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Class: 11
Subject: Mathematics
Topic: ASK15E11UT05
No. of Questions: 30

Q1. The number of normals drawn from a point (3, 0) to the parabola $y^2 = 4x$ is /are

(A) 1 (B) 2
(C) 3 (D) none of these

Sol. (C)
$y = mx - 2m - m^3$, this passes through (3, 0) hence
$0 = 3m - 2m - m^3 \Rightarrow m^3 - m = 0 \Rightarrow m = 0, m = \pm 1$
Hence three normal can be drawn.

Q2. If a normal chord to the parabola $y^2 = 4x$ is drawn at (1, 2), then the chord meets the parabola again at:

(A) (9, 6) (B) (9, –6)
(B) (6, 6) (D) none of these

Sol. (B)
For point (1, 2) value of t = 1,
The value of t_2 for other end of the normal is $t_2 = -1 - \frac{2}{1} = -3$.
So point is $(at^2, 2at) = (9, -6)$

Q3. The area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum is

(A) 12 sq. units (B) 16 sq. units
(C) 18 sq. units (D) 24 sq. units

Sol. (C)
Required area is given by $\Delta = \frac{1}{2}(12x3) = 18$ sq. units

Q4. If the line $x + y - 1 = 0$ touches the parabola $y^2 = kx$, then the value of k is

(A) 4 (B) –4
(C) 2 (D) –2

Sol. (B)

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Any tangent to $y^2 = kx$ is ;
$y = mx + k/4m$
Comparing it with the given line $y = 1 - x$,
we get, $m = -1$ and $k/4m = 1 \Rightarrow k = -4$
Alternative Solution:
If $x + y - 1 = 0$ touches $y^2 = kx$, then $y^2 = k(1-y)$ would have equal roots
$\Rightarrow k^2 + 4k = 0$
$\Rightarrow k = 0$ or -4 but $k \neq 0$,
Hence $k = -4$

Q5. The coordinates of the point on the parabola $y = x^2 + 7x + 2$, which is nearest to the straight line $y = 3x - 3$ are

(A) $(-2, -8)$ (B) $(1, 10)$
(C) $(2, 20)$ (D) $(-1, -4)$

Sol. (A) Any point on the parabola is $(x, x^2 + 7x + 2)$
Its distance from the line $y = 3x - 3$ is given by

$$P = \left|\frac{3x - (x^2 + 7x + 2) - 3}{\sqrt{9+1}}\right| = \left|\frac{x^2 + 4x + 5}{\sqrt{10}}\right|$$

$$= \frac{x^2 + 4x + 5}{\sqrt{10}}$$ (as $x^2 + 4x + 5 > 0$ for all $x \in R$)

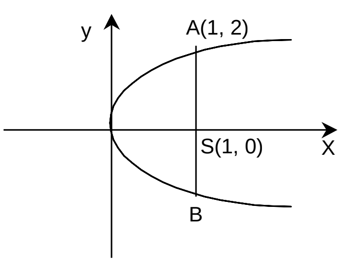
$$\frac{dP}{dx} = 0 \Rightarrow x = -2$$

The required point $\equiv (-2, -8)$

Q6. The point (1, 2) is one extremity of focal chord of parabola $y^2 = 4x$. The length of this focal chord is

(A) 2 (B) 4
(C) 6 (D) none of these

Sol. (B)
The parabola $y^2 = 4x$. Here $a = 1$ and focus is (1, 0).
The focal chord is ASB. This is clearly latus rectum of parabola, its value = 4.



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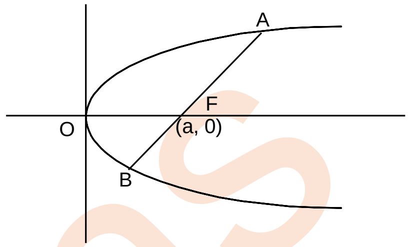
Q7. If AFB is a focal chord of the parabola $y^2 = 4ax$ and AF = 4, FB = 5, then the latus-rectum of the parabola is equal to

(A) $\frac{80}{9}$ (B) $\frac{9}{80}$

(C) 9 (D) 80



Sol. (A)

FA = 4, FB = 5

We know that $\frac{1}{a} = \frac{1}{AF} + \frac{1}{FB}$

$\Rightarrow a = \frac{20}{9} \Rightarrow 4a = \frac{80}{9}$.

