

Class: XII
Subject: Mathematics
Topic: ASK15E11UT03
No. of Questions: 30
Duration: 90 Min
Maximum Marks: 90

1. If the third term of a G.P. is equal to 4, then product of it's first five terms is equal to:

- (A) 2^6 (B) 2^{10}
(C) 2^8 (D) None of these

Sol. (A)

The r^{th} term of the series is given by $T_r = (n - r + 1)r$

$$\text{Sum of the series} = \sum_{r=1}^n T_r = (n+1) \sum_{r=1}^n r - \sum_{r=1}^n r^2$$

$$\Rightarrow S_n = \frac{(n+1)n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$$

$$\Rightarrow S_n = \frac{n(n+1)}{2} \left[(n+1) - \frac{2n+1}{3} \right] = \frac{n(n+1)(n+2)}{6}$$

2. If a, b, c be in AP and a^2, b^2, c^2 be in HP, then $-\frac{a}{2}, b, c$ are in:

- (A) A.P. (B) G. P.
(C) H. P. (D) can't be said

Sol. (B)

$$\begin{aligned}\frac{1}{b-a} + \frac{1}{b-c} &= \frac{2b-(a+c)}{(b-a)(b-c)} \\ &= \frac{2b-(a+c)}{b^2 - b(a+c) + ac} \\ &= \frac{2b-(a+c)}{b^2 - 2ac + ac} \\ &= \frac{2b-(a+c)}{b^2 - ac} \\ &= \frac{2b - \frac{2ac}{b}}{b^2 - ac} = \frac{2}{b}\end{aligned}$$

3. Value of $\frac{1}{1.3.5} + \frac{1}{3.5.7} + \frac{1}{5.7.9} + \dots$ 'n' terms is equal to:

(A) $\frac{4n^2 + 8n}{12(2n+1)(2n+3)}$

(B) $\frac{1}{4} \left(\frac{1}{(2n+1)(2n+3)} - \frac{1}{3} \right)$

(C) $\frac{4n^2 + 6n}{12(2n+1)(2n+3)}$

(D) None of these

Sol. (A)

$a_1, a_2, a_3, \dots, a_n$ are in H. P. Then $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \dots, \frac{1}{a_n}$ are in A.P.

$$\Rightarrow \frac{1}{a_2} - \frac{1}{a_1} = \frac{1}{a_3} - \frac{1}{a_2} = \frac{1}{a_4} - \frac{1}{a_3} = \dots = \frac{1}{a_n} - \frac{1}{a_{n-1}} = d \text{ (say)} \dots (1)$$

$$\Rightarrow a_1 a_2 = \frac{a_1 - a_2}{d}, a_2 a_3 = \frac{a_2 - a_3}{d}, a_3 a_4 = \frac{a_3 - a_4}{d}, \dots, a_{n-1} a_n = \frac{a_{n-1} - a_n}{d}$$

$$a_1 a_2 + a_2 a_3 + a_3 a_4 + \dots + a_{n-1} a_n = \frac{1}{d} (a_1 - a_n) \dots (2)$$

If we add all (n-1) terms of (1), we get

$$\frac{1}{a_n} - \frac{1}{a_1} = (n-1) d \Rightarrow \frac{a_1 - a_n}{d} = (n-1) a_1 a_n$$

Thus from (2) $a_1 a_2 + a_2 a_3 + a_3 a_4 + \dots + a_{n-1} a_n = (n-1) a_1 a_n$.

4. If a, b, c and d are distinct positive numbers in H.P., then

- (A) $a+b > c+d$ (B) $a+c > b+d$
 (C) $a+d > b+c$ (D) none of these

Sol. (C)

$$A_1 = \frac{a+b}{2}, G_1 = a \cdot \left(\frac{b}{a}\right)^{1/3}, G_2 = a \cdot \left(\frac{b}{a}\right)^{2/3}$$

$$G_1^3 = a^2 b, G_2^3 = b^2 a, G_1 G_2 = a^2 \cdot \left(\frac{b}{a}\right) = ab$$

$$\Rightarrow \frac{G_1^3 + G_2^3}{G_1 G_2 A_1} = \frac{ab(a+b) \cdot 2}{ab(a+b)} = 2$$

