CBSE Board
Class XI Mathematics
Sample Paper – 1

Time: 3 hrs **Total Marks: 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 26 questions divided into three sections A, B and C. Section A comprises of 06 questions of one mark each, section B comprises of 13 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted.

SECTION – A

1. Find the derivative of sin(x + 1).

2. Find the truth value of p: 'Every real number is either prime or composite.'

3. Simplify: $\frac{1+3i}{1-2i}$

4. A coin is tossed twice. Find the probability of getting atleast one head.

5. Find the new co-ordinates of the point (9, 4) if the origin is shifted to the point (1, 2) by translation of axes.

6. Identify the conic section represented by the equation $4x^2 + y^2 = 100$ and draw its rough graph.

SECTION – B

7. A and B are two sets such that $n(A - B) = 14 + x$, $n(B - A) = 3x$ and $n(A \cap B) = x$, draw a Venn diagram to illustrate the information. If $n(A) = n(B)$, then find the value of x .

8. If the power sets of two sets are equal, then show that the sets are also equal.

9. If f and g are two functions: $R \to R$; $f(x) = 2x - 1$, $g(x) = 2x + 3$, then evaluate

(i) $(f+g)(x)$ (ii) $(f-g)(x)$ (iii) $(fg)(x)$ (iv) $\left(\frac{f}{g}\right)(x)$

10. Let R be a relation from N to N defined by $R = \{(a, b) \in N \text{ and } a = b^4\}$. Determine if the relation is

(i) Reflexive (ii) Symmetric (iii) Transitive (iv) Equivalence

11. In a ΔABC, if $a = 3$, $b = 5$, $c = 7$, find cosA, cosB and cosC.

12. Find the square root of the complex number $5 - 12i$.

13. Find the probability such that when 7 cards are drawn from a well shuffled deck of 52 cards, all the aces are obtained.

14. Find the sum to infinity of the series: $\frac{1}{3}+\frac{1}{5^2}+\frac{1}{3^3}+\frac{1}{5^4}+\frac{1}{3^5}+\frac{1}{5^6}+\ldots$

15. In how many ways can the letters of the word 'Mathematics' be arranged so that the (i) vowels are together (ii) vowels are not together

OR

In how many ways can 5 girls and 3 boys be seated in a row with 11 chairs so that no two boys sit together?

16. A point M with x-coordinate 4 lies on the line segment joining the points P(2, −3, 4) and Q (8, 0, 10). Find the co-ordinates of the point M.

OR

Find the equation of the set of points such that the sum of the square of its distance from the points (3, 4, 5) and (−1, 3,−7) is a constant.

17. Solve for x: $\tan 2x + \sec^2 2x - 1 = 0$

OR

Solve for x: $\sin x + \sin 2x + \sin 3x = 0$

18. Evaluate: $\lim\limits_{x \to 0} \frac{\log 10 + \log\left(x + \frac{1}{10}\right)}{x}$

OR

Find the derivative of the given function

$$y = \frac{x}{\sin^n x}$$

19. Write down the binomial expression $(1 + x)^{n+1}$, when x = 8. Deduce that $9^{n+1} - 8n - 9$ is divisible by 64, when n is an integer.

SECTION – C

20. If $\frac{\pi}{2} \le x \le \pi$ and $\tan x = -\frac{4}{3}$, find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$, $\tan\frac{x}{2}$.

21. Find the mean deviation about the median for the following data:

Marks	No. of students
0–10	5
10–20	10
20–30	20
30–40	5
40–50	10

22. Prove by the principle of Mathematical Induction that every even power of every odd integer greater than one when divided by 8 leaves one as the remainder.

23. Solve the following system of inequalities graphically:
$x + 2y \le 10$; $x + y \ge 1$; $x - y \le 0$; $x \ge 0$; $y \ge 0$

OR

For the purpose of an experiment an acid solution between 4% and 6% is required. 640 liters of 8% acid solution and a 2% acid solution are available in a laboratory. How many liters of the 2% solution needs to be added to the 8% solution?

24. The first three terms in the binomial expansion of $(a + b)^n$ are given to be 729, 7290 and 30375 respectively. Find a, b and n.

25. A student wants to buy a computer for Rs. 12,000. He has saved up to Rs. 6000 which he pays as cash. He is to pay the balance in annual installments of Rs. 500 plus an interest of 12% on the unpaid amount. How much will the computer cost him?

