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Class: 11
Subject: Mathematics
Topic: ASK15E11UT04
No. of Questions: 30

Q1. Tangents to the circle $x^2 + y^2 = a^2$ cut the circle $x^2 + y^2 = 2a^2$ at P and Q. The tangents at P and Q to the circle $x^2 + y^2 = 2a^2$ intersect at

(A) right angles (B) 60°

(C) can't be determined (D) none of the above.

Sol. (A)

Equation of tangent at any point $(a\cos\theta, a\sin\theta)$ is

$x\cos\theta + y\sin\theta = a$ ……(1)

Let the point of intersection of the tangents at P and Q be (h, k). If the tangents at P and Q intersect at right angles, then locus of (h, k) will be director circle of $x^2 + y^2 = 2a^2$ i.e. $x^2 + y^2 = 4a^2$. PQ is chord of contact of the circle $x^2 + y^2 = 2a^2$ w.r.t. the point (h, k) i.e. equation of PQ is $hx + ky = 2a^2$ ……(2)

(1) and (2) are same equation $\dfrac{\cos\theta}{h} = \dfrac{\sin\theta}{k} = \dfrac{a}{2a^2} = \dfrac{1}{2a}$

$\cos^2\theta + \sin^2\theta = 1 \Rightarrow h^2 + k^2 = 4a^2$

$\therefore$ locus of (h,k) is $x^2 + y^2 = 4a^2$ which is director circle to $x^2 + y^2 = 2a^2$

Q2. If $\frac{x}{a}+\frac{y}{b}=1$ touches the circle $x^2+y^2=r^2$, then prove that the point $\left(\frac{1}{a}, \frac{1}{b}\right)$ lies.

(A) on a circle (B) in a circle

(C) on a straight line (D) none of these.

Sol. (A)

$\frac{x}{a}+\frac{y}{b}=1$ touches $x^2+y^2=r^2$

$$\Rightarrow \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}=r \Rightarrow \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{r^2}$$

$\Rightarrow \left(\frac{1}{a}, \frac{1}{b}\right)$ lies on $x^2+y^2=\frac{1}{r^2}$, which is a circle.

Q3. If the line $y=\sqrt{3}x$ cuts the curve $x^3+ax^2+bx-72=0$ at A, B and C, then OA. OB.OC (Where 'O' is origin) is

(A) 576 (B) –576
(C) a + b –c – 576 (D) a + b + c – 576

Sol. (A)

The line $y=\sqrt{3}x$ is passing through the origin and slope is $\sqrt{3}$, hence in parametric form the equation of given line can be written as

$$\frac{x}{1/2}=\frac{y}{\sqrt{3}/2}=r \qquad \text{....(1)}$$

Any point on the line (1) is $\left(\frac{r}{2}, \frac{\sqrt{3}r}{2}\right)$. If the line cuts the given curve, then

$\frac{r^3}{8}+\frac{ar^2}{4}+\frac{br}{2}-72=0$. This is a cubic equation in r. Roots of this equation r_1, r_2, r_3 will represent OA, OB and OC.

Therefore OA.OB.OC = $|r_1||r_2||r_3|$ = 576

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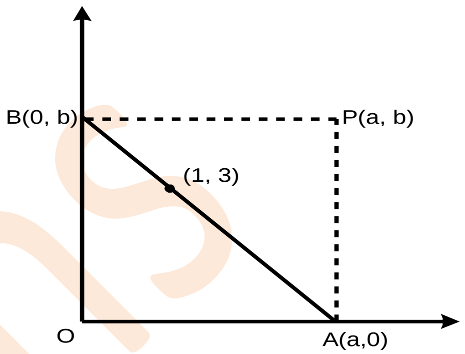
Q4. A variable line drawn through the point (1, 3) meets the x-axis at A and y-axis at B. If the rectangle OAPB is completed, where 'O' is the origin, then locus of 'P' is

(A) $\frac{1}{y}+\frac{3}{x}=1$ (B) x + 3y = 1

(C) $\frac{1}{x}+\frac{3}{y}=1$ (D) 3x + y = 1



Sol. (C)

Let the line be $\frac{x}{a}+\frac{y}{b}=1$. Since the line is passing through (1, 3), hence $\frac{1}{a}+\frac{3}{b}=1$.