5. If $a, b, c \in \mathbb{R}^+$ such that $a + b + c = 18$, then the maximum value of $a^2 b^3 c^4$ is equal to:

- (A) $2^{18} \cdot 3^2$ (B) $2^{18} \cdot 3^3$
 (C) $2^{19} \cdot 3^2$ (D) $2^{19} \cdot 3^3$

Sol. (B)

$$S_n = 5n^2 + 2n, S_{n-1} = 5(n-1)^2 + 2(n-1)$$

$$\Rightarrow T_n = S_n - S_{n-1} = 10n - 3 \Rightarrow T_2 = 20 - 3 = 17$$

6. H_1 is the H.M. and G_1 is the G.M. of positive real numbers 'a' and 'b'. If $H_1 : G_1 = 4 : 5$ then a : b is:

(A) 5 : 4

(B) 1 : 4

(C) 1 : 5

(D) None of these

Sol. (B)

$$2 \angle B = \angle A + \angle C$$

$$\Rightarrow \angle B = \frac{\pi}{3}$$

$$\Rightarrow \cos \frac{\pi}{3} = \frac{a^2 + c^2 - b^2}{2ac} = \frac{1}{2}$$

$$\Rightarrow a^2 + c^2 - ac = b^2$$

7. $A_i(x_i, y_i)$, $i = 1, 2, \dots, n$ is a point on the curve $y = 2\sqrt{x}$. If x_1, x_2, \dots, x_n are in G.P. with $x_1 = 1, x_2 = 2$, then y_n is equal to:

(A) $(\sqrt{2})^n$

(B) $(\sqrt{2})^{n+1}$

(C) 2^n

(D) None of these

Sol. (C)

$$a, b, c \text{ are in H.P.} \Rightarrow b = \frac{2ac}{a+c} \Rightarrow \frac{b}{a} = \frac{2c}{a+c} \Rightarrow \frac{b+a}{b-a} = \frac{3c+a}{c-a} \dots\dots (A)$$

Again a, b, c are in H.P.

$$\Rightarrow b = \frac{2ac}{a+c} \Rightarrow \frac{b}{c} = \frac{2a}{a+c} \Rightarrow \frac{b+c}{b-c} = \frac{3a+c}{a-c} \dots\dots (B)$$

$$\text{From (A) and (B)} \quad \frac{b+a}{b-a} + \frac{b+c}{b-c} = \frac{3c+a}{c-a} + \frac{3a+c}{a-c} = 2.$$

8. The least value of the expression $5^{\sin x-1} + 5^{-\sin x-1}$

- (A) $2/5$ (B) $1/5$
 (C) 5 (D) $5/2$

Sol. (C, D)

Let α be the first and β be the $(2n-1)$ terms of an A.P., G.P. and H.P., then α, a, β will be in A.P., α, b, β will be G.P., α, c, β will be in H.P.

Hence a, b, c are respectively A.M., G.M. and H.M. of α and β .

Since $A.M. \geq G.M. \geq H.M.$, $a \geq b \geq c$.

$$\text{Again } a = \frac{\alpha + \beta}{2}, b = \alpha\beta \text{ and } c = \frac{2\alpha\beta}{\alpha + \beta}. \text{ Hence } ac - b^2 = 0.$$

9. Sum of first 'n' terms of the sequence 5, 7, 11, 17, 25,..... is equal to:

- (A) $\frac{2n(n^2 + 4)}{3}$ (B) $\frac{n^2(n+4)}{2}$
 (C) $\frac{n}{6}(n^2 + 29)$ (D) $\frac{n}{6}(2n^2 + 28)$

Sol. (A)

$$\text{Using } A.M. \geq G.M. \quad \frac{1}{3} \left(\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \right) \geq \left[\left(\frac{a}{b} \right) \left(\frac{b}{c} \right) \left(\frac{c}{a} \right) \right]^{1/3} \Rightarrow \frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq 3.$$