OR

Find the value of $\frac{1\times 2^2 + 2\times 3^2 + 3\times 4^2 + \text{....uptill the nth term}}{1^2\times 2 + 2^2\times 3 + 3^2\times 4 + \text{....uptill the nth term}}$

26. Show that the equation of the line passing through the origin and making an angle of θ with the line y= mx + c is $\frac{y}{x} = \frac{m + \tan\theta}{1 - m\tan\theta}$ or $\frac{y}{x} = \frac{m - \tan\theta}{1 + m\tan\theta}$

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Class XI Mathematics
Sample Paper – 1 Solution

SECTION – A

1. [sin(x+1)]’ = cos (x+1) .1 = cos (x+1)

2. Giving one counter example is enough to prove the falsehood of a statement. Here counter example is: The real number 1 is neither prime nor composite. So the statement is false.

3.

$$\frac{1+3i}{1-2i}\times\frac{1+2i}{1+2i}=\frac{1-6+3i+2i}{(1)^2-(2i)^2}=\frac{-5+5i}{1-4i^2}=\frac{-5+5i}{1+4}=\frac{-5+5i}{5}=-1+i$$

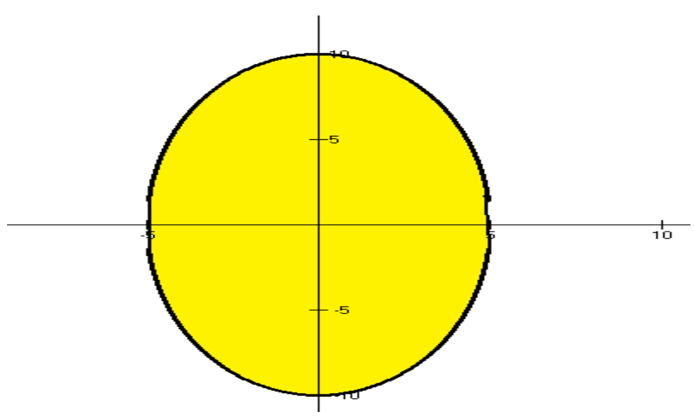
4. Sample space S = {HH, HT, TH, TT} i.e. total number of cases = 4
Favourable cases for atleast one head are {HH, HT, TH}.
Required probability = $\frac{3}{4}$

5. Let the new origin be (h, k) = (1, 2) and (x, y) = (9, 4) be the given point.
Therefore new co-ordinates (X, Y) of (9, 4) are given by
x = X + h and y = Y + k i.e. 9 = X + 1 and 4 = Y + 2
This gives, X = 8 and Y = 2. Thus the new co-ordinates are (8, 2).

6. $4x^2 + y^2 = 100$

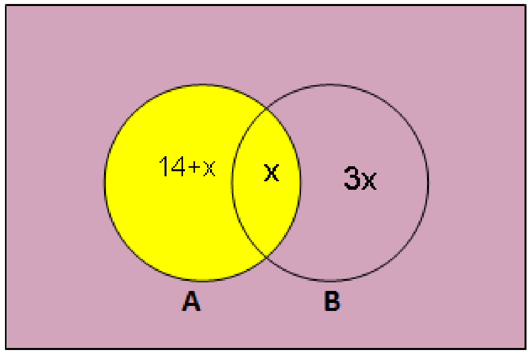
$$\frac{x^2}{25}+\frac{y^2}{100}=1$$

This is the equation of an ellipse with major axis along the y-axis



SECTION – B

7. $n(A - B) = 14 + x$, $n(B - A) = 3x$ and $n(A \cap B) = x$



$n(A) = n(B)$

$n(A) = n(A - B) + n(A \cap B)$;

$n(B) = n(B - A) + n(A \cap B)$

$\Rightarrow n(A - B) + n(A \cap B) = n(B - A) + n(A \cap B)$

$\Rightarrow 14 + x + x = 3x + x$

$\Rightarrow 14 = 2x \Rightarrow x = 7$

8. Let a be any element which belongs to set A, i.e $a \in A$

P(A) is the set of all subsets of the set A. Therefore {a} belongs to P(A)

i.e $\{a\} \in P(A)$

But P(A) = P(B) [Given]

$\therefore \{a\} \in P(B)$

$\Rightarrow a \in B$

So $a \in A \Rightarrow a \in B$, Hence $A \subseteq B$

Similarly, we can prove that $A \subseteq B$

$\Rightarrow A = B$

9. $f(x) = 2x - 1, g(x) = 2x + 3; x \in R$

$(f + g)(x) = f(x) + g(x) = (2x - 1) + (2x + 3) = 4x + 2; x \in R$

$(f - g)(x) = f(x) - g(x) = (2x - 1) - (2x + 3) = -4$

$(fg)(x) = f(x)g(x) = (2x - 1)(2x + 3) = 4x^2 - 2x + 6x - 3 = 4x^2 + 4x - 3$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x - 1}{2x + 3}; x \in R - \left\{-\frac{3}{2}\right\}$

10.