Now A = (a, 0), B = (0, b) $\Rightarrow$ P = (a, b)

Thus locus of 'P' is $\frac{1}{x}+\frac{3}{y}=1$

Q5. If the straight lines ax + by + P = 0 and x cosα + y sinα = P are inclined at an angle $\pi/4$ and concurrent with straight line x sinα - ycos α = 0, then the value of $a^2 + b^2$ is

(A)1 (B) 0
(C)2 (D) 13

Sol. (C)

ON = distance of origin from the line x sinα + y cosα = P
OM = Perpendicular distance of (0, 0) from the line ax + by + P = 0

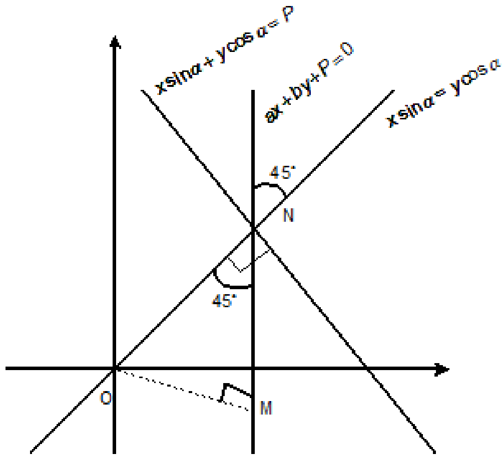
$$\Rightarrow OM = -\frac{P}{\sqrt{a^2+b^2}}$$

Now OMN is a right angle triangle with $\angle$ONM = $\pi/4$

$\Rightarrow$ OM = ON sin $\pi/4$ = $\frac{P}{\sqrt{2}}$

$\Rightarrow a^2 + b^2 = 2$

Hence (C) is the correct answer.



Q6. The least value of $|\alpha|$, for which the lines $x = \alpha + m$, $y = -2$ and $y = mx$ are concurrent, is

(A) $\sqrt{2}$
(B) $2\sqrt{2}$
(C) 0
(D) 1

Sol. (B)

Since lines are concurrent, point of intersection of lines $x = \alpha + m$ and $y = -2$ which is $(\alpha + m, -2)$ will also satisfy $y = mx$

$\Rightarrow m^2 + m\alpha + 2 = 0$

Since m is real $\alpha^2 - 8 \geq 0 \Rightarrow |\alpha| \geq 2\sqrt{2}$.

Hence the least value of $|\alpha| = 2\sqrt{2}$.

Q7. If the sum of the distances of a point from two perpendicular lines in a plane is 1, then its locus is

(A) square
(B) circle
(C) straight line
(D) two intersection lines

Sol. (A)

Let the perpendicular line be $x = 0$ and $y = 0$.

According to given condition $|x| + |y| = 1$

Which represent a square.

Q8. If the line y – 2 = m (x – 1) cuts the circle $x^2 + y^2 = 9$ at two real point, then the number of possible values of m is

(a) 1 (b) 2
(c) infinite (d) 0

Sol. (C)

Since the given line always passes through (1, 2), the interior point of the circle, so m can have any real value.

Q9. The range of values of λ for which the circle $x^2 + y^2 = 4$ and $x^2 + y^2 - 4\lambda x + 9 = 0$ have two common tangent is

(a) $\lambda \in \left[-\frac{13}{8}, \frac{13}{8}\right]$ (b) $\lambda > \frac{13}{8}$ or $\lambda < \frac{13}{8}$
(c) $k\lambda < \frac{13}{8}$ (d) none

Sol. Key concept: There will exist two common tangents when both the circles are intersecting. Hence apply the condition.

(A)

Solving the equation $4 - 4\lambda x + 9 = 0 \Rightarrow x = \frac{13}{4\lambda}$

$\Rightarrow x^2 + \left(\frac{13}{4\lambda}\right)^2 = 4$ or $x = \pm\sqrt{4 - \left(\frac{13}{4\lambda}\right)^2}$. It should have two real and distinct values so $4 - \left(\frac{13}{4\lambda}\right)^2 > 0 \Rightarrow \lambda \in \left(-\frac{13}{8}, -\frac{13}{8}\right)$

Q10. The equation of a circle of radius 1 touching the circles $x^2 + y^2 - 2|x| = 0$ is

(a) $x^2 + y^2 + 2\sqrt{3}x - 2 = 0$ (b) $x^2 + y^2 - 2\sqrt{3}y + 2 = 0$

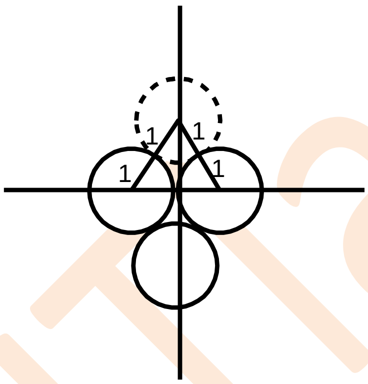
(c) $x^2 + y^2 + 2\sqrt{3}y + 2 = 0$ (d) Both B and C

Approach: First simplify the equation of the given circle and then plot its graph.

