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CBSE Board
Class XI Mathematics
Sample Paper – 2

Q1. A function f has domain $[-1, 2]$ and range $[0, 1]$. Find the domain and range respectively of the function g defined by $g(g) = 1 - f(x + 1)$.

(a) $[-2, 1]$; $[0,1]$
(b) $[-2, 2]; [1, 2]$
(c) $[-2, 3]$; $[2, 3]$
(d) $[-2, 4]$; $[3, 4]$

Sol. (a)

$g(x) = 1 - f\ (x + 1)$

Domain of $f(x)$ is given as $[-1, 2]$

$-1 \leq x + 1 \leq 2 \qquad \Rightarrow \qquad -2 \leq X \leq 1$

Domain of $f(x) = [-2, 1]$

Range of $f(x) = [0, 1]$

$g(x) = 1 = f(x + 1)$

Range of $g(x) = [0, 1]$

Q2. The function f is not defined for x = 0, but for all non-zero real numbers x, $f(x) + 2f\ \left(\frac{1}{x}\right) = 3x$.

(a) 2
(b) 3
(c) 4
(d) 5

Sol. (a)

$f(x) + 2f\left(\frac{1}{x}\right) = 3x$ ..... (1)

Replacing x by $\frac{1}{x}$

$$f\left(\frac{1}{x}\right) + 2f(x) = 3\ \left(\frac{1}{x}\right)$$

$2f\ \left(\frac{1}{x}\right) + 4f(x) = 6\ \left(\frac{1}{x}\right)$ ...... (2)

(2) – (1) we get

$$3f(x) = \frac{6}{x} - 3\,x$$

Now if $f(x) = if\ (-x)$

$$\frac{6}{x} - 3x = \frac{6}{x} + 3x$$

$\frac{12}{x} = 6x$

$x^2 = 2$

$x = \pm\sqrt{2}$

Q3. If $S = \sum_{k=1}^{\infty} \frac{1}{2^{k+1}} . \tan\frac{\pi}{2^{k+1}}$ then find $\log_{\pi}(S)$.

(a) -1
(b) -2
(c) -3
(d) -4

Sol. (a)

$S = \sum_{k=1}^{\infty} \frac{1}{2^{k+1}} . \tan\frac{\pi}{2^{k+1}}$

Now taking

$$\tan\left(\frac{\pi}{2^{k+1}}\right) = \frac{\sin\left(\frac{\pi}{2^{k+1}}\right)}{\cos\left(\frac{\pi}{2^{k+1}}\right)} . \frac{\sin\left(\frac{\pi}{2^{k+1}}\right)}{\sin\left(\frac{\pi}{2^{k+1}}\right)} = \frac{\cos^2\frac{\pi}{2^{k+1}} + \sin^2\frac{\pi}{2^{k+1}} - \cos^2\frac{\pi}{2^{k+1}}}{\cos\frac{\pi}{2^{k+1}} . \sin\frac{\pi}{2^{k+1}}}$$

$$= \cot\frac{\pi}{2^{k+1}} - 2\cot\frac{\pi}{2^k}$$

$S = \sum_{k=1}^{\infty} \frac{1}{2^{k+1}} \left[\cot\frac{\pi}{2^{k+1}} - 2\cot\frac{\pi}{2^k}\right]$

$S = \frac{1}{4}\left(\cot\frac{\pi}{4} - 2\cot\frac{\pi}{2}\right) + \frac{1}{8}\left(\cot\frac{\pi}{8} - 2\cot\frac{\pi}{2}\right) + \frac{1}{16}\left(\cot\frac{\pi}{16} - 2\cot\frac{\pi}{8}\right) + \frac{1}{32}\left(\cot\frac{\pi}{32} - 2\cot\frac{\pi}{16}\right) + ..$

$\dots\dots + \frac{1}{2^{k+1}}\left(\cot\frac{\pi}{2^{k+1}} - 2\cot\frac{\pi}{2k}\right)$

$S = \lim_{n\to\infty} \frac{1}{2^{k+1}} = \lim_{n\to\infty} \frac{\left(\frac{1}{2^{k+1}}\right)}{\tan\left(\frac{\pi}{2^{k+1}}\right)} = \lim_{n\to\infty} \frac{\left(\frac{\pi}{2^{k+1}}\right)}{\tan\left(\frac{\pi}{2^{k+1}}\right)} . \frac{1}{\pi}$

$S = \frac{1}{\pi} \quad \log_p S = \log_{\pi}\left(\frac{1}{\pi}\right) = -1$

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Q4. If $\alpha + ß + \gamma = \frac{\pi}{2}$, then prove that $\cot \alpha = \frac{\tan ß + \tan \gamma}{1 - \tan \ \tan \gamma}$.