10. If $\log_4 x + \log_4 \sqrt{x} + \log_4 \sqrt[4]{x} + \log_4 \sqrt[8]{x} + \dots + \log_4 \sqrt[n]{x} = \log_2 x$, then values of n is
- (A) 1 (B) 2
(C) 3 (D) infinite

Sol. (D)

$$2b = a + c$$

$$\Rightarrow 8b^3 = (a+c)^3 = a^3 + c^3 + 3ac(a+c)$$

$$\Rightarrow 8b^3 = a^3 + c^3 + 3ac(2b)$$

$$\Rightarrow a^3 + c^3 - 8b^3 = -6abc$$

11. If $\frac{b+c-2a}{a}, \frac{c+a-2b}{b}, \frac{a+b-2c}{c}$ are in A.P., then a, b, c are in
- (A) A.P (B) G.P
(C) H.P (D) none of these

Sol. (A)

$$S = \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \dots \infty = \sum_{r=1}^{\infty} \frac{1}{(3r-2)(3r+1)} = \frac{1}{3} \sum_{r=1}^{\infty} \left(\frac{1}{3r-2} - \frac{1}{3r+1} \right)$$
$$= \frac{1}{3} \left[\frac{1}{1} - \frac{1}{4} + \frac{1}{4} - \frac{1}{7} + \dots \right] = \frac{1}{3}$$

12. If a, b, c, d are in H.P., then $ab + bc + cd$ is equal to:

- (A) $a(2c + d)$ (B) $c(2a + d)$
(C) $b(2c + d)$ (D) None of these

Sol. (B)

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\Rightarrow \frac{a-b}{ab} = \frac{b-c}{bc}$$

$$\Rightarrow \frac{a-b}{b-c} = \frac{a}{c}$$

13. The co-efficient of y in the expansion of $(y^2+c/y)^5$ is

- (A) $10 c^3$ (B) $20 c^2$
(C) $10 c$ (D) $20 c$

Sol. (A)

$$(r + 1)^{\text{th}} \text{ terms} = {}^5C_r y^{10-2r} \cdot C^r \cdot y^{-r}$$

$$\text{power of } y = 1$$

$$\Rightarrow 10 - 3r = 1 \Rightarrow r = 3$$

$$\text{Required coefficient} = {}^5C_2 \cdot x^3 = 10 x^3$$

14. If the coefficients of x^2 and x^3 in the expansion of $(3 + kx)^9$ are equal, then the value of k is

- (A) $-\frac{9}{7}$ (B) $\frac{9}{7}$
(C) $\frac{7}{9}$ (D) None of these.

Sol. (B)

$$\begin{aligned} T_{r+1} \ln(3+kx)^9 &= {}^9C_r 3^{9-r} (kx)^r \\ &= {}^9C_r 3^{9-r} k^r x^r \end{aligned}$$

$$\therefore \text{Coefficient of } x^r = {}^9C_r 3^{9-r} k^r.$$

Now coefficient of x^2 = coefficient of x^3

$$\therefore {}^9C_2 3^{9-2} k^2 = {}^9C_3 3^{9-3} k^3$$

$$\Rightarrow 36 \times 3^7 k^2 = 84 \times 3^6 k^3$$

$$\Rightarrow 36 = 28k \Rightarrow k = \frac{9}{7}.$$

15. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$, then

$$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_n}{n+1} \text{ is equal to}$$

(A) 2^{n+1} (B) $\frac{2^{n+1} - 1}{n+1}$

(C) $\frac{2^{n+1}}{n+1}$ (D) $2^{n+1} - 1$

Sol. $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_n}{n+1}$

$$= 1 + \frac{1}{2} + \frac{1 \cdot 2}{3} + \dots + \frac{1}{n+1}$$

$$= 1 + \frac{n}{2!} + \frac{n(n-1)}{3!} + \dots + \frac{1}{n+1}$$

$$\begin{aligned}
 &= \frac{1}{n+1} \left[(n+1) + \frac{(n+1)n}{2!} + \frac{(n+1)n(n-1)}{3!} + \dots + 1 \right] \\
 &= \frac{1}{n+1} \left[{}^{n+1}C_1 + {}^{n+1}C_2 + {}^{n+1}C_3 + \dots + {}^{n+1}C_{n+1} \right] \\
 &= \frac{1}{n+1} \left[2^{n+1} - {}^{n+1}C_0 \right] = \frac{2^{n+1} - 1}{n+1}.
 \end{aligned}$$

Hence (B) is the correct answer.