Q8. Vertex of the parabola whose parametric equation is $x = t^2 - t + 1,\ y = t^2 + t; t \in R, is$

(A) (1, 1) (B) (2, 2)

(C) (1/2, 1/2) (D) (3, 3)

Sol. (A)

$x = t^2 - t + 1,\ y = t^2 + t + 1$

$\Rightarrow$ $x + y = 2(t^2 + 1)$ and $y - x = 2t$

$\Rightarrow$ $\frac{x+y}{2} = 1 + \left(\frac{y-x}{2}\right)^2$

$\Rightarrow$ $(y - x)^2 = 2(x + y) - 4$

$\Rightarrow$ $(y - x)^2 = 2(x + y - 2)$.

Vertex will be the point where lines y – x = 0 and x + y – 2 = 0 meet, i.e., the point (1,1).

Q9. If the vertex of the parabola $y = x^2 - 8x + c$ lies on x-axis, then the value of c is

(A) –16 (B) –4

(C) 16 (D) 4

Sol. (C)

The parabola is $y = (x - 4)^2 + c - 16$. So the vertex is (4, c – 16). As vertex is on x-axis, c = 16.

Q10. If (2, 0) is the vertex and y-axis the directrix of a parabola, then its focus is

(A) (2, 0) (B) (-2, 0)

(C) (4, 0) (D) (-4, 0)

Sol. (C)

a = distance of vertex from focus = distance of vertex from directrix = 2.

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So, focus is at (4, 0).

Q11. If the straight line y = mx + c (m > 0) touches the parabola $y^2 = 8(x + 2)$, then the minimum value taken by c is

(A) 12 (B) 8
(C) 4 (D) 6

Sol. (C)

The tangent of slope m must be of the form $y = m(x + 2) + \frac{a}{m}$.

So, $2m + \frac{2}{m} = c \Rightarrow c = 2\left(m + \frac{1}{m}\right) \geq 2 \times 2$. So $c_{min} = 4$.

Q12. If the latus rectum of an ellipse is equal to half the minor axis then its eccentricity is equal to

(A) 1/2 (B) $\frac{\sqrt{3}}{2}$
(C) 1/4 (D) 3/4

Sol. (B)

Latus rectum is $\frac{2b^2}{a}$

Given $\frac{2b^2}{a} = b \Rightarrow \frac{b}{a} = \frac{1}{2}$

$\Rightarrow e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

Q13. Asymptotes of the hyperbola xy = 5x + 4y are

(A) x = 4 , y = 5 (B) x = 5 , y = 4
(C) x = 4, y = 2 (d) x = 5 , Y = 2

Sol. (A)

Let the pair of asymptotes be $xy - 5x - 4y + k = 0$

This should represent a pair of straight line

$\Rightarrow k = 20$

Hence equation is $xy - 5x - 4y + 20 = 0$

Hence the equations are $x = 4$ and $y = 5$

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Q14. If the equation $4x^2 + ky^2 = 18$ represents a rectangular hyperbola , then k is equal to

(A) 4 (B) -4
(C) 3 (D) None of these

Sol. (B)
Clearly for $4x^2 + ky^2 = 18$ to represent a rectangular hyperbola k = 4

Q15. A tangent is drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at point P such that it intersect the auxiliary circle at points A and B, if S and S' are the focii of the ellipse, then the minimum value of S'A + SB is

(A) b (B) 2b
(C) 2a (D) none of these

Sol. (B)

We know S'A.S'B = b^2
Applying AM ≥ GM

$$\frac{S'A+S'B}{2} \geq \sqrt{S'A.S'B}$$

Clearly S'A + S'B ≥ 2b

Q16. Equation of a rectangular hyperbola whose asymptotes are x = 3 and y = 5 and passing through (7, 8) is

(A) xy –3y + 5x +3 = 0 (B) xy + 3y + 5x +3 = 0
(C) xy –3y + 5x –3 = 0 (D) xy –3y – 5x +3 = 0

Sol. (D)
Equation of hyperbola will be (x – 3)(y – 5) + k =0
This passes through (7, 8)
$\Rightarrow k = -12$
Hence equation is xy – 3y – 5x + 3 = 0

Q17. The line l x + my + n =0 will be a normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1, if$