$\{(a, b), a = b^4, a, b \in N\}$

(i) $(a, a) \in R \Rightarrow a = a^4$,
which is true for a = 1 only, not for other values of $a \in N$
$\therefore$ Relation is not reflexive

(ii) $\{(a, b), a = b^4, a, b \in N\}$ and $\{(b, a), b = a^4, a, b \in N\}$
$a = b^4$ and $b = a^4$ cannot be true simultaneously
$\therefore$ Relation is not symmetric.

(iii) $\{(a, b), a = b^4, a, b \in N\}$; $\{(b, c), b = c^4, b, c \in N\}$
$\Rightarrow a = b^4 = c^{16}$
So $a \neq c^4$
$\therefore (a, c) \notin R$
$\therefore$ Relation is not transitive.

(iv) Since the relation is not reflexive, not symmetric, and also not transitive,
$\Rightarrow$ Relation is not an equivalence relation

11. We know,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{25 + 49 - 9}{2(5)(7)} = \frac{65}{70} = \frac{13}{14}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{9 + 49 - 25}{2(3)(7)} = \frac{33}{42}$$

12. Let $\sqrt{5 - 12i} = x + yi$

$$\Rightarrow 5 - 12i = \left(x^2 - y^2\right) + 2xy\, i$$

Equating real and imaginary parts, we get
$x^2 - y^2 = 5$.......(i) and
$2xy = -12$.........(ii)

Now $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2 = 5^2 + 12^2 = 169$

$\therefore x^2 + y^2 = 13$ ....(iii)

From (i) and (iii), we get
$2x^2 = 18 \Rightarrow x = \pm 3$
and $y = \pm 2$
From equation (ii) we can say that xy is negative.
As xy is negative $\Rightarrow$ when $x = 3, y = -2$
and when $x = -3, y = 2$

The required square roots are $(3 - 2i)$ and $(-3 + 2i)$

or $\pm (3 - 2i)$

13. Total number of possible sets of 7 cards = ${}^{52}C_7$

Number of sets of 7 with all 4 aces = ${}^{4}C_4 \times {}^{48}C_3$

(4 aces from among 4 aces and other 3 cards must be chosen from the rest 48 cards)

Hence the probability that the 7 cards drawn contain 4 aces = $\dfrac{{}^{4}C_4 \times {}^{48}C_3}{{}^{52}C_7}$

$= \dfrac{1}{7735}$

14. We have

$$\frac{1}{3}+\frac{1}{5^2}+\frac{1}{3^3}+\frac{1}{5^4}+\frac{1}{3^5}+\frac{1}{5^6}+----=\left[\frac{1}{3}+\frac{1}{3^3}+\frac{1}{3^5}+-----\right]+\left[\frac{1}{5^2}+\frac{1}{5^4}+\frac{1}{5^6}+-----\right]$$

$$=\frac{1/3}{1-\frac{1}{3^2}}+\frac{\frac{1}{5^2}}{1-\frac{1}{5^2}}=\frac{1}{3}\times\frac{9}{8}+\frac{1}{25}\times\frac{25}{24}$$

$$=\frac{3}{8}+\frac{1}{24}=\frac{10}{24}=\frac{5}{12}$$

15. In MATHEMATICS, there are 11 letters of which there are 2 Ms, 2 As, and 2 Ts, so the total arrangements are $\dfrac{11!}{2!.2!.2!}$ = 4,989,600

(i) In MATHEMATICS, there are 4 vowels: 2 As, 1 E and 1 I.

Since they must be together so 'AAEI' is treated as a single unit.

So there are 8 objects which include 2 Ms, 2 As, and 2 Ts.

So the required number of arrangements are $\dfrac{8!}{2!.2!.2!}$ = 5040

(ii) Number of arrangements with vowels never together =

Total arrangements – arrangements in which vowels are always together

= 4,989,600-5040 = 4984560

OR

First the 5 girls are arranged in 5! ways as shown below:

$(G_1 - G_2 - G_3 - G_4 - G_5)$

Now there are 6 places in which the boys can be arranged.