Sol. (D)

The given circles are $x^2 + y^2 - 2x = 0$, $x > 0$ and $x^2 + y^2 + 2x = 0$, $x < 0$

From the figure the centre of the required circle will be $(0, \sqrt{3})$ and $(0, -\sqrt{3})$.



Q11. Radius of smaller circle that touches the line $y = x$ ot (1, 1) and also touches the x–axis is :

(a) $2 - \sqrt{2}$ (b) $2 + \sqrt{2}$

(c) $\sqrt{2} - 1$ (d) $1 + \sqrt{2}$

Sol. (A)

Since $OA = OP \Rightarrow p \in (\sqrt{2}, 0)$

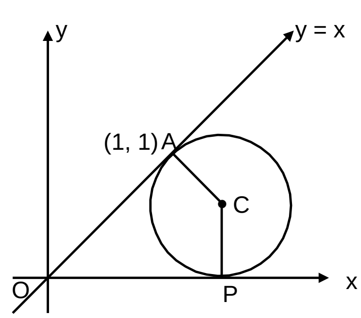
equation of AC $(y - 1) = -1 (x - 1)$

i.e. $y = -x + 2$

equation of PC $x = \sqrt{2}$

$\Rightarrow C = (\sqrt{2}, 2 - \sqrt{2})$

Thus radius is $2 - \sqrt{2}$



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Q12. A circle touches the lines $y = \frac{x}{\sqrt{3}}, y = \sqrt{3}x$ and has unit radius. If the centre of this circle lies in the first quadrant then one possible equation of this circle is

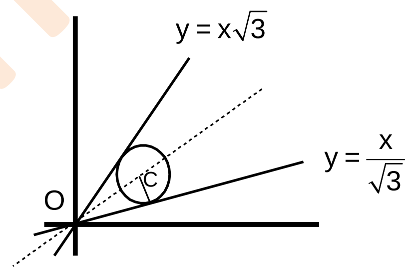
(a) $x^2 + y^2 - 2x(\sqrt{3}+1) - 2y(\sqrt{3}+1) + 8 + 4\sqrt{3} = 0$

(b) $x^2 + y^2 - 2x(\sqrt{3}+1) - 2y(\sqrt{3}+1) + 5 + 4\sqrt{3} = 0$

(c) $x^2 + y^2 - 2x(\sqrt{3}+1) - 2y(\sqrt{3}+1) + 7 + 4\sqrt{3} = 0$

(d) $x^2 + y^2 - 2x(\sqrt{3}+1) - 2y(\sqrt{3}+1) + 6 + 4\sqrt{3} = 0$

Sol. (c)



Angle between lines is 60° – 30° = 30°. Thus equation of their acute angle bisector is y tan (30° + 15°). x i.e., y = x .

let C = (h, h) then $\frac{|h - h\sqrt{3}|}{2} = 1 \Rightarrow (\sqrt{3}-1)h = 2$

$$\Rightarrow h = \frac{2}{(\sqrt{3}-1)} = \frac{2(\sqrt{3}+1)}{2} = (\sqrt{3}+1)$$

thus equation of circle is

$$\left(x - (\sqrt{3}+1)\right)^2 + \left(y - (\sqrt{3}+1)\right)^2 = 1$$

$$\Rightarrow x^2 + y^2 - 2x(\sqrt{3}+1) - 2y(\sqrt{3}+1) + 7 + 4\sqrt{3} = 0$$

Q13. If the circles $x^2 + y^2 - 8x + 2y + 8 = 0$ and $x^2 + y^2 - 2x - 6y - a^2 = 0$ have exactly two common tangents, then

(a) 1<|a|<8 (b) 2<|a|<8

(c) 3<|a|<8 (d) 4<|a|<8

Sol. (b)