(a) $\frac{1}{2} \leq a \leq 1$
(b) $\frac{1}{2} \leq a \leq 2$
(c) $\frac{1}{2} \leq a \leq 3$
(d) $\frac{1}{2} \leq a \leq 4$

Sol. (a)

$\sin^4 x + \cos^4 x = a$

$(\sin^2 x + \cos^2 x) - 2\sin^2 x \cos^2 x = a$

$1 - 2\sin^2 x \cos^2 x = a$

$1 = \frac{\sin^2 2x}{2} = a$

Since, $0 \leq \sin^2 2x \leq 1$

$\frac{1}{2} \leq a \leq 1$

Q5. If in a Δ ABC, $a = 6, b = 3 \ and \cos(A - B) = 45$ then find its area.

(a) 8
(b) 9
(c) 7
(d) 6

Sol. (b)

$\cos(A - B) = \frac{1 - \tan^2\left(\frac{A-B}{2}\right)}{1 + \tan^2\left(\frac{A-B}{2}\right)} = \frac{4}{5}$

$\Rightarrow \quad \tan^2\left(\frac{A-B}{2}\right) = \frac{1}{9} \Rightarrow \tan\left(\frac{A-B}{2}\right) = \frac{1}{3} \qquad (\because a > b \Rightarrow A > B)$

$\frac{a-b}{a+b}\cot\left(\frac{C}{2}\right) = \frac{1}{3} \Rightarrow \frac{6-3}{6+3}\cot\left(\frac{C}{2}\right) = 1 \Rightarrow \frac{C}{2} = 45° \Rightarrow c = 90°$

Area d the triangle $= \frac{1}{2}ab = \frac{1}{2}.6.3 = 9 \ unit^2$.

Q6. In a triangle ABC if $a^2 + b^2 = 101\ c^2$ then find the value of $\frac{\cot C}{\cot A + \cot B}$.

(a) 55

(b) 50
(c) 65
(d) 70

Sol. (b)

$$\frac{\cot C}{\cot A+\cot B}=\frac{\frac{\cos C}{\sin C}}{\frac{\cos A}{\sin A}+\frac{\cos B}{\sin B}}=\frac{\left(\frac{a^2+b^2-c^2}{2ab}\right)\cdot\frac{1}{\left(\frac{e}{k}\right)}}{\left(\frac{b^2+c^2-a^2}{2bc}\right)\frac{1}{\left(\frac{a}{k}\right)+\left(\frac{a^2+c^2-b^2}{2ac}\right)\frac{1}{\left(\frac{b}{k}\right)}}}$$

$\because$ $$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=k=\frac{a^2+b^2-c^2}{(b^2+c^2-a^2)+(a^2+c^2-b^2)}$$

$$=\frac{a^2+b^2-c^2}{2c^2}=\frac{101\,c^2-c^2}{2c^2}=50.$$

Q7. Find the number of solutions of the equation $\cos 3x+\cos 2x=\sin\frac{3x}{2}$ lying in the interval $0\leq x\leq 2\pi$.

(a) 6
(b) 5
(c) 7
(d) 8

Sol. (b)

$$\cos 3x+\cos 2x=\sin\left(\frac{3x}{2}\right)+\sin\left(\frac{x}{2}\right)$$

$$2\cos\frac{5x}{2}\cos\frac{x}{2}=2\sin(x)\cos\frac{x}{2}$$

$\Longrightarrow$ $\cos\frac{x}{2}=0$

$\Longrightarrow$ $\frac{x}{2}=(2n+1)\frac{\pi}{2}$

$\Longrightarrow$ $x=(2n+1)\,\pi$

Or $\cos\frac{5x}{2}=\sin x$

$$\frac{5x}{2}=2n\pi\pm\left(\frac{\pi}{2}-x\right)$$

Taking positive sign

$\frac{7x}{2} = 2n\pi + \frac{\pi}{2}$

$x = (4n+1)\frac{\pi}{7}$

Taking negative sign

$\frac{3x}{2} = 2n\pi - \frac{\pi}{2} \implies x = (4n-1)\frac{\pi}{3}$

$\therefore \quad 0 \le x \le 2\pi, \quad x = \pi, \frac{\pi}{7}, \frac{5\pi}{7}, \frac{9\pi}{7}, \frac{13\pi}{7}$