This can be done in ${}^{6}P_3$ ways

$\Rightarrow$ Total ways = $5! \times {}^{6}P_3$

= 120 x 6 x 5 x 4 = 14 , 400

16. Let M divide PQ in the ratio k : 1.

The co-ordinates of the point M are given by

$$\left(\frac{8k+2}{k+1},\frac{-3}{k+1},\frac{10k+4}{k+1}\right)$$

here the x-coordinate = $\frac{8k+2}{k+1}=4$.

$\Rightarrow 8k+2=4k+4 \Rightarrow 4k=2 \Rightarrow k=\frac{2}{4}=\frac{1}{2}$

Putting k back in the x, y and z co-ordinate of the point M, we have (4,–2,6)

OR

Let the given points be A(3, 4, 5) and B (−1, 3, −7).

Let the required point be P: P(x, y, z)

Given: $PA^2 + PB^2 = k^2$ (k^2 is a constant)

$\Rightarrow (x-3)^2 + (y-4)^2 + (z-5)^2 + (x+1)^2 + (y-3)^2 + (z+7)^2 = k^2$

$\Rightarrow x^2 - 6x + 9 + y^2 - 8y + 16 + z^2 - 10z + 25 + x^2 + 2x + 1 + y^2$
$- 6y + 9 + z^2 + 14z + 49 = k^2$

$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 - k^2 = 0$

This is the equation of the set of points P that satisfy the condition.

17. $\tan 2x + \sec^2 2x - 1 = 0$

$\Rightarrow \tan 2x + 1 + \tan^2 2x - 1 = 0$

$\Rightarrow \tan 2x + \tan^2 2x = 0$

$\Rightarrow \tan 2x[1 + \tan 2x] = 0$

$\Rightarrow \tan 2x = 0 \text{ or } 1 + \tan 2x = 0$

$\Rightarrow \tan 2x = 0 \text{ or } \tan 2x = -1$

$\tan 2x = 0 \Rightarrow 2x = n\pi \Rightarrow x = \frac{n\pi}{2}$

$\tan 2x = -1 \Rightarrow \tan 2x = -\tan\frac{\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right) = \tan\frac{3\pi}{4}$

$\Rightarrow 2x = n\pi + \frac{3\pi}{4}, n \in Z$

$\Rightarrow x = \frac{n\pi}{2} + \frac{3\pi}{8}, n \in Z$

OR

Again

sinx + sin2x + sin3x = 0

sinx + sin3x + sin2x = 0

$$2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)+\sin 2x=0$$

2sin2x cos(−x) + sin2x =0

sin2x(2cosx +1) = 0

$$\sin 2x=0 \text{ or} \cos\ x=\frac{-1}{2}$$

sin2x = 0 ⇒ 2x = nπ

$$\text{or} \cos x=\frac{-1}{2}\Rightarrow x=2n\pi\pm\left(\pi-\frac{\pi}{3}\right)$$

$$x = \frac{n\pi}{2} \text{ or } x = 2n\pi\pm\left(\pi-\frac{\pi}{3}\right)$$

18.

$$\lim_{x\to 0}\frac{\log 10+\log\left(x+\frac{1}{10}\right)}{x}$$

$$=\lim_{x\to 0}\frac{\log\left[10\left(x+\frac{1}{10}\right)\right]}{x}$$

$$=\lim_{x\to 0}\frac{\log\left(10x+1\right)}{x}$$

$$=10\lim_{x\to 0}\frac{\log\left(10x+1\right)}{10x}$$

$$=10\times 1 \qquad \left(\because \lim_{x\to 0}\frac{\log\left(x+1\right)}{x}=1\right)$$

$$=10$$

OR

The derivative can be obtained as follows:

$$y = \frac{x}{\sin^n x}$$

$$\frac{dy}{dx} = \frac{\sin^n x \frac{d}{dx}(x) - x\frac{d}{dx}(\sin^n x)}{\sin^{2n} x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin^n x.1 - xn(\sin^{n-1} x).\cos x}{\sin^{2n} x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(\sin^{n-1} x)\left[\sin x - xn.\cos x\right]}{\sin^{2n} x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\left[\sin x - xn.\cos x\right]}{\sin^{2n-n+1} x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin x - nx\cos x}{\sin^{n+1} x}$$

19. We have,

$(1 + x)^{n+1} = {}^{n+1}C_0 + {}^{n+1}C_1x + {}^{n+1}C_2x^2 + {}^{n+1}C_3x^3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + {}^{n+1}C_{n+1}x^{n+1}$

