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Class: XI
Subject:
Topic: ASK15E11HY01
No. of Questions: 30
Duration: 90 Miutes
Maximum Marks: 90

Q1. If $(\log_5 x)^2 + \log_5 x < 2$, then x belong to:

(A) $\left(\frac{1}{25}, 5\right)$

(B) $\left(\frac{1}{5}, \frac{1}{\sqrt{5}}\right)$

(C) $(1, \infty)$

(D) None of these

Sol: (A)

We have $(\log_5 x)^2 + \log_5 x < 2$

Put $\log_5 x = a$ then $a^2 + a < 2$

$\Rightarrow a^2 + a - 2 < 0 \Rightarrow (a + 2)(a - 1) < 0$

$\Rightarrow -2 < a < 1$ or $-2 < \log_5 x < 1$

$\therefore 5^{-2} < x < 5$

i.e. $1/25 < x < 5$

Q2. If $A + B + C = 180^o$ then the value of tanA + tanB + tanC is :

(A) $\geq 3\sqrt{3}$

(B) $\geq 2\sqrt{3}$

(C) $> 3\sqrt{3}$

(D) $> 2\sqrt{3}$

Sol:

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tan(A + B) = tan(180^o – C)

or, $\frac{\tan A + \tan B}{1 - \tan A \tan B}$ = tanC

or, tanA + tanB + tanC = tana tanB tanC

$\frac{\tan A + \tan B + \tan C}{3} \geq \sqrt[3]{\tan A \tan B \tan C}$ [since A.M. $\geq$ G.M.]

or, tanA tanB tanC $\geq$ $\sqrt[3]{\tan A \tan B \tan C}$

or, $\tan^2 A \tan^2 B \tan^2 C \geq 27$ [cubing both sides]

or tanA tanB tanC $\geq 3\sqrt{3}$

$\Rightarrow$ tanA + tanB + tanC $\geq 3\sqrt{3}$.

Hence (A) is the correct answer.

Q3. Let 0 < A, B < $\frac{\pi}{2}$ satisfying the equalities 3 $\sin^2$ A + 2 $\sin^2$ B = 1 and 3sin2A – 2sin2B = 0. Then A + 2B = :

(A) $\frac{p}{4}$ (B) $\frac{p}{3}$

(C) $\frac{\pi}{2}$ (D) None of these.

Sol: (C)

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From the second equation, we have

$\sin 2B = \frac{3}{2}\sin 2A$ ... (1)

and from the first equality

$3\sin^2 A = 1 - 2\sin^2 B = \cos 2B$ ... (2)

Now cos (A + 2B) = cosA. cos2B – sinA . sin2B

$= 3\cos A . \sin^2 A - \frac{3}{2} . \sin A . \sin 2A$

$= 3\cos A. \sin^2 A - 3\sin^2 A . \cos A = 0$

$\Rightarrow A + 2B = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

Given that $0 < A < \frac{\pi}{2}$ and $0 < B < \frac{\pi}{2} \Rightarrow 0 < A + 2B < \pi + \frac{\pi}{2}$

Hence $A + 2B = \frac{\pi}{2}$.

Hence (C) is the correct answer.

Q4. If $a\cos^3\theta + 3a\cos\theta\sin^2\theta = x$ and $a\sin^3\theta + 3a\cos^2\theta\sin\theta = y$, then $(x + y)^{2/3} + (x - y)^{2/3}$ =

(A) $2a^{2/3}$ (B) $a^{2/3}$

(C) $3a^{2/3}$ (D) $2a^{1/3}$

Sol: (A)

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$a\cos^3\theta + 3a\cos\theta\sin^2\theta = x$

$a\sin^3\theta + 3a\cos^2\theta\sin\theta = y$

$x + y = a[\sin^3\theta + \cos^3\theta + 3\sin\theta\cos\theta(\sin\theta + \cos\theta)] = a(\sin\theta + \cos\theta)^3$

$\left(\frac{x+y}{a}\right)^{1/3} = \sin\theta + \cos\theta$ ......(1)