Centres of the circles are (4, –1) and (1, 3). Their radii are 3 and $|a|$ respectively. They will have exactly two common tangents if they meet in two distinct points. That means

$$|3 - |a|| < \sqrt{3^2 + 4^2} < 3 + |a|$$

$$\Rightarrow |3 - |a|| < 5 < 3 + |a|$$

From $|3 - |a|| > 5$ we get a $\in (-\infty, -2) \cup (2, \infty)$

From $|3 + |a|| < 5$ we get a $\in (-8, 8)$

$$a \in (-8, -2) \cup (2, 8) \text{ i.e. } 2 < |a| < 8$$

Q14. If (a, 0) is a point on a diameter of the circle $x^2+y^2=4$, then $x^2 - 4x - a^2 = 0$ has

(a) exactly one real root in (-2, -1]

(b) exactly one real root in [2, 5]

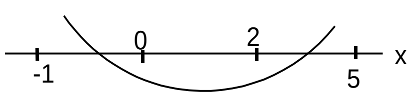
(c) distinct roots greater than 1

(d) distinct roots less than -1

Sol. (b)

Since (a, 0) is a point on the diameter of the circle $x^2 + y^2 = 4$,

so maximum value of a^2 is 4



Let $f(x) = x^2 - 4x - a^2$
clearly $f(-1) = 5 - a^2 > 0$, $f(2) = -(a^2 + 4) < 0$

$f(0) = -a^2 < 0$ and $f(5) = 5 - a^2 > 0$

so graph of f(x) will be as shown

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Q15. If A is a point on the circle $x^2 + y^2 - 4x + 6y - 3 = 0$. which is farthest from the point (7,2) , then

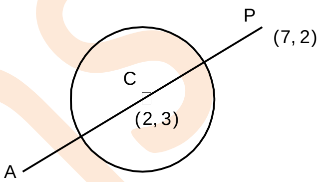
(a) $(2 - 2\sqrt{2}, -3 - 2\sqrt{2})$ (b) $(2 + 2\sqrt{2}, -3 - 2\sqrt{2})$

(c) $(2 + 2\sqrt{2}, -3 + 2\sqrt{2})$ (d) $(2 - 2\sqrt{2}, -3 + 2\sqrt{2})$

Sol. (a)

Given circle is $(x-2)^2 + (y+3)^2 = 16$

slope of PA $= \dfrac{2+3}{7-2} = 1$



Parametric equation of line PC is

$$\frac{x-2}{\frac{1}{\sqrt{2}}} = \frac{y+3}{\frac{1}{\sqrt{2}}} = r \Rightarrow (x, y) = \left(2 + \frac{r}{\sqrt{2}}, -3 + \frac{r}{\sqrt{2}}\right)$$

Hence the point is $\left(2 - 2\sqrt{2}, -3 - 2\sqrt{2}\right)$

Q16. Length of the common chord of circles $x^2 + y^2 - 2x - 4y + 1 = 0$ and $x^2 + y^2 - 4x - 2y + 1 = 0$ is

(a) $\sqrt{14}$ (b) $\sqrt{15}$

(c) 4 (d) $\sqrt{17}$

Sol. (a)

Equation of common chord is $s1 - s2 = 0$

$2x - 2y = 0 \Rightarrow x - y = 0$

$$PP' = \frac{\sqrt{P'C^2 - PC^2}}{4 - \frac{1}{2}}$$

Length of chord $= \dfrac{2\sqrt{7}}{\sqrt{2}} = \sqrt{14}$

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Q17. The straight lines $L_1 \equiv 4x - 3y + 2 = 0$, $L_2 \equiv 3x + 4y - 4 = 0$

and $L_3 \equiv x - 7y + 6 = 0$

(A) form a right angled triangle
(B) form a right angled isosceles triangle
(C) are concurrent
(D) none of these

Sol. (B)

Since the line L_1 and L_2 are perpendicular and angle between L_1 and L_2 is 45°

Hence the triangle will be right angled isosceles.

Q18. If a,b,c are in H.P. then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point, that point is

(A) (-1,-2)
(B) (-1, 2)
(C) (1, -2)
(D) (1,-1/2)

Sol. (C)

Since a, b, c are in H.P.

$$\Rightarrow \frac{2}{b}=\frac{1}{a}+\frac{1}{c}$$

Hence $\frac{x}{a}+\frac{y}{b}+\frac{2}{b}-\frac{1}{a}=0$

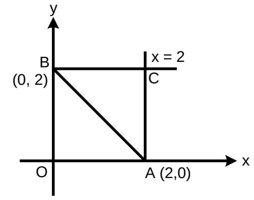
$$\Rightarrow (x-1)+\frac{a}{b}(y+2)=0$$

Q19. The extremitas of the base of an isosceles triangle are (2, 0) and (0, 2). If the equation of one of the equal sides is x = 2 then equation of the other equal side is

(A) x = y = 2
(B) x – y + 2
(C) y = 2
(D) 2x + y = 2

Sol. (c)

Adjacent figure represents the given isoceles triangle. Clearly the equation of other equal side is y = 2.