Putting x = 8, we get

$(1 + 8)^{n+1} = {}^{n+1}C_0 + {}^{n+1}C_1(8)^1 + {}^{n+1}C_2(8)^2 + {}^{n+1}C_3(8)^3 + \ldots\ldots\ldots\ldots + {}^{n+1}C_{n+1}(8)^{n+1}$ ..... (i)

$9^{n+1} = 1 + (n + 1) \times 8 + {}^{n+1}C_2(8)^2 + {}^{n+1}C_3(8)^3 + \ldots\ldots\ldots\ldots + {}^{n+1}C_{n+1}(8)^{n+1}$

$9^{n+1} = 1 + 8n + 8 + 8^2\{{}^{n+1}C_2 + {}^{n+1}C_3(8)^1 + \ldots\ldots\ldots\ldots + {}^{n+1}C_{n+1}(8)^{n-1}\}$

$9^{n+1} = 8n + 9 + 8^2\{{}^{n+1}C_2 + {}^{n+1}C_3(8)^1 + \ldots\ldots\ldots\ldots + {}^{n+1}C_{n+1}(8)^{n-1}\}$

$9^{n+1} - 8n + 9 = 64\{{}^{n+1}C_2 + {}^{n+1}C_3(8)^1 + \ldots\ldots\ldots\ldots + {}^{n+1}C_{n+1}(8)^{n-1}\}$

$9^{n+1} - 8n + 9 = 64 \times$ an integer

$\therefore$ $9^{n+1} - 8n + 9$ is divisible by 64.

SECTION – C

20. $\tan x = -\frac{4}{3}$; $\frac{\pi}{2} \le x \le \pi$

$\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$ $\left(\because \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}\right)$

$\Rightarrow -\frac{4}{3} = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \Rightarrow 4\left(1-\tan^2\frac{x}{2}\right) = -6\tan\frac{x}{2}$

$\Rightarrow 4\tan^2\frac{x}{2} - 6\tan\frac{x}{2} - 4 = 0$

$\Rightarrow 2\tan^2\frac{x}{2} - 3\tan\frac{x}{2} - 2 = 0$

The equation is quadratic in $\tan\frac{x}{2}$

$\Rightarrow \tan\frac{x}{2} = \frac{-(-3)\pm\sqrt{9+16}}{2.2} = \frac{3\pm 5}{4} = 2, -\frac{1}{2}$

Given $\frac{\pi}{2} \le x \le \pi \Rightarrow \frac{\pi}{4} \le \frac{x}{2} \le \frac{\pi}{2} \Rightarrow \frac{x}{2} \in I^{st}$ quadarnt

In I^{st} quadrant, $\tan\frac{x}{2} \ge 0 \Rightarrow \tan\frac{x}{2} = 2$

We know, $1+\tan^2\theta = \sec^2\theta$

$\Rightarrow$ $1+\tan^2\frac{x}{2} = \sec^2\frac{x}{2} \Rightarrow 1+(2)^2 = \sec^2\frac{x}{2}$

$\Rightarrow \sec^2\frac{x}{2} = 5 \Rightarrow \sec\frac{x}{2} = \pm\sqrt{5} \Rightarrow \cos\frac{x}{2} = \pm\frac{1}{\sqrt{5}}$

In I^{st} quadrant, $\cos\frac{x}{2} \ge 0 \Rightarrow \cos\frac{x}{2} = \frac{1}{\sqrt{5}}$

We know $\sin\theta = \pm\sqrt{1-\cos^2\theta}$

$\sin\frac{x}{2} = \pm\sqrt{1-\cos^2\frac{x}{2}} = \pm\sqrt{1-\frac{1}{5}} = \pm\sqrt{\frac{4}{5}} = \pm\frac{2}{\sqrt{5}}$

In I^{st} quadrant, $\sin\frac{x}{2} \ge 0 \Rightarrow \sin\frac{x}{2} = \frac{2}{\sqrt{5}}$

$\therefore$ (i) $\sin\frac{x}{2} = \frac{2}{\sqrt{5}}$ (ii) $\cos\frac{x}{2} = \frac{1}{\sqrt{5}}$ (iii) $\tan\frac{x}{2} = 2$

21.