$x - y = a[\cos^3\theta - \sin^3\theta + 3\cos\theta\sin^2\theta - 3\cos^2\theta\sin\theta] = a[\cos\theta - \sin\theta]^3$

$\left(\frac{x-y}{a}\right)^{1/3} = \cos\theta - \sin\theta$ ...... (2)

$(\sin\theta + \cos\theta)^2 + (\cos\theta - \sin\theta)^2 = \frac{(x+y)^{2/3} + (x-y)^{2/3}}{a^{2/3}}$

$2(\sin^2\theta + \cos^2\theta) = \frac{(x+y)^{2/3} + (x-y)^{2/3}}{a^{2/3}}$

$(x + y)^{2/3} + (x - y)^{2/3} = 2a^{2/3}$.

Q5. For the equation $3x^2 + px + 3 = 0$, $p > 0$, if one of the roots is the square of the other, then p is equal to

(A) 1/3 (B) 1

(C) 3 (D) 2/3

Sol: (C)

$\alpha + \alpha^2 = -\frac{p}{3}$ ... (1)

$\alpha^3 = 1 \Rightarrow \alpha = 1, \omega, \omega^2$

From (1), $p = -3(\alpha + \alpha^2) = -6, 3, 3$

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Q6. If the roots of equation $ax^2 + bx + 10 = 0$ are not real and distinct, where $a, b \in R$ and m and n are values of a and b respectively for which $5a + b$ is minimum, then the family of lines $(4x + 2y + 3) + n(x - y - 1) = 0$ are concurrent at

(A) $(1, -1)$ (B) $(-1/6, -7/6)$

(C) $(1, 1)$ (D) none of these

Sol: (B)

Let $f(x) = ax^2 + bx + 10$

Since equation $f(x) = 0$ has no real and distinct roots

$\therefore$ f(x) will have same sign for all real x

But $f(0) = 10 > 0$

$\therefore f(x) \geq 0, \forall x \in R$

$\therefore f(5) \geq 0 \Rightarrow 5(5a + b) + 10 \geq 0$

$\Rightarrow 5a + b \geq -2$

Minimum value of $5a + b = -2$

According to question

$5m + n = -2 \Rightarrow n = -5m - 2$

Given family of lines is

$m(4x + 2y + 3) - (5m + 2)(x - y - 1) = 0$

or $2(x - y - 1) + m(-x + 7y + 8) = 0$

Clearly this family of lines pass through the fixed point $\left(-\frac{1}{6}, -\frac{7}{6}\right)$

Q7. The values of a and b so that $x^4 + 12x^3 + 46x^2 + ax + b$ is square of quadratic expression are respectively

(A) 60, 25 (B) 45, 25

(C) 60, 30 (D) 25, 45

Sol: (A)

Given $x^4 + 12x^3 + 46x^2 + ax + b = (x^2 + cx + d)^2$

Can paring coefficient of x both the sides, we get $12 = 2c$, $46 = 2d + c^2$, $a = 2cd$

and $b = d^2$

$\Rightarrow c = 6$ and $d = 5$

$\Rightarrow a = 60$ and $b = 25$

Q8. If $a^2 + b^2 - ab - a - b + 1 \leq 0$, a, b $\leq$ R, then a + b is equal to:

(A) 5 (B) 2
(C) 9 (D) 4

Sol: (B)

Given that $a^2 + b^2 - ab - a - b + 1 \leq 0$

$\Rightarrow 2a^2 + 2b^2 - 2ab - 2a - 2b + 2 \leq 0$

$(a - b)^2 + (a - 1)^2 + (b - 1)^2 \leq 0$

$\Rightarrow a = 1$ and $b = 1$

Hence (B) is the correct answer.

Q9. Which of the following is a singleton set ?

(A) $(x : |x| = 5, x \in N)$ (B) $(x : |x| = 6, x \in Z)$
(C) $(x : x^2 + 2x + 1 = 0, x \in N)$ (D) $(x : x^2 = 7, x \in N)$

Sol: $|x| = 5 \Rightarrow x = 5$ $(\because x \in N)$

Given set is singleton.