Q20. If coordinate axes are the angle bisectors of the pair of lines $ax^2 + 2hxy + by^2$, then

(A) a – b = 0
(B) $a^2 + b^2 = 0$
(C) h = 0
(D) $b^2 + a = 0$

Sol. (C)

Equations of angle bisectors of the given lines is $\dfrac{x^2 - y^2}{a - b} = \dfrac{xy}{h}$

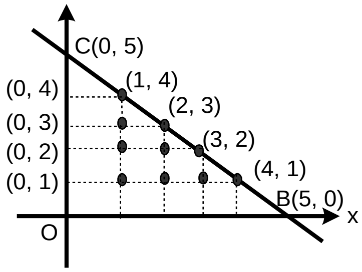
It should be same as xy = 0 $\Rightarrow h = 0$

Q21. P (x, y) is called a good point if x, y $\in$ N. Total number of good points lying inside the quadrilateral formed by the line 2x + y = 2, x = 0, y = 0 and x + y = 5, is equal to

(A) 4
(B) 2
(C) 6
(D) 1

Sol. (C)

Adjacent figure indicates that there are exactly six good points inside the quadrilateral ABCD.



Q22. Consider a family of straight lines (x + y) + λ (2x – y + 1) = 0. Equation of the straight line belonging to this family that is farthest form (1, –3) is

(A) 13y + 6x = 7 (B) 15y + 6x = 7
(C) 13y – 6x = 7 (D) 15y – 6x = 7

Sol. (D)

Given family is concurrent at $'P'\left(-\frac{1}{3}, \frac{1}{3}\right)$.If Q = (1, –3) then $m_{PQ} = \frac{-3-\frac{1}{3}}{1+\frac{1}{3}} = -\frac{5}{2}$.

Now member of the family that is farthest from 'Q' will have it's slope as $\frac{2}{5}$.

$$\Rightarrow \frac{2}{5} = -\frac{(1+2\lambda)}{(1-\lambda)} \Rightarrow \lambda = -\frac{7}{8}$$

Thus equation of required line is

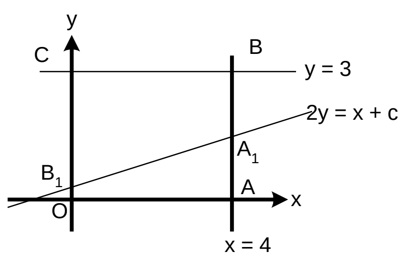
$$(x+y) - \frac{7}{8}(2x - y + 1) = 0$$

i.e. 15y – 6x – 7 = 0

Q23. The sides of a rectangle are x = 0, y = 0, x = 4, and y = 3. The equation of straight line having slope $\frac{1}{2}$ that divides the rectangle in to two equal halves is

(A) 2y = x + 1 (B) 2x = y + 1
(C) 2y + x = 1 (D) 2x + y = 1

Sol. (A)



Let the required line be 2y = x + c.

We have $A_1\left(4, \frac{4+C}{2}\right)$, $B_1\left(0, \frac{c}{2}\right)$ Area of rectangle ABCD is 12 sq. units

Thus area of trapezium OAA_1B is

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$$6 = \frac{1}{2}(4)\left(\frac{c}{2} + \frac{4+C}{2}\right) \Rightarrow c = 1$$

Thus required line is 2y = x + 1

Q24. If the line y = $\sqrt{3}$ x cuts the curve $x^3 + y^2 + 3x^2 + 8 = 0$ at the points A, B, C, then OA.OB.OC (O being origin) equals

(A) -32 (B) -64

(C) 108 (D) -100

Sol. (B)

Parametric equation of line

$$y = \sqrt{3}x \text{ is } \frac{x}{\frac{1}{2}} = \frac{y}{\frac{\sqrt{3}}{2}} = r$$