(A) $\frac{a^2}{l^2}+\frac{b^2}{m^2}=\left(\frac{a^2+b^2}{b}\right)^2$ (B) $\frac{a^2}{l^2}-\frac{b^2}{m^2}=\left(\frac{a^2+b^2}{b}\right)^2$

(C) $\frac{a^2}{l^2}+\frac{b^2}{m^2}=\left(\frac{a^2+b^2}{n}\right)^2$ (D) $\frac{a^2}{l^2}-\frac{b^2}{m^2}=\left(\frac{a^2+b^2}{n}\right)^2$

Sol. (D)

Equation of normal at any point P(θ) is ax cosθ + by cotθ = $a^2 + b^2$If the line lx + my + n = 0 is a normal, then

$$\frac{a\cos\theta}{l} = \frac{b\cos\theta}{m} = \frac{a^2+b^2}{-n}$$

$$\Rightarrow \sec\theta = -\frac{an}{l(a^2+b^2)},\ \tan\theta = \frac{-bn}{m(a^2+b^2)}$$

$$\Rightarrow \frac{a^2n^2}{l^2(a^2+b^2)^2} - \frac{b^2n^2}{m^2(a^2+b^2)^2} = 1$$

$$\Rightarrow \frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2+b^2)^2}{n^2}$$

Q18. Angle between tangents drawn from the point $p\left(\sqrt{3}, 1\right)$ *to the hyperbola* $\frac{x^2}{9} - \frac{y^2}{5} = 1$ *is equal to*

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

Sol. (D)

Distance of 'P' from origin is 2 units, which is equal to radius of director circle of the hyperbola. Thus, required angle is $\frac{\pi}{2}$.

Q19. A common tangent of $9x^2 - 16y^2 = 144$ and $x^2 + y^2 = 9$ is

(A) $y\sqrt{7} = 3x\sqrt{2} + 15$ (B) $y\sqrt{7} = 3x\sqrt{2} - 15$

(C) $y\sqrt{2} = 3x\sqrt{7} + 15$ (D) None of these

Sol. (D)

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Any tangent to $\frac{x^2}{16}-\frac{y^2}{9}=1$ is $y = mx + c$ where, $c^2 = 16m^2 - 9$.

If it touch the circle $x^2 + y^2 = 9$, then

$$\frac{|c|}{\sqrt{1+m^2}}=3$$

$\Rightarrow c^2 = 9(1 + m^2) = 16m^2 - 9 \Rightarrow m^2 = \frac{18}{7}$

$\Rightarrow c^2 = 16.\frac{18}{7} - 9 = \frac{225}{7}$

$\Rightarrow$ Hence, common tangents are

$y \pm 3x\sqrt{\frac{2}{7}} \pm \frac{15}{\sqrt{7}}$

i.e. $y\sqrt{7} = \pm 3x\sqrt{2} \pm 15$

Q20. The equation of tangents drawn from the point (0, 4) to the hyperbola $x^2 - 4y^2 = 36$ are

(A) $5x - 6y + 24$ and $5x + 6y - 24 = 0$

(B) $x - 4y + 16 = 0$ and $x + 4y - 16 = 0$

(C) $2x - y + 4 = 0$ and $2x + y - 4 = 0$

(D) None of these

Sol. (A)

Any tangent will be in the form $y = mx + c$, where $c^2 = 36m^2 - 9$. If it passes through (0, 4) then $4 = c \Rightarrow 16 = 36m^2 - 9 \Rightarrow m \pm \frac{5}{6}$.

Hence tangent are $6y = 5x + 24$, $6y = -5x + 24$

Q21. The locus of the mid-point of the chords of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, which pass through a fixed point $P(x_1, y_1)$ is

(A) A circle (B) An ellipse

(C) A hyperbola (D) None of these

Sol. (C)

Let Q(h, k) be the mid–point of drawn chord. Equation o this chord will be $T = S_1$ i.e.