Hence (A) is the correct answer.

Q10: The number of solutions of the equation $x^3 + 2x^2 + 5x + 2\cos x = 0$ in $[0, 2\pi]$ is:

(A) 0 (B) 1

(C) 2 (D) 3

Sol: (A)

Let $f(x) = x^3 + 2x^2 + 5x + 2\cos x$

$\Rightarrow f'(x) = 3x^2 + 4x + 5 - 2\sin x$

$= 3\left(x + \frac{2}{3}\right)^2 + \frac{11}{3} - 2\sin x$

Now $\frac{11}{3} - 2\sin x > 0 \ \forall x$ (as $-1 \le \sin x \le 1$)

$\Rightarrow f'(x) > 0 \ \forall x$

$\Rightarrow$ f(x) is an increasing function.

Now $f(0) = 2$

$\Rightarrow f(x) = 0$ has no solution in $[0, 2\pi]$.

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Q11: The value of $\sum_{r=1}^{5} \cos(2r-1)\frac{\pi}{11}$ is :

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{1}{4}$ (D) $\frac{1}{6}$

Sol: (A)

$$\sum_{r=1}^{5} \cos(2r-1)\frac{\pi}{11}$$

$$\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11}$$

$$= \frac{2\sin\frac{\pi}{11}}{2\sin\frac{\pi}{11}}\left(\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11}\right)$$

$$= \frac{\left(\sin\frac{2\pi}{11} + \sin\frac{4\pi}{11} - \sin\frac{2\pi}{11} + \sin\frac{6\pi}{11} - \sin\frac{4\pi}{11} + \sin\frac{8\pi}{11} - \sin\frac{6\pi}{11} + \sin\frac{10\pi}{11} - \sin\frac{8\pi}{11}\right)}{2\sin\frac{\pi}{11}}$$

$$= \frac{\sin\frac{10\pi}{11}}{2\sin\frac{\pi}{11}} = \frac{\sin\left(\pi - \frac{\pi}{11}\right)}{2\sin\frac{\pi}{11}} = \frac{1}{2}$$

Q12. If tanx = n. tany, $n \in \boldsymbol{R}^+$, then maximum value of $\boldsymbol{sec^2}$ (x – y) is equal to:

(A) $\frac{(n+1)^2}{2n}$ (B) $\frac{(n+1)^2}{n}$

(C) $\frac{(n+1)^2}{2}$ (D) $\frac{(n+1)^2}{4n}$

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Sol: (D)

tanx = n tany, cos(x – y)

= cosx. cosy + sinx.siny.

$\Rightarrow$ cos(x – y) = cosx.cosy (1 + tanx.tany)

= cosx. cosy (1 + n $\tan^2 y$)

$$\Rightarrow \sec^2(x-y) = \frac{\sec^2 x \sec^2 y}{(1+n\tan^2 y^2)}$$

$$= \frac{(1+\tan^2 x)(1+\tan^2 y)}{(1+n\tan^2 y)^2}$$

$$= \frac{(1+n^2\tan^2 y)(1+\tan^2 y)}{(1+n\tan^2 y)^2}$$

$$= 1 + \frac{(n-1)^2\tan^2 y}{(1+n\tan^2 y)^2}$$

Now, $\left(\frac{1+n\tan^2 y}{2}\right)^2 \geq n\tan^2 y.$

$$\Rightarrow \frac{\tan^2 y}{(1+n\tan^2 y)^2} \leq \frac{1}{4n}$$

$$\Rightarrow \sec^2(x-y) \leq 1 + \frac{(n-1)^2}{4n} = \frac{(n+1)^2}{4n}$$

Q13. The number of real roots of equation $x^8 - x^5 + x^2 - x + 2 = 0$ is

(A) 2 (B) 4

(C) 6 (D) 0

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Sol: (D)

Given equation is $x^8 - x^5 + x^2 - x + 2 = 0$. Clearly given equation will have no negative root. Now given equation can be written as $x^5(x^3 - 1) + x(x - 1) + 2 = 0$.