Marks	Frequency (f_i)	Cumulative frequency (cf)
0-10	5	5
10-20	10	15
20-30	20	35
30-40	5	40
40-50	10	50
Total	N= 50	

Median (M) = $\ell + \dfrac{\frac{n}{2} - cf}{f} x h$

l = lower limit of the median class, n = number of observations,

cf = cumulative frequency of the class preceding the median class, h = class size and

f = frequency of the median class

Substituting the values we get

Median (M) = 20 + $\dfrac{(25-15)\times 10}{20}$

Median (M) = 25

xi	fi	$\lvert di \rvert = \lvert x_i - M \rvert$	$f_i \lvert d_i \rvert$
5	5	20	100
15	10	10	100
25	20	0	0
35	5	10	50
45	10	20	200
	50		450

$\therefore \sum_{i=1}^{n} \lvert d_i \rvert f_i = 450, \quad n = \sum_{i=1}^{n} f_i = 50$

$\therefore \text{ M.D (M)} = \dfrac{\sum_{i=1}^{n} \lvert d_i \rvert f_i}{n} = \dfrac{450}{50} = 9$

22. The first odd integer > 1, is 3 .

The general term for odd number > 1 is (2r + 1)

P(n): $(2r + 1)^{2n} = 8m + 1$ where m, n are natural numbers

i.e. P(n): $(2r + 1)^{2n} - 1$ is divisible by 8

Here P(1): $(2r + 1)^{2.1}$ and −1 is divisible by 8.

Consider $(2r + 1)^2 - 1 = 4r^2 + 4r = 4r(r + 1)$

r(r + 1) being the product of consecutive natural numbers is even,

So, 4r(r + 1) is divisible by 8.

Therefore, P(1) is true.

Let us assume P(k) to be true

P(k): $(2r + 1)^{2k} - 1$ is divisible by 8.

Using this assumption, we will prove P(k + 1) to be true

P(k + 1): $(2r + 1)^{2(k+1)} - 1$ is divisible by 8.

Consider $(2r + 1)^{2(k+1)} - 1 = (2r + 1)^{2k}(2r + 1)^2 - 1 = (8m + 1)(8p + 1) - 1$ [using P(1) and P(k), where m and p are integers]

$(2r + 1)^{2(k+1)} - 1 = 64\,mp + 8(m + p) + 1 - 1 = 64\,mp + 8(m + p)$, which is divisible by 8.

Thus P(k + 1) is true whenever P(k) is true, also P(1) is true.

⇒P(n) is true for every natural number n.

23.

x + 2y = 10 or x = 10 – 2y

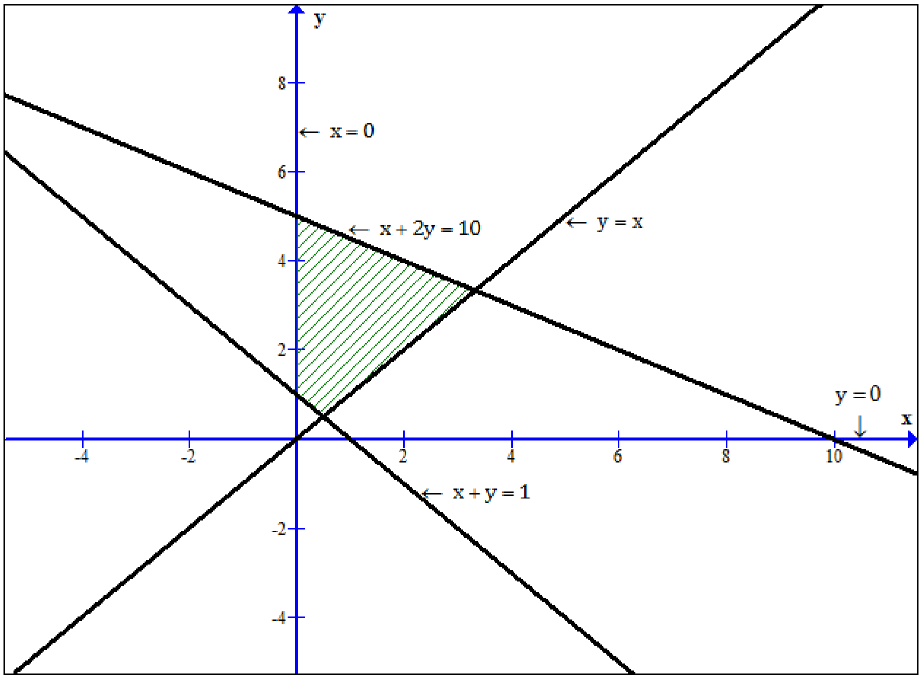
x	14	10	6
y	-2	0	2

x + y = 1 or y = 1 – x

x	−2	0	3
y	3	1	−2

x – y = 0 or y = x

x	−2	0	2
y	−2	0	2



OR

The amount of acid in 640 litres of the 8% solution= 8% of 640= $\frac{8\times 640}{100}$