Q8. Solve the equation $3^{\sin 2x + 2\cos^2 x} + 3^{1-\sin 2x + 2\sin^2 x} = 28.$

(a) $(2n+1)\frac{\pi}{2}, m\pi - \frac{\pi}{4}$
(b) $(2n+1)\frac{\pi}{2}, m\pi - \frac{\pi}{5}$
(c) $(3n+1)\frac{\pi}{2}, m\pi - \frac{\pi}{4}$
(d) $(2n+1)\frac{\pi}{4}, m\pi - \frac{\pi}{4}$

Sol. (a)

Given equation is $3^{\sin 2x + 2\cos^2 x} + 3^{1-\sin 2x + 2\sin^2 x} = 28$

Or $3^{\sin 2x + 2\cos^2 x} + 3^{3-(\sin 2x + 2\cos^2 x)} = 28$

Let $3^{\sin 2x + 2\cos^2 x} = y$

Then $y + \frac{3^3}{y} = 28$

Or $(y-1)(y-27) = 0$

$\implies$ either $y - 1 = 0$

$\therefore \quad y = 1$

Or $3^{\sin 2x + 2\cos^2 x} = 1$

$\implies \sin 2x + 2\cos^2 x = 0$

Or $2\cos x(\sin x + \cos x) = 0$

$\implies$ either $\cos x = 0$

i.e. $x = (2n+1)\frac{\pi}{2}$

or $\tan x = -1 = \tan\left(-\frac{\pi}{4}\right)$

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i.e. $x = n\pi - \frac{\pi}{4}$

Hence $x = (2n+1)\frac{\pi}{2}$

Or $3^{\sin 2x + 2\cos^2 x} + 3^{3}.3^{-(\sin 2x + 2\cos^2 x)} = 28$

Or $y - 28y + 27 = 0$

$\therefore$ $y = 27$

Or $3^{\sin 2x + 2\cos^2 x} = 3$

$\Longrightarrow$ $\sin 2x + 2\cos^2 x = 3$

$\Longrightarrow$ $\sin 2x + \cos 2x = 2$

$\Longrightarrow$ $\sin 2x = 1 \; and \; \cos 2x = 1$

(both satisfied simultaneously)

Which is not possible for any value of x.

Or $n\pi - \frac{\pi}{4}$

Q9. In a GP the ratio of the sum of the first eleven terms to the sum of the last eleven terms is 1/8 and the ratio of the sum of all the terms without the first nine to the sum of all the terms without the last nine is 2. Find the number of terms in the GP.

(a) 36
(b) 37
(c) 38
(d) 39

Sol. (c)

Let the total no. of terms in a G.P. is in the sum of first eleven terms is S_{11} and sum of last eleven is S_{11} then

$$\frac{S_{11}}{S_{11}} = \frac{1}{8} = \frac{a(r^{11}-1)\,(1-1/r)}{(r-1)\,(ar^{n-1})\,(1-1/r^{11})}$$

Or $r^{11-n} = \frac{1}{8}$ .....(1)

$\Longrightarrow$ $\frac{S_n - S_9}{S_n - S_9} = 2$

$$\Rightarrow \frac{\frac{a(r^n-1)}{(r-1)} - \frac{a(r^a-1)}{(r-1)}}{\frac{a(r^n-1)}{(r-1)} - \frac{ar^{n9-1}(r^9-1)}{r^9.(r-1)}.r} \Rightarrow \frac{(r^n-1)-(r^9-1)}{(r^n-1)-r^{n-9}(r^9-1)}$$

$\Rightarrow$ $r^9 = 2$ ..... (2)

By (1) & (2)

Or $r^{11-n+27} = 1 = r^{\circ}$

Or $n = 38.$

Q10. The sequence $a_1, a_2, a_3, \ldots \ldots \ldots a_{98}$ satisfies the relation $a_{n+1} = a_n + 1$ for $n = 1, 2, 3, \ldots \ldots . .97$ and has the sum equal to 4949. Evaluate $\sum_{k=1}^{49} a_{2k}$