Clearly no value of x will satisfy the given equation

Q14. If $3\sin\theta + 5\cos\theta = 5$, then the value of $5\sin\theta - 3\cos\theta$ is equal to

(A) 5 (B) 3

(C) 4 (D) none of these

Sol: (B)

$3\sin\theta = 5(1 - \cos\theta) = 5 \times 2\sin^2\theta/2 \Rightarrow \tan\theta/2 = 3/5$

$$5\sin\theta - 3\cos\theta = 5\times\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} - 3\frac{\left(1-\tan^2\frac{\theta}{2}\right)}{1+\tan^2\frac{\theta}{2}} = 5\times\frac{2\times\frac{3}{5}}{1+\frac{9}{25}} - \frac{3\times\left(1-\frac{9}{25}\right)}{1+\frac{9}{25}} = 3$$

Q15. In a ΔABC, if cotA cotB cotC > 0, then the Δ is

(A) acute angled (B) right angled

(C) obtuse angled (D) does not exist

Sol: (A)

Since cotA cotB cotC > 0

cotA, cotB, cotC are positive $\Rightarrow \Delta$ is acute angled

Q16. The value of k for which the equation $x^2 + 2(k - 1)x + k + 5 = 0$ possess at least one positive root as

(A) $[4, \infty)$ (B) $(-\infty, -1) \cup [4, \infty)$

(C) $[-1, 4]$ (D) $(-\infty, -1)$

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Sol: (D)

Let $f(x) = x^8 - x^5 + x^2 - x + 1$

For $x < 0$, $f(x) > 0$

For $0 < x < 1$, $f(x) = x^8 + (x^2 - x^5) + (1 - x) > 0$

For $x > 1$, $f(x) = (x^8 - x^5) + (x^2 - x) + 1 > 0$

Hence $f(x) = 0$ has no real root.

Q17. Let $f(x) = (1 + b^2)x^2 + 2bx + 1$ and let m(b) be the minimum value of f(x). As b varies, the range of m(b) is

(A) [0, 1] (B) $\left[0, \frac{1}{2}\right]$

(C) $\left[\frac{1}{2}, 1\right]$ (D) (0, 1]

Sol: $1 + 2\log_2 x + \log_2 x(\log_2 x + 1) + 2\log_2^2 x + (\log_2 x)^3 = 1$

$t(t^2 + 3t + 3) = 0$

$\log_2 x = 0$

$x = 1$ rejected.

Q18. If a, b, c be positive real numbers forming a H.P., then $\frac{1}{b-a} + \frac{1}{b-c}$ is always equal to

(A) 2/a (B) 2/b

(C)2/c (D) None of these

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Sol: (B)

$$\frac{1}{b-a}+\frac{1}{b-c}=\frac{2b-(a+c)}{(b-a)(b-c)}$$

$$=\frac{2b-(a+c)}{b^2-b(a+c)+ac}$$

$$=\frac{2b-(a+c)}{b^2-2ac+ac}$$

$$=\frac{2b-(a+c)}{b^2-ac}$$

$$=\frac{2b-\frac{2ac}{b}}{b^2-ac}=\frac{2}{b}$$

Q19. If $a_1, a_2, a_3 \ldots a_n$ are in H. P, then $a_1a_2 + a_2a_3 + a_3a_4 + \ldots + a_{n-1}a_n$ is equal to

(A) $(n-1)\,a_1a_n$ (B) na_1a_n

(C) $\frac{n}{2}(a_1 + a_n)$ (D) none of these.