Any point on this line $\left(\frac{r}{2}, \frac{\sqrt{3}r}{2}\right)$

At point of intersection this point will also satisfy $x^3 + y^2 + 3x^2 + 8 = 0$

$$\Rightarrow \frac{r^3}{8} + \frac{3r^2}{4} + \frac{3r^2}{4} + 8 = 0$$

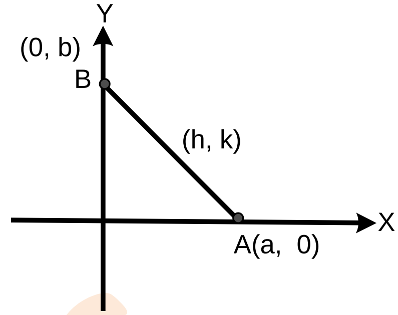
$$\Rightarrow r^3 + 12r^2 + 64 = 0$$

$$\Rightarrow OA.OB.OC = -64$$

Q25. Two points A and B move on the x –axis and the y – axis respectively such that the distance between the two points is always same. The locus of middle point of AB is

(A) a straight line (B) a circle

(C) a parabola (D) an ellipse



Sol. (B)

Given $a^2 + b^2 = P^2$ (P is constant)

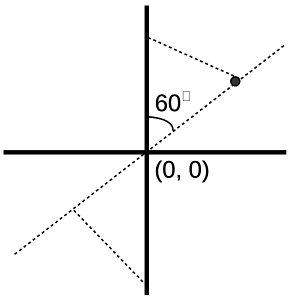
$$h = \frac{a}{2},\ k = \frac{b}{2}$$

Hence $4h^2 + 4k^2 = P^2$

$$\Rightarrow x^2 + y^2 = \frac{P^2}{4}$$

Q26. If one vertex of an equilateral triangle of side 2 is the origin and another vertex lies on the line $x = \sqrt{3}\,y$ then third vertex can be

(A) (0, 2)
(B) $(-\sqrt{3}, -1)$
(C) (0, -2)
(D) $(\sqrt{3}, 1)$



Sol. (A)

Clearly the third vertex will lie on the y-axis. Hence the points are (0, 2) or (0, –2)

Q27. A line passing through the point (2, 2) and the axes enclose an area λ. The intercepts on the axes made by the line are given by the two roots of

(A) $x^2 - 2|\lambda|x + |\gamma| = 0$
(B) $x^2 + |\lambda|x + 2|\gamma| = 0$
(C) $x^2 - |\lambda|x + 2|\gamma| = 0$
(D) none of these

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Sol. (C)

Area $\frac{1}{2}ab = |\lambda|$ ……(i)

Equation of line passing through (2, 2)

$\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{2}{a} + \frac{2}{b} = 1$ ……(ii)

2(a + b) = ab = $2|\lambda|$

$\Rightarrow a + b = |\lambda|$

$a(|\lambda| - a) = 2|\lambda|$

$\Rightarrow a^2 - a|\lambda| + 2|\lambda| = 0$

Here intercepts on axes made by the line are given by $x^2 - |\lambda| x + 2|\lambda| = 0$

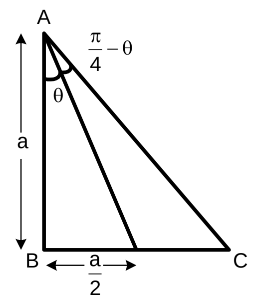
Q28. In an isosceles right angled triangle, a straight line drawn from the mid-point of one of equal sides to the opposite angle. It divides the angle into two parts, θ and $\frac{\pi}{4} - \theta$. Then $\tan\theta$ and $\tan\frac{\pi}{4} - \theta$ are equal to

(A) ½,1/3 (B) 1/3,1/4

(C) 1/5,1/6 (D) none of these

Sol. (A)

Clearly $\tan\theta = \frac{1}{2}$ and $\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1}{3}$



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Q29. Area of the triangle formed by the line x + y = 3 and angle bisector of the pair of straight lines $x^2 - y^2 + 2y = 1$ is

(A) 2 sq.units (B) 4 sq.units

(C) 6 sq.units (D) 8 sq.units

Sol. (A)

The equation $x^2 - y^2 + 2y = 1$ represents x – y + 1 = 0 and x + y -1 = 0

Hence equation of their angle bisector are x = 0 and y = 1.

Q30. Let P ≡ (-1, 0), Q ≡ (0, 0) and R ≡ (3, 3√3) be three points. Then the equation of the bisector of the angle PQR is

(A) $\frac{\sqrt{3}}{2}.x + y = 0$ (B) $x + \sqrt{3}.y = 0$

(C) $\sqrt{3}x + y = 0$ (D) $x + \frac{\sqrt{3}}{2}y = 0$

Sol. (C)

Inclination as QR is 60°, clearly angle bisector will be inclined at an angle of 120° with the x-axis.