16. The term independent of x in the expansion of $\left(2x + \frac{1}{3x}\right)^6$ is
- (A) $\frac{160}{9}$ (B) $\frac{80}{9}$
 (C) $\frac{160}{27}$ (D) $\frac{80}{3}$

Sol. $T_{r+1} = {}^6C_r (2x)^{6-r} \left(\frac{1}{3x}\right)^r = {}^6C_r \frac{2^{6-r}}{3^r} x^{6-2r}$

Let T_{r+1} be independent of x .

$$\therefore 6 - 2r = 0 \text{ or } r = 3$$

$$\therefore T_{r+1} = T_{3+1} = {}^6C_3 \frac{2^{6-3}}{3^3} x^{6-2(3)}$$

$$= \frac{20 \cdot 8}{27} = \frac{160}{27}$$

Hence (C) is the correct answer.

17. The middle term in the expansion of $(1+x)^{2n}$ is

- (A) ${}^{2n}C_n$ (B) ${}^{2n}C_{n+1}x^{n+1}$
 (C) ${}^{2n}C_{n-1}x^{n-1}$ (D) $\frac{1.3.5\dots(2n-1)}{n!} \times 2^n x^n$

Sol. (D)

$2n$ is even.

\therefore Middle term

$$= T_{\frac{2n+2}{2}} = T_{n+1} = {}^{2n}C_n 1^{2n-n} x^n = {}^{2n}C_n x^n$$

$$= \frac{2n!}{n!n!} x^n = \frac{1.2.3.4.5.6\dots 2n}{n!n!} x^n$$

$$= \frac{1.3.5\dots(2n-1) 2^n \cdot n!}{n!n!} x^n$$

18. If the binomial expansion of $(a+bx)^{-2}$ is $\frac{1}{4} - 3x + \dots$, where $a > 0$, then (a, b) is

- (A) (2, 12) (B) (2, 8)
 (C) (-2, 12) (D) None of these.

Sol. (A)

$$(a+bx)^{-2} = a^{-2} \left(1 + \frac{b}{a}x\right)^{-2}$$

$$= \frac{1}{a^2} \left[1 + (-2) \left(\frac{b}{a}x\right) + \dots\right] = \frac{1}{a^2} - \frac{2b}{a^3}x + \dots$$

Also, $(a + bx)^{-2} = \frac{1}{4} - 3x + \dots$

$\therefore \frac{1}{a^2} = \frac{1}{4} \dots (1)$ and $-\frac{2b}{a^3} = -3 \dots (2)$

(1) $\Rightarrow a^2 = 4 \Rightarrow a = 2$ and from (2) $b = 12$

19. $(4 - 5x^2)^{-1/2}$ can be expanded as a power series of x if

- (A) $|x| < \sqrt{5}/2$ (B) $|x| < 2/\sqrt{5}$
 (C) $-1 < x < 1$ (D) None of these

Sol. (B)

$$(4 - 5x^2)^{-1/2} = 4^{-1/2} \left(1 - \frac{5}{4}x^2\right)^{-1/2}$$

$$= \frac{1}{2} \left(1 + \left(-\frac{5}{4}x^2\right)\right)^{-1/2}$$

$$\left| -\frac{5}{4}x^2 \right| < 1 \text{ or } \left| -\frac{5}{4}x^2 \right| |x^2| < 1 \text{ or } \frac{5}{4}x^2 < 1$$

$$\text{or } x^2 < \frac{4}{5} \text{ or } |x| < \frac{2}{\sqrt{5}}$$

20. If the coefficient of m th, $(m + 1)$ th and $(m + 2)$ th terms in the expansion $(1 + x)^n$ are in A.P., then

(A) $n^2 + 4(4m + 1) + 4m^2 - 2 = 0$

(B) $n^2 + n(4m + 1) + 4m^2 + 2 = 0$

(C) $(n - 2m)^2 = n + 2$

(D) $(n + 2m)^2 = n + 2$

Sol. (C)