Sol: (A)

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$a_1, a_2, a_3, \ldots\ldots, a_n$ are in H. P. Then $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots\ldots, \frac{1}{a_n}$ are in A.P.

$$\Rightarrow \frac{1}{a_2} - \frac{1}{a_1} = \frac{1}{a_3} - \frac{1}{a_2} = \frac{1}{a_4} - \frac{1}{a_3} = \ldots\ldots = \frac{1}{a_n} - \frac{1}{a_{n-1}} = d \quad \text{(say)} \; \ldots. \text{(A)}$$

$$\Rightarrow a_1a_2 = \frac{a_1 - a_2}{d}, a_2a_3 = \frac{a_2 - a_3}{d}, a_3a_4 = \frac{a_3 - a_4}{d}, \ldots, a_{n-1}a_n = \frac{a_{n-1} - a_n}{d}$$

$$a_1a_2 + a_2a_3 + a_3a_4 + \ldots. + a_{n-1}a_n = \frac{1}{d}(a_1 - a_n). \ldots.. \quad \text{(B)}$$

If we add all (n-1) terms of (A), we get

$$\frac{1}{a_n} - \frac{1}{a_1} = (n-1)\, d \Rightarrow \frac{a_1 - a_n}{d} = (n-1)a_1a_n.$$

Thus from (B) $a_1a_2 + a_2a_3 + a_3a_4 + \ldots\ldots + a_{n-1}a_n = (n-1)\, a_1a_n$.

Q20. If A_1 be the A.M. and G_1, G_2 be two G.Ms between two positive numbers a and b, then $\frac{G_1^3 + G_2^3}{G_1G_2A_1}$ is equal to

(A) 2 (B) 1

(C) 2 (D) None of these

Sol: (C)

$$A_1 = \frac{a+b}{2}, G_1 = a.\left(\frac{b}{a}\right)^{1/3}, G_2 = a.\left(\frac{b}{a}\right)^{2/3}$$

$$G_1^3 = a^2b,\ G_2^3 = b^2a,\ G_1G_2 = a^2.\left(\frac{b}{a}\right) = ab$$

$$\Rightarrow \frac{G_1^3 + G_2^3}{G_1G_2A_1} = \frac{ab(a+b).2}{ab(a+b)} = 2$$

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Q21. If the sum to n terms of a series be $5n^2 + 2n$, then second term is

(A) 15 (B) 17

(C) 10 (D) 5

Sol: (B)

$S_n = 5n^2 + 2n$, $S_{n-1} = 5(n-1)^2 + 2(n-1)$

$\Rightarrow T_n = S_n - S_{n-1} = 10n - 3 \Rightarrow T_2 = 20 - 3 = 17$

Q22. $\angle A$, $\angle B$, $\angle C$ of a triangle ABC are in A.P. If a, b, c are the corresponding sides, then

(A) $b^2 + c^2 - bc = a^2$ (B) $a^2 + c^2 - ac = b^2$

(C) $b^2 + a^2 - ab = c^2$ (D) None of these

Sol: (B)

$2\angle B = \angle A + \angle C$

$\Rightarrow \angle B = \frac{\pi}{3}$

$\Rightarrow \cos\frac{\pi}{3} = \frac{a^2 + c^2 - b^2}{2ac} = \frac{1}{2}$

$\Rightarrow a^2 + c^2 - ac = b^2$

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Q23. Coefficient of x^5 in the expansion of $(1 + x^2)^5(1 + x)^4$ is

(A) 61 (B) 59

(C) 0 (D) 60

Sol: (D)

$(1 + x^2)^5(1 + x)^4 = (1 + 5x^2 + 10x^4 + ...)(1 + x)^4$

$\Rightarrow$ Coefficient of $x^5 = 5 \times {}^4C_3 + 10\times {}^4C_1 = 20 + 40 = 60$.

Q24. The sixth term in the expansion of $\left(\frac{1}{x^{8/3}} + x^2 \log_{10} x\right)^8$ is 5600 when x is equal to

(A) 10 (B) $\log_e 10$

(C) 1 (D) none of these

Sol: (A)

$T_6 = {}^8C_5\left(\frac{1}{x^{8/3}}\right)^3 (x^2 \log_{10} x)^5 \Rightarrow 56x^2(\log_{10} x)^5 = 5600$

$\Rightarrow x^2 (\log_{10} x)^5 = 100$, obviously x = 10 satisfies the above equation.