(a) 2498
(b) 2497
(c) 2496
(d) 2499

Sol. (d)

Let $\sum_{k=1}^{49} a_{2k} = a_2 + a_4 + a_6 + \cdots + a_{98} = k$ ......(1)

Now according to given condition

$a_{k+1} = a_k + 1$

$a_2 = a_1 + 1$

$a_4 = a_3 + 1$

$\therefore$ $a_1 + a_3 + a_5 + \cdots + a_{97} + 49 = k$

$a_1 + a_3 + a_5 + \cdots + a_{97} = k - 49$ ......(2)

Equation by (1) and (2)

$a_1 + a_2 + a_3 + \cdots + a_{98} = 2k - 49$

$\therefore$ $2k - 49 = 4949$ (Given)

$\therefore$ $k = 2499.$

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Q11. Find coefficient of x^4 in the expansion of

$(1+x+x^2+x^3)^{11}$

(a) 890
(b) 790
(c) 690
(d) 990

Sol. (d)

$(1+x+x^2+x^3)^{11}$

$\left((1+x)+x^2(1+x)^{11}\right)$

$\underbrace{(1+x)^{11}}_{I}\underbrace{(1+x^2)^{11}}_{II}$

Coefficient of x in

I	II
0	$4 = {}^{11}C_0 \,.\, {}^{11}C_2 = 55$
2	$2 = {}^{11}C_2.\, {}^{11}C_1 = 55 \times 11 = 605$
4	$0 = {}^{11}C_4.\ {}^{11}C_0 = 330$

$\therefore$ Coefficient of $x^4 = 55 + 605 + 330 = 990$

Q12. Find x for which sixth term I the expansion of

$E = \left(3^{\log_3 \sqrt{9|x-2|}} + 7^{\frac{1}{5}\log_7[4.3\ j|x-2|-9]^7}\right)$ is 567.

(a) 3 or I
(b) 4 or I
(c) 5 or I
(d) 6 or I

Sol. (a)

As $\sqrt{9^{|x-2|}} = 3^{|x-2|}$

$E = \left(3^{|x-2|} + \left(3^{|x-2|-9}\right)^{\frac{1}{5}}\right)^7$

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$T_{r+1} = {}^nC_r.x^{n-r}.y^r$ (put r = 5)

Now, $T_6 = 567 = {}^7C_5\left(3^{|x-2|-2}\right)\left(3^{|x-2|-9}\right) \Rightarrow 27 = 3^{2|x-2|}.\left(3^{|x-2|-9}\right)$

$\Rightarrow \quad |x-2| = 4 \Rightarrow x = 6 \; or -2$

Hence, sum of possible values of $x = 6 - 2 = 4.$

Q13. Find the equation of the straight lines passing through $(-2,-7)$ & having an intercept of length 3 between the straight lines $4x + 3y = 12, 4x + 3y = 3.$

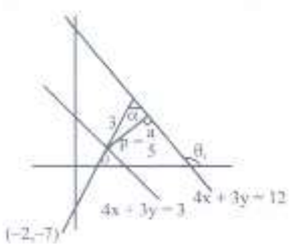
(a) $7x + 24y + 182 = 0$
(b) $7x + 24y - 182 = 0$
(c) $7x - 24y + 182 = 0$
(d) $7x - 24y - 182 = 0$

Sol. (a)

$p = \frac{12-3}{5} = \frac{9}{5}$

$\text{Sin}\,\alpha \frac{9}{5\times3} = \frac{3}{5}$

$\text{Tan}\,\theta_1 = -\frac{4}{3} \quad \therefore \qquad \tan\alpha = \frac{3}{4} \; or -\frac{3}{4}$



$\theta_1 = \alpha + \theta$

$\therefore \quad \theta = \theta_1 - \alpha$

$tan\theta = \frac{\tan\theta_1 - \tan\alpha}{1+\tan\theta_1\tan\alpha}$

$\tan\theta = \frac{-\frac{4}{3}-\frac{3}{4}}{1-\frac{4}{3}\times\frac{3}{4}} \rightarrow \infty$

$\therefore \quad$ Line is $x = -2$

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Or $\tan\theta = \frac{-\frac{4}{3}+\frac{3}{4}}{1+\frac{4}{3}\times\frac{3}{4}} = \frac{-16+9}{12\times 2} = -\frac{7}{24}$

∴ Line is $y + 7 = -\frac{7}{24}(x+2)$

$24y + 168 - 7x - 14$

$7x + 24y + 182 = 0$

Q14. A straight line is drawn from the point (1, 0) to the curve $x^2 + y^2 + 6x - 10y + 1 = 0$, such that the intercept made on it by the curve subtends a right angle at the origin. Find the equations of the line.