$\frac{xh}{a^2}-\frac{yk}{b^2}=\frac{h^2}{a^2}-\frac{k^2}{b^2}$. If it's passes through $P(x_1, y_1)$ then $=\frac{h^2}{a^2}-\frac{k^2}{b^2}-\frac{x_1h}{a^2}+\frac{y_1k}{b^2} = 0$. Thus, locus is $\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=0$

Which is clearly a hyperbola

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Q22. The line y = x + p (p is parameter) cuts the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at P and Q, then mid point of PQ lies on

(A) $a^2y + b^2x$ (B) $a^2y + b^2x = 0$
(C) ay + bx = 0 (D) none of these

Sol. (D)

Solving the line and ellipse, we get

$$\frac{x^2}{a^2}+\frac{(x+p)^2}{b^2}=1$$

$\Rightarrow$ $(a^2 + b^2)x^2 + 2a^2px + a^2p^2 - a^2b^2 = 0$

$\Rightarrow$ $\bar{x}\frac{x_1+x_2}{2}=\frac{-a^2p}{a^2+b^2}$ and $\bar{y}\frac{y_1+y_2}{2}=\frac{b^2p}{a^2+b^2}$

$\Rightarrow$ $\bar{y}=\frac{-b^2}{a^2}\bar{x}$ $\Rightarrow$ mid-point lies on y = $\frac{-b^2}{a^2}x$

Q23. Normals drawn to the ellipse $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, at point 'P' meets the coordinate axes at points A and B respectively. Locus of mid point of segment AB is

(A) $4x^2a^2 + 4y^2b^2 = (a^2 - b^2)^2$
(B) $4x^2b^2 + 4y^2a^2 = (a^2 - b^2)$
(C) $16x^2a^2 + 16y^2b^2 = (a^2 - b^2)^2$
(D) $16x^2b^2 + 16y^2a^2 = (a^2 - b^2)$

Sol. (A)

Let P = (acosθ, bsinθ). Equation of normal is, $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$

$\Rightarrow$ $A\equiv\left(\frac{a^2-b^2}{a}\cos\theta, 0\right)$ and $B\equiv\left(0, \frac{a^2-b^2}{a}\sin\theta\right)$

It (h, k) be the mid– point of segment AB then 2h = $\frac{a^2-b^2}{a}\cos\theta$, 2k = $\frac{a^2-b^2}{a}\sin\theta$

$\Rightarrow$ $4h^2a^2 + 4k^2b^2 = (a^2 - b^2)^2$

Q24. A common tangent to $9x^2 + 16y^2 = 144$, $y^2 = x - 4$ and $(x - 6)^2 + y^2 = 4$ is

(A) y = 3 (B) x = 4
(C) y = –3 (D) x = –4

Sol. (B)

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Given curves are $\frac{x^2}{16}+\frac{y^2}{9}=1$, $y^2 = (x - 4)$ and $(x - 6)^2 + y^2 = 4$. x = 4 is indeed the common tangent of these given curves.

Q25. The distance between the directrices of the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is

(A) $\frac{9}{\sqrt{5}}$ (B) $\frac{24}{\sqrt{5}}$

(C) $\frac{18}{\sqrt{5}}$ (D) None of these

Sol. (C)

$4 = 9(1-e^2) \Rightarrow e=\sqrt{5}/3$

Distance between the directrices = $\frac{2b}{e}=\frac{2\times 3\times 3}{\sqrt{5}}=\frac{18}{\sqrt{5}}$

Q26. If 'z' be any complex number such that $|3z-2|+|3z+2|=4$, then locus of 'z' is

(A) A circle (B) An ellipse

(C) A line segment (D) None of these

Sol: (C)

$|3z-2|+|3z+2|=4$

$\Rightarrow \left|z-\frac{2}{3}\right|+\left|z+\frac{2}{3}\right|=\frac{4}{3}$ ....(i)

Let P (z), A = $\left(\frac{2}{3}, 0\right)$, B = $\left(-\frac{2}{3}, 0\right)$ then (i) represents PA + PB = 4/3.

Clearly AB = 4/3 $\Rightarrow$ PA + PB = AB

Thus P is any point on the line segment AB.