Hence (A) is the correct answer.

Q25. The term independent of x in $\left[\sqrt{\frac{x}{3}} + \sqrt{\left(\frac{3}{2x^2}\right)}\right]^{10}$ is

(A) 1 (B) 5/12

(C) ${}^{10}C_1$ (D) None of these

Sol: (D)

General term in the expansion is $^{10}C_r\left(\frac{x}{3}\right)^{\frac{r}{2}}\left(\frac{3}{2x^2}\right)^{\frac{10-r}{2}} = {}^{10}C_r\, x^{\frac{3r}{2}-10}\cdot\frac{3^{5-r}}{2^{\frac{10-r}{2}}}$

For constant term, $\frac{3r}{2} = 10 \Rightarrow r = \frac{20}{3}$

which is not an integer. Therefore, there will be no constant term.

Hence (D) is the correct answer.

Q26. If $(1+x+x^2)^n = a_0 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n}$, then the value of $a_0 + a_3 + a_6 + \ldots$ is

(A) $a_1 + a_4 + a_7 + \ldots$

(B) $a_1 + a_2 + a_3 + \ldots$

(C) 2^{n+1}

(D) none of these.

Sol:

$(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$

Put $x = w, w^2$ we get

$0 = (a_0 + a_3 + a_6 + \ldots) + w(a_1 + a_4 + a_7 + \ldots) + w^2(a_2 + a_5 + a_8 + \ldots)$ ... (1)

$0 = (a_0 + a_3 + a_6 + \ldots) + w^2(a_1 + a_4 + a_7 + \ldots) + w(a_2 + a_5 + a_8 + \ldots)$ ... (2)

From (1) and (2) we get,

$a_0 + a_3 + a_6 \ldots = a_1 + a_4 + a_7 + \ldots$

Q27. Without changing the direction of coordinates axes, origin is transferred to (α, β) so that the linear terms in the equation $x^2 + y^2 + 2x - 4y + 6 = 0$ are eliminated. The point (α, β) is

(A) (-1, 2)

(B) (1, -2)

(C) (1, 2)

(D) (-1, -2)

Sol: (A)

The given equation is $x^2 + y^2 + 2x - 4y + 6 = 0$ .....(1)
Putting $x = x' + \alpha$ and $y = y' + \beta$ in (1) we get $x'^2 + y'^2 + x'(2\alpha + 2) + y'(2\beta - 4) + (\alpha^2 + \beta^2 + 2\alpha - 4\beta + 6) = 0$. To eliminate linear term, we should have $2\alpha + 2 = 0$ and $2\beta - 4 = 0$
$\Rightarrow \alpha = -1$ and $\beta = 2$
$\Rightarrow (\alpha, \beta) \equiv (-1, 2)$

Q28. Equation $ax^2 + 2hxy + by^2 = 0$ represents a pairs of lines, combined equation of lines that can be obtained by taking the mirror of lines about the x-axis is

(A) $ax^2 + 2hxy + by^2 = 0$
(B) $bx^2 + 2hxy + ay^2 = 0$
(C) $bx^2 - 2hxy + ay^2 = 0$
(D) none of these

Sol: (D)

Let the lines represented by $ax^2 + by^2 + 2hxy = 0$ be $y = m_1x$ and $y = m_2x$, then $m_1 + m_2 = \frac{-2h}{b}$, $m_1m_2 = a/b$. If these lines reflected about the x-axis, there equation becomes $y + m_1x = 0$ and $y + m_2x = 0$ and their combined equation is $(y + m_1x)(y + m_2x) = 0$
$\Rightarrow y^2 + xy(m_1 + m_2) + m_1m_2x^2 = 0$
$\Rightarrow by^2 - 2hxy + ax^2 = 0$

Q29. If the line $= \sqrt{3}\,xy$ cuts the curve $x^3 + ax^2 + bx - 72 = 0$ at A, B and C, then OA. OB.OC (Where 'O' is origin) is