We have ${}^nC_{m-1}, {}^nC_m, {}^nC_{m+1}$ in A.P.

$$\Rightarrow 2 {}^nC_m = {}^nC_{m-1} + {}^nC_{m+1}$$

$$\Rightarrow \frac{2(n!)}{m!(n-m)!} = \frac{n!}{(m-1)!(n-m+1)!} + \frac{n!}{(m+1)!(n-m+1)!}$$

$$\Rightarrow \frac{2}{m(n-m)} = \frac{1}{(n-m+1)(n-m)} + \frac{1}{m(m+1)}$$

$$\Rightarrow 2(m+1)(n-m+1) = m(m+1) + (n-m+1)(n-m)$$

On simplification, we get

$$n^2 - 4mn + 4m^2 - n - 2 = 0 \Rightarrow (n - 2m)^2 = n + 2.$$

21. If n is a positive integer which of the following will always be integers?

I. $(\sqrt{2} + 1)^{2n} + (\sqrt{2} - 1)^{2n}$

II. $(\sqrt{2} + 1)^{2n} - (\sqrt{2} - 1)^{2n}$

III. $(\sqrt{2} + 1)^{2n+1} + (\sqrt{2} - 1)^{2n+1}$

IV. $(\sqrt{2} + 1)^{2n+1} - (\sqrt{2} - 1)^{2n+1}$

(A) only I and III

(B) only I and II

(C) only I and IV

(D) only II and III'

Sol. (C)

In I and IV only even powers of $\sqrt{2}$ occurs whereas in II and III only odd powers of $\sqrt{2}$ occurs.

22. Coefficient of x^5 in the expansion of $(1 + x^2)^5(1 + x)^4$ is
- (A) 61 (B) 59
 (C) 0 (D) 60

Sol. (D)

$$(1 + x^2)^5(1 + x)^4 = (1 + 5x^2 + 10x^4 + \dots)(1 + x)^4$$

$$\Rightarrow \text{Coefficient of } x^5 = 5 \times {}^4C_3 + 10 \times {}^4C_1 = 20 + 40 = 60.$$

23. The term independent of x in $\left[\sqrt{\frac{x}{3}} + \sqrt{\left(\frac{3}{2x^2}\right)} \right]^{10}$ is
- (A) 1 (B) 5/12
 (C) ${}^{10}C_1$ (D) None of these

Sol. General term in the expansion is ${}^{10}C_r \left(\frac{x}{3}\right)^{\frac{r}{2}} \left(\frac{3}{2x^2}\right)^{\frac{10-r}{2}} = {}^{10}C_r x^{\frac{3r}{2}-10} \cdot \frac{3^{5-r}}{2^{\frac{10-r}{2}}}$

For constant term, $\frac{3r}{2} = 10 \Rightarrow r = \frac{20}{3}$

Which is not an integer? Therefore, there will be no constant term.

Hence (D) is the correct answer.

24. If $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + \dots + a_{2n}x^{2n}$, then the value of $a_0 + a_3 + a_6 + \dots$ is
- (A) $a_1 + a_4 + a_7 + \dots$ (B) $a_1 + a_2 + a_3 + \dots$
 (C) 2^{n+1} (D) none of these.