Let x litres of the 2% solution be added to obtain a solution between 4% and 6%

The amount of acid in x litres of the 2% solution $=\frac{2\times x}{100}$

The resultant amount = 640 +x

The amount of acid in $(640+x)$ litres solution is $=\frac{8\times 640}{100}+\frac{2\times x}{100}$

Acid percentage of the solution now $=\frac{\frac{8\times 640}{100}+\frac{2\times x}{100}}{640+x}\times 100$

$$\Rightarrow 4<\frac{\frac{8\times 640}{100}+\frac{2\times x}{100}}{640+x}\times 100<6$$

$$\Rightarrow \frac{4(640+x)}{100}<\frac{8\times 640}{100}+\frac{2\times x}{100}<\frac{6(640+x)}{100}$$

$$\Rightarrow 4(640+x)<5120+2x<6(640+x)$$

$$\Rightarrow 2(640+x)<2560+x<3(640+x)$$

$$\Rightarrow 2(640+x)<2560+x \text{ and } 2560+x<3(640+x)$$

$$\Rightarrow 1280+2x<2560+x \text{ and } 2560+x<1920+3x$$

$$\Rightarrow x<1280 \text{ and } 320<x$$

$$\Rightarrow 320<x<1280$$

Hence, the number of litres of 2% of acid which must be added should be more than 320 but less than 1280.

24. The first three terms in the binomial expansion $(a+b)^n$, ie t_1, t_2, t_3 are given.

$\Rightarrow t_1 = {}^nC_0 a^n b^0 = 729 \ldots (i);$

$t_2 = {}^nC_1 a^{n-1} b^1 = 7290 \ldots (ii);$

$t_3 = {}^nC_2 a^{n-2} b^2 = 30375 \ldots (iii)$

Now, $t_1 = {}^nC_0 a^n b^0 = 729 \Rightarrow 1 \times a^n \times 1 = 729 \Rightarrow a^n = 729 \ldots (iv)$

Dividing (ii) by (i), we have

$$\frac{t_2}{t_1} = \frac{{}^nC_1 a^{n-1} b^1}{{}^nC_0 a^n b^0} = \frac{7290}{729} = 10 \Rightarrow \frac{na^{n-1}b}{a^n} = 10 \Rightarrow \frac{nb}{a} = 10 \ldots (v)$$

Multiplying (iii) by (i), we have

$$t_3 \times t_1 = {}^nC_2 a^{n-2} b^2 \times {}^nC_0 a^n b^0 = \frac{n(n-1)}{2} a^{2n-2} b^2 = 729 \times 30375 \ldots (vi)$$

Squaring (ii), we have

$$\left[{}^nC_1 a^{n-1} b^1\right]^2 = [7290]^2 \Rightarrow n^2 a^{2n-2} b^2 = 7290 \times 7290 \ldots (vii)$$

Dividing (vi) by (vii), we have

$$\frac{\frac{n(n-1)}{2} a^{2n-2} b^2}{n^2 a^{2n-2} b^2} = \frac{729 \times 30375}{7290 \times 7290}$$

$$\Rightarrow \frac{(n-1)}{2n} = \frac{30375}{7290 \times 10}$$

$$\Rightarrow \frac{(n-1)}{2n} = \frac{5}{12}$$

$\Rightarrow 12n - 12 = 10n \Rightarrow 2n = 12 \Rightarrow n = 6$

Putting n = 6 in (iv), we have

$a^6 = 729 \Rightarrow a = 3$

Putting n = 6, a = 3 in (v), we have

$$\frac{6b}{3} = 10 \Rightarrow b = 5$$

Hence, $a = 3, b = 5, n = 6$

25. Interest to be paid with Installment 1 (S.I. on Rs. 6000 for 1 year) = $\frac{6000 \times 12 \times 1}{100} = 720$

Interest to be paid with Installment 2 (S.I. on Rs. 5500 for 1 year) = $\frac{5500 \times 12 \times 1}{100} = 660$

Interest to be paid with Installment 3 (S.I. on Rs. 5000 for 1 year) = $\frac{5000 \times 12 \times 1}{100} = 600$

Interest to be paid with Installment 1^{st} (S.I. On Rs 500 for 1 year) = $\frac{500 \times 12 \times 1}{100} = 60$

Total interest paid = 720 + 660 + 600 + …. + 60

This forms an A.P., with a = 720 and d = −60

$Sn = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow Sn = \frac{12}{2}\left[2 \times 720 + (12-1)(-60)\right] = 4680$

