $a_s > a_r$, it follows that $v_s > v_r$, which is choice (2).

Q16. Three thin metal rods, each of mass M and length L are welded to form an equilateral triangle. The moment of inertia of the composite structure about an axis passing through the centre of mass of the structure and perpendicular to its plane is

a. $\frac{ML^2}{2}$

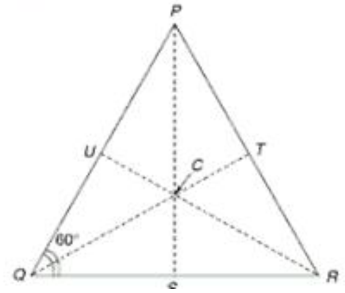
b. $\frac{ML^2}{4}$

c. $\frac{ML^2}{8}$

d. $\frac{ML^2}{12}$

Soln: a

Given $PQ = QR = RP = L$. The centre of mass is located at centroid C which cuts lines PS, QT and UR in the ratio 2 : 1. Let $h = CS = CT = UC$. In ΔPQS, we have



$$PS = PQ \sin 60° = L \sin 60° = \frac{\sqrt{3}}{2} L.$$

$$\therefore \quad h = \frac{PS}{3} = \frac{1}{3} \times \frac{\sqrt{3}}{2} L = \frac{L}{2\sqrt{3}}$$

Since the structure consists of three identical rods, its moment of inertia about an axis passing through its centre of mass C and perpendicular to its plane is, from parallel axes theorem,

$$I_c = 3\,(I + Mh^2)$$

where I is the moment of inertia of each rod about the axis passing through its centre and perpendicular to its length, which is given by

$$I = \frac{M\,L^2}{12}$$

Also $$Mh^2 = M\left(\frac{L}{2\sqrt{3}}\right)^2 = \frac{M\,L^2}{12}$$

$$\therefore \quad I_c = 3\left(\frac{M\,L^2}{12} + \frac{M\,L^2}{12}\right)$$

$$= 3 \times \frac{M\,L^2}{6} = \frac{M\,L^2}{2}$$

Hence the correct choice is (1).

Q17. A solid sphere is rotating about its diameter. Due to increase in room temperature, its volume increases by 0.5%. If no external torque acts, the angular speed of the sphere will

a. increase by nearly $\frac{1}{3}\%$

b. decrease by nearly $\frac{1}{3}\%$

c. increase by nearly $\frac{1}{2}\%$

d. decrease by nearly $\frac{2}{3}\%$

Soln: b

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$V = \frac{4}{3}\pi r^3$ or $\log V = \log\left(\frac{4\pi}{3}\right) + 3\log r$.

Differentiating, we have

$$\frac{\delta V}{V} = 3\frac{\delta r}{r} \text{ or } \frac{\delta r}{r} = \frac{1}{3}\frac{\delta V}{V} = \frac{1}{3}\times 0.5\% = \frac{1}{6}\%$$

Since no external torque acts, $I\omega$ = constant or $\frac{2}{5}mr^2\omega$ = constant or $r^2\omega$ = constant (c)

or $2\log r + \log\omega = \log c$. Differentiating, we have

$$\frac{2\delta r}{r} + \frac{\delta\omega}{\omega} = 0$$

or $$\frac{\delta\omega}{\omega} = -2\frac{\delta r}{r} = -2\times\frac{1}{6}\% = -\frac{1}{3}\%$$

The negative sign indicates that ω decreases. Hence the correct choice is (2).

Q18. A uniform rod of length L is suspended from one end such that it is free to rotate about an axis passing through that end and perpendicular to the length. What minimum speed must be imparted to the lower end, so that the rod completes one full revolution?

a. $\sqrt{2gL}$
b. $\sqrt{gL}$
c. $\sqrt{6gL}$
d. $2\sqrt{2gL}$

Soln: c

In one full revolution the increase in PE = MgL, where M is the mass of the rod. Therefore,

$$MgL = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$$

or $\omega = \sqrt{\frac{6g}{L}}$. Now $v = L\omega = L\sqrt{\frac{6g}{L}} = \sqrt{6gL}$

Hence the correct choice is (3).

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Q19. A thin uniform rod AB of mass M and length L is hinged at one end A to the horizontal floor. Initially, it stands vertically. It is allowed to fall freely on the floor in the vertical plane. The angular velocity of the rod when its end B strikes the floor is

a. $\sqrt{\frac{g}{L}}$

b. $\sqrt{\frac{2g}{L}}$

c. $\sqrt{\frac{3g}{L}}$

d. $2\sqrt{\frac{g}{L}}$

Soln: c

Loss in PE = gain in rotational KE. As the centre of mass of the rod falls through a distance $L/2$, the loss in PE = $\frac{MgL}{2}$. Gain in KE = $\frac{1}{2}\ I\omega^2$ = $\frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$

Equating the two, we have

$$\frac{MgL}{2} = \frac{ML^2\omega^2}{6}$$

or $\omega = \sqrt{\frac{3g}{L}}$, which is choice (3).

Q20. The moment of inertia of a thin rod of mass M and length L about an axis passing through the point at a distance L/4 from one of its ends and perpendicular to the rod is

a. $\frac{7ML^2}{48}$

b. $\frac{ML^2}{12}$

c. $\frac{ML^2}{9}$

d. $\frac{ML^2}{3}$

Soln: a

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Using the parallel axes theorem, $I = I_{CM} + Mx^2$ where x is the distance of the axis of rotation from the C.M. (centre of mass) of the rod, which is $x = \frac{L}{2} - \frac{L}{4} = \frac{L}{4}$.

Also $I_{C.M.} = \frac{ML^2}{12}$. Hence

$I = \frac{ML^2}{12} + \frac{ML^2}{16} = \frac{7ML^2}{48}$, which is choice (1).