Sol. (A)

$$(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$$

put $x = w$, w^2 we get

$$0 = (a_0 + a_3 + a_6 + \dots) + w (a_1 + a_4 + a_7 + \dots) + w^2 (a_2 + a_5 + a_8 + \dots) \quad \dots (1)$$

$$0 = (a_0 + a_3 + a_6 + \dots) + w^2 (a_1 + a_4 + a_7 + \dots) + w (a_2 + a_5 + a_8 + \dots) \quad \dots (2)$$

from (1) and (2) we get,

$$a_0 + a_3 + a_6 \dots = a_1 + a_4 + a_7 + \dots$$

25. The value of ${}^{2n}C_n - {}^nC_1 \cdot {}^{2n-2}C_n + {}^nC_2 \cdot {}^{2n-4}C_n - \dots$ is equal to
- (A) 3^n (B) 4^n
 (C) 5^n (D) none of these

Sol (D)

$$\begin{aligned} & {}^{2n}C_n - {}^nC_1 \cdot {}^{2n-2}C_n + {}^nC_2 \cdot {}^{2n-4}C_n - \dots \\ &= \text{coefficient of } x^n \text{ in } [{}^nC_0 (1+x)^{2n} - {}^nC_1 (1+x)^{2n-2} + {}^nC_2 (1+x)^{2n-4} - \dots] \\ &= \text{coefficient of } x^n \text{ in } [1 - (1+x)^2]^n = 2^n \end{aligned}$$

26. If $|x| < 1$, then the coefficient of x^n in the expansion of $(1 + x + x^2 + x^3 + \dots)^2$ is
- (A) n (B) $n-1$
 (C) $n+2$ (D) $n+1$

Sol. (D)

$$(1 + x + x^2 + x^3 + \dots)^2 = \left(\frac{1}{1-x}\right)^2 = (1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots$$

coefficient of $x^n = (n + 1)$

27. If $(1+ax)^n = 1+8x+24x^2+\dots$ then

(A) $a=3$

(B) $a=5$

(C) $a=2$

(D) none of these

Sol. (C)

$$(1 + ax)^n = 1 + nax + \frac{n(n-1)}{2} a^2 x^2 + \dots = 1 + 8x + 24x^2 + \dots \Rightarrow na = 8$$

$$n(n-1)a^2 = 48 \Rightarrow n = 4, a = 2$$

28. The two successive terms in the expansion of $(1+x)^{24}$ whose coefficients are in the ratio 4 : 1 are

(A) 3rd and 4th

(B) 4th and 5th

(C) 5th and 6th

(D) 6th and 7th

Sol: (C)

Let the coefficient of successive terms be ${}^{24}C_r$ and ${}^{24}C_{r+1}$, then

$$\frac{{}^{24}C_r}{{}^{24}C_{r+1}} = 4 \Rightarrow \frac{r+1}{(24-r)} = 4 \Rightarrow r = 19$$

$${}^{24}C_{19}, {}^{24}C_{20} \Rightarrow {}^{24}C_5, {}^{24}C_4 \Rightarrow 6^{\text{th}} \text{ and } 5^{\text{th}} \text{ terms}$$

29. The co-efficient of x^k ($0 \leq k \leq n$) in the expansion of $E = 1+(1+x) + (1+x)^2 + \dots + (1+x)^n$ is
- (A) ${}^{n+1}C_{k+1}$ (B) nC_k
(C) ${}^{n+1}C_{n-k-1}$ (D) none of these

Sol. (A)

$$E = \frac{(1+x)^{n+1} - 1}{(1+x) - 1} = \frac{{}^{n+1}C_0 + {}^{n+1}C_1x + {}^{n+1}C_2x^2 + \dots - 1}{x}$$
$$= {}^{n+1}C_1 + {}^{n+1}C_2 x + {}^{n+1}C_3 x^2 + \dots$$

Coefficient of $x^k = {}^{n+1}C_{k+1}$

30. The coefficient of x^n in $\left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots + \frac{(-1)^n x^n}{n!}\right)^2$ is
- (A) $\frac{(-n)^n}{n!}$ (B) $\frac{(-2)^n}{n!}$
(C) $\frac{1}{(n!)^2}$ (D) $-\frac{1}{(n!)^2}$

Sol. (B)

$$\text{Coefficient of } x^n \text{ in } \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots + \frac{(-1)^n x^n}{n!} \right)^2$$

$$\text{Coefficient of } x^n \text{ in } \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots \right)^2$$

$$\text{Coefficient of } x^n \text{ in } (e^{-x})^2$$

$$\text{Coefficient of } x^n \text{ in } e^{-2x} = \frac{(-2)^n}{n!}$$

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