The computer costed the student = 12000+ 4680= 16680

OR

$\frac{1 \times 2^2 + 2 \times 3^2 + 3 \times 4^2 + \text{....uptill the nth term}}{1^2 \times 2 + 2^2 \times 3 + 3^2 \times 4 + \text{....uptill the nth term}}$

Consider Numerator = $1 \times 2^2 + 2 \times 3^2 + 3 \times 4^2 + ... + \text{uptill the nth term}$

The nth term is $n(n+1)^2$

$\therefore 1 \times 2^2 + 2 \times 3^2 + 3 \times 4^2 + \text{....uptill the nth term}$

$= \sum n(n+1)^2$

$= \sum n\,(n^2+1+2n)$

$= \sum\,(n^3+n+2n^2) = \sum n^3 + 2\sum n^2 + \sum n$

$= \left[\frac{n(n+1)}{2}\right]^2 + 2\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + 2\frac{(2n+1)}{3} + 1\right]$

$= \frac{n(n+1)}{12}\left[3n^2 + 3n + 8n + 4 + 6\right]$

$= \frac{n(n+1)}{12}\left[3n^2 + 11n + 10\right]$

$= \frac{n(n+1)}{12}\left[(3n+5)(n+2)\right]$

Consider Denominator $=1^2\times 2+2^2\times 3+3^2\times 4+$....uptill the nth term

The nth term is $n^2\ (n+1)$

$\therefore 1^2\times 2+2^2\times 3+3^2\times 4+$....uptill the nth term

$=\sum n^2\ (n+1)$

$=\sum\ (n^3+n^2)=\sum n^3+\sum n^2$

$=\left[\frac{n(n+1)}{2}\right]^2+\frac{n(n+1)(2n+1)}{6}$

$=\frac{n(n+1)}{2}\left[\frac{n(n+1)}{2}+\frac{(2n+1)}{3}\right]$

$=\frac{n(n+1)}{12}\left[3n^2+3n+4n+2\right]$

$=\frac{n(n+1)}{12}\left[(3n+1)(n+2)\right]$

The given expression $=\dfrac{\frac{n(n+1)}{12}\left[(3n+5)(n+2)\right]}{\frac{n(n+1)}{12}\left[(3n+1)(n+2)\right]}=\frac{3n+5}{3n+1}$

26. Of the given line y = mx + c, the slope is m

Let m_1 be the slope of the required line

$\tan\theta=\left|\frac{m_1-m}{1+mm_1}\right|\Rightarrow\tan\theta=\pm\frac{m_1-m}{1+mm_1}$

Case I When $,\tan\theta=\frac{m_1-m}{1+mm_1}$

$\Rightarrow\tan\theta(1+mm_1)=(m_1-m)$

$\Rightarrow\tan\theta+mm_1\tan\theta=m_1-m$

$\Rightarrow m_1(1-m\tan\theta)=(m+\tan\theta)$

$\Rightarrow m_1=\frac{(m+\tan\theta)}{(1-m\tan\theta)}$

The equation of the line through the origin is $y-0=\left[\frac{(m+\tan\theta)}{(1-m\tan\theta)}\right](x-0)$

$\Rightarrow y=\left[\frac{(m+\tan\theta)}{(1-m\tan\theta)}\right]x\Rightarrow\frac{y}{x}=\frac{(m+\tan\theta)}{(1-m\tan\theta)}$

Case II When , $\tan\theta = -\dfrac{m_1 - m}{1 + mm_1}$

$\Rightarrow \tan\theta(1 + mm_1) = (m - m_1)$

$\Rightarrow m_1(m\tan\theta + 1) = (m - \tan\theta)$

$\Rightarrow m_1 = \dfrac{(m - \tan\theta)}{(1 + m\tan\theta)}$

The equation of the line , through the origin is

$y - 0 = \left[\dfrac{(m - \tan\theta)}{(1 + m\tan\theta)}\right](x - 0)$

$\Rightarrow y = \left[\dfrac{(m - \tan\theta)}{(1 + m\tan\theta)}\right]x$

$\Rightarrow \dfrac{y}{x} = \dfrac{m - \tan\theta}{1 + m\tan\theta}$

Thus the equation of the line is $\dfrac{y}{x} = \dfrac{m + \tan\theta}{1 - m\tan\theta}$ or $\dfrac{y}{x} = \dfrac{m - \tan\theta}{1 + m\tan\theta}$