(a) $x + y = 1; x + 9y = 1$
(b) $x + y = 2; x + 9y = 1$
(c) $x - y = 1; x + 9y = 1$
(d) $x + y = 1; x - 9y = 1$

Sol. (a)

Let the line be

$y - 0\, m(x-1)$

$y = mx - m$

∴ $mx - y = m$

$\frac{mx-y}{m} = 1$

Homogenising the curve

$x^2 + y^2 + 6x\left(\frac{mx-y}{m}\right) - 10y\left(\frac{mx-y}{m}\right) + 1\left(\frac{mx-y}{m}\right)^2 = 0$

Coefficients of x^2 $Coefficients\ of\ y^2 = 0$

$1 + 1 + 6 + \frac{10}{m} + \frac{m^2+1}{m^2} = 0$

$8m^2 + 10m + m^2 + 1 = 0$

$9m^2 + 10m + 1 = 0 \quad \Longrightarrow \quad 9m^2 + 9m + m + 1 = 0$

$(9m+1)(m+1) = 0$

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$m = -1$ or $m = -\frac{1}{9}$

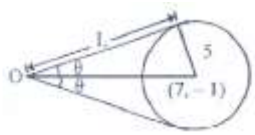
$\therefore$ Line is $x + y = 1$ *and* $x + 9y = 1$

Q15. Find the angle between the two tangents from the origin to the circle

$(x-7)^2 + (y+1)^2 = 25.$

(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$
(d) $\frac{\pi}{5}$

Sol. (a)



$L = \sqrt{(-7)^2 + (1)^2 - 25} \Rightarrow$

$\therefore \tan\theta = 1$ *or* $\theta = 45°$

$\therefore$ Angle between the two tangents is $\left(\frac{\pi}{2}\right)$.

Q16. There are two circles through origin which touch the lines $x + 2y - 9 = 0, 2x - y + 2 = 0$. If radius of one of them is $\sqrt{5}$ and the radius of other one is $\frac{k}{\sqrt{5}}$, then find the value of k.

(a) 0015
(b) 0016
(c) 0017
(d) 0018

Sol. (c)

Origin must lie on bisector bisecting the angle between the lines containing origin. Therefore, if the centres are (h, k) *then* $\frac{-h-2k+9}{\sqrt{5}} = \frac{2h-k+5}{\sqrt{5}}$

Or $3h + k = 7$ .....(i)

Equation of circles must be the form

$$(k-h)^2 + (y-k)^2 = \left(\frac{2h-k+2}{\sqrt{5}}\right)^2 \qquad \because \frac{2h-k+2}{\sqrt{5}}$$

Is radius of both circles.

Since it passes through $(0, 0)$, we have

$$h^2 + k^2 = \left(\frac{2h-k+2}{\sqrt{5}}\right) \qquad \text{.....(ii)}$$

On solving (i) and (ii), we get

$$h = 2 \; or \frac{22}{5} \; hence \; k = 1 \; or -\frac{31}{5}$$

The first circle has radius $\sqrt{5}$.

The radius of other circle is perpendicular distance of $\left(\frac{22}{5}, \frac{-31}{5}\right)$ from any of the lines $2x - y + 2 = 0$ which is equal to $\left|\frac{2\times\frac{22}{5}+\frac{31}{5}+2}{\sqrt{5}}\right| = \frac{17}{\sqrt{5}} \quad \Rightarrow \quad k = 17$

Q17. Three normal form a point to the parabola $y^2 = 4ax$ meet the axis of the parabola in points whose abscissa are in A.P. Find the locus of the point.