(A) 576
(B) –576
(C) a + b –c – 576
(D) a + b + c – 576

Sol: (A)

The line $y = \sqrt{3}x$ is passing through the origin and slope is $\sqrt{3}$, hence in parametric form the equation of given line can be written as

$$\frac{x}{1/2} = \frac{y}{\sqrt{3}/2} = r \qquad \text{.... (1)}$$

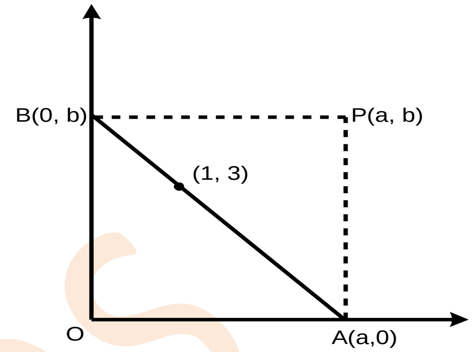
Any point on the line (1) is $\left(\frac{r}{2}, \frac{\sqrt{3}r}{2}\right)$. If the line cuts the given curve, then

$\frac{r^3}{8} + \frac{ar^2}{4} + \frac{br}{2} - 72 = 0$. This is a cubic equation in r. Roots of this equation r_1, r_2, r_3 will represent OA, OB and OC.
Therefore OA.OB.OC = $|r_1||r_2||r_3|$ = 576

Q30. A variable line drawn through the point (1, 3) meets the x-axis at A and y-axis at B. If the rectangle OAPB is completed, where 'O' is the origin, then locus of 'P' is

(A) $\frac{1}{y}+\frac{3}{x}=1$ (B) x + 3y = 1

(C) $\frac{1}{x}+\frac{3}{y}=1$ (D) 3x + y = 1



Sol: (C)

Let the line be $\frac{x}{a}+\frac{y}{b}=1$. Since the line is passing through (1, 3), hence $\frac{1}{a}+\frac{3}{b}=1$.

Now A = (a, 0), B = (0, b) $\Rightarrow$ P = (a, b)

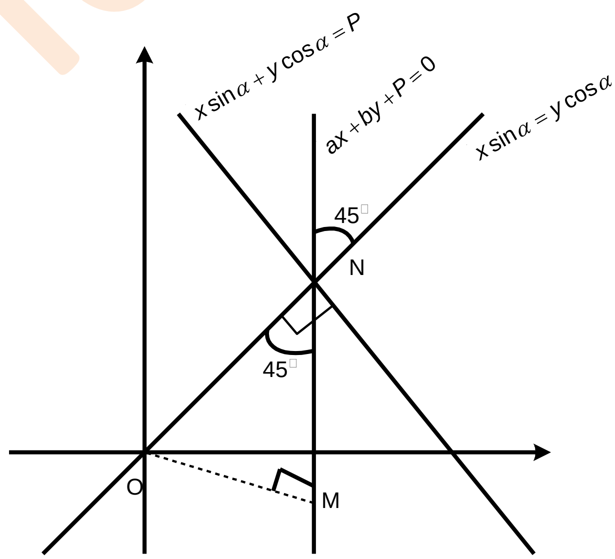
Thus locus of 'P' is $\frac{1}{x}+\frac{3}{y}=1$

14. If the straight lines ax + by + P = 0 and x cosα + y sinα = P are inclined at an angle $\pi/4$ and concurrent with straight line x sinα - ycos α = 0, then the value of $a^2 + b^2$ is

(A)1 (B) 0

(C)2 (D) 13

Sol (C)



ON = distance of origin from the line x sinα + y cosα = P

OM = Perpendicular distance of (0, 0) from the line ax + by + P = 0

$$\Rightarrow OM = -\frac{P}{\sqrt{a^2+b^2}}$$

Now OMN is a right angle triangle with $\angle ONM = \pi/4$

$$\Rightarrow OM = ON \sin \pi/4 = \frac{P}{\sqrt{2}}$$

$\Rightarrow a^2 + b^2 = 2$