Q27. The value of the expression $2\left(1+\frac{1}{\omega}\right)\left(1+\frac{1}{\omega^2}\right)+3\left(2+\frac{1}{\omega}\right)\left(2+\frac{1}{\omega^2}\right)+4\left(3+\frac{1}{\omega}\right)\left(3+\frac{1}{\omega^2}\right)$ $+ \ldots + (n+1)\left(n+\frac{1}{\omega}\right)\left(n+\frac{1}{\omega^2}\right)$, where ω is an imaginary cube root of unity, is

(A) $\frac{n(n^2+2)}{3}$ (B) $\frac{n(n^2-2)}{3}$

(C) $\frac{n^2(n+1)^2+4n}{4}$ (D) none of these

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Sol: (C)

$t_n = (n+1)\left(n+\frac{1}{\omega}\right)\left(n+\frac{1}{\omega^2}\right) = n^3 + n^2\left(\frac{1}{\omega^2}+\frac{1}{\omega}+1\right)+n\left(1+\frac{1}{\omega^2}+\frac{1}{\omega}\right)+1$

$= n3 + n^2(\omega + \omega^2 + 1) + n(\omega + \omega^2 + 1) + 1 = n^3 + 1$

$\therefore S_n = \sum_{r=1}^{n} t_r = \sum_{r=1}^{n}(r^3+1) = \frac{n^2(n+1)^2}{4} + n$.

Q28. If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ equals

(A) $128\,\omega$ (B) $-128\,\omega$

(C) $128\,\omega^2$ (D) $-128\,\omega^2$

Sol: (D)

We have $(1 + \omega + \omega^2)^7 = -(\omega^2 - \omega^2)^7$

$= (-2)^7 (\omega^2)^7 = 128\,\omega^{14} = -128\,\omega^2$

Hence (D) is the correct answer.

Q29. Let 'z' be a complex number and 'a' be a real parameter such that $z^2 + az + a^2 = 0$, then locus of z is

(A) a pair of straight lines (B) a circle

(C) an ellipse (D) none of these

Sol: (A)

$z^2 + az + a^2 = 0$

$\Rightarrow z = a\omega, a\omega^2$ (where 'ω' is non-real root of unity)

$\Rightarrow$ Locus of z is a pair of straight lines

Q30. The equation $| z + i | - | z - i | = k$ represents a hyperbola if

(A) $-2 < k < 2$ (B) $k > 2$

(C) $0 < k < 2$ (D) none of these

Sol: (A)

$|z + i| - |z - i| = k$ represents a hyperbola if $4 - \frac{16}{k^2} < 0$ i.e. $k^2 < 4$.

Q31. If $\left|\frac{z_1}{z_2}\right| = 1$ and $\arg(z_1 z_2) = 0$, then

(A) $z_1 = z_2$ (B) $|z_2|^2 = z_1 z_2$

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(C) $z_1z_2 = 1$ (D) none of these.

Sol: (B)

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ then $\left|\frac{z_1}{z_2}\right| = 1$

$\Rightarrow |z_1| = |z_2| \Rightarrow |z_1| = |z_2| = r_1$.

Now arg $(z_1 z_2) = 0 \Rightarrow \arg(z_1) + \arg(z_2) = 0 \Rightarrow \arg(z_2) = -\theta_1$

Therefore, $z_2 = r_1(\cos(-\theta_1) + i\sin(-\theta_1)) = r_1(\cos\theta_1 - i\sin\theta_1) = \bar{z}_1$

$\Rightarrow \bar{z}_2 = \left(\bar{\bar{z}}_1\right) = z_1 \Rightarrow |z_2|^2 = z_1 z_2$.

Q32. If $|z| < 4$, then $|iz + 3 - 4i|$ is less than

(A) 4 (B) 5

(C) 6 (D) 9

Sol: (D)

$|iz + (3 - 4i)| \le |iz| + |3 - 4i| = |z| + 5 < 4 + 5 = 9$.

Q33. If z is a complex number, then $z^2 + \bar{z}^2 = 2$ represents

(A) a circle (B) a straight line

(C) a hyperbola (D) an ellipse

Sol. (C)

Let $z = x + iy$, then $z^2 + \bar{z}^2 = 2 \Rightarrow x^2 - y^2 = 1$, which represents a hyperbola.

Q34. If $\frac{1 - i\alpha}{1 + i\alpha} = A + iB$, then $A^2 + B^2$ equals to

(A) 1 (B) α^2

(B) -1 (D) $-\alpha^2$

Sol: (A)

$A + iB = \frac{1-i\alpha}{1+i\alpha} \Rightarrow A - iB = \frac{1+i\alpha}{1-i\alpha} \Rightarrow (A+iB)(A-iB) = \frac{(1-i\alpha)(1+i\alpha)}{(1+i\alpha)(1-i\alpha)} = 1$

$\Rightarrow A^2 + B^2 = 1$.