(a) $27\,ay^2 = 2(x-2a)^3$
(b) $27\,ay^2 = 2(x+2a)^3$
(c) $27\,ay^2 = 4(x-2a)^3$
(d) $27\,ay^2 = 5(x-2a)^3$

Sol. (a)

The equation of any normed to the parabola is

$y = mx - 2am - am^3$

It passes through the point (h, k) then

$am^3 + m(2a-h) + k = 0 \qquad$ ......(1)

The normal cuts the axis of the parabola viz., y =0 at point where x = 2a + am^2

Hence the abscissa of the points in which the normal through (h, k) meet the axis of the parabola are

$x_1 = 2a + am_1^2, x_2 = 2a + ma_2^2, x_3 = 2a + am_3^2$

Since x_1, x_2, x_3 $are\ in\ A.P.$

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$(2a + am_1^2) + (2a + am_3^2) = 2\,(2a + am_2^2)$

$\Longrightarrow \quad m_1^2 + m_3^2 = 2m_2^2$ ....(2)

Also, from (1), $m_1 + m_2 + m_3 = 0$ ....(3)

$m_2m_3 + m_3m_1 + m_1m_2 = \frac{2a-h}{a}$ ......(4)

And $m_1m_2m_3 = -\frac{k}{a}$ ......(5)

From (3)

$(m_1 + m_3)^2 = m_2^2 \Longrightarrow m_1^2 + 2m_1m_3 = m_2^2$

$\Longrightarrow \quad 2m_2^2 - 2\frac{k}{am_2} = m_2^2 \Longrightarrow m_2^2 = \frac{2k}{am_2}$

$\Longrightarrow \quad am_2^2 = 2k$ ...... (6)

Since $m_2 is\ a\ root\ of\ (1)$.

$am_2^3 + m_2(2a - h) + k = 0$

$\Longrightarrow \quad 2k + m_2(2a - h) + k = 0$

$\Longrightarrow \quad \{m_2(h - 2a)\}^3 = 27k^3 \Longrightarrow \frac{2k}{a}\,(h - 2a)^3 = 27k^3$

$\Longrightarrow \quad 27ak^2 = 2(h - 2a)^3$.

Hence the locus of (h, k) is $27\,ay^2 = 2(x - 2)^3$.

Q18. Suppose x and y are real numbers and that $x^2 + 9y^2 - 4 + 6y + 4 = 0$ then find the maximum value of $(4x - 9y)$.

(a) 14
(b) 15
(c) 16
(d) 17

Sol. (c)

$(x - 2)^2 + (3y + 1)^2 = 1$ (Converting the given expression)

$(x - 2)^2 + 9\left(y + \frac{1}{3}\right) = 1$ (This is the equation of ellipse)

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Standard form is $(x-2)^2+\frac{\left(y+\left(\frac{1}{3}\right)\right)^2}{\left(\frac{1}{9}\right)}=1$; Let $x-2=x, y+\frac{1}{3}=Y; X^2+\frac{Y^2}{\left(\frac{1}{9}\right)}=1$

Let $X=\cos\theta;$ $Y=\frac{1}{3}\sin\theta$

Now $E=4x-9y$

Or $E=4(X+2)-9\left(Y-\frac{1}{3}\right)=4X-9Y+11=4\cos\theta-3\sin\theta+11$

$$\left[\because -\sqrt{a^2+b^2}\le a\cos\theta+b\sin\theta\le \sqrt{a^2+b^2}\right]$$

$E_{max}=11+5=16$

Q19. Find z if

$(3x-2iy)\,(2+i)^2=10\,(1+i)$

(a) $z=\frac{15}{15}+i\frac{1}{4}$
(b) $z=\frac{15}{15}-i\frac{1}{5}$
(c) $z=\frac{15}{15}+i\frac{1}{5}$
(d) $z=\frac{15}{15}+i\frac{1}{4}$

Sol. (c)

$(3x-2iy)=\frac{10(1+i)}{(3+4i)} \Longrightarrow \frac{10(1+i)\,(3-4i)}{25}=\frac{2}{5}(7-i)$

$\therefore x=\frac{14}{15}\,\&\, y=\frac{1}{5}.\, z=\frac{14}{15}+i\frac{1}{5}$

Q20. Find the solution(s) of the following equations

$|z+1|=z+2+2i$

(a) $z=\frac{1}{2}+2i$
(b) $z=\frac{1}{2}-2i$
(c) $z=\frac{1}{2}-3i$
(d) $z=\frac{1}{2}+2i$

Sol. (b)

Let z = x + iy

$\Longrightarrow\ |x+1+iy|=(x+2)+(y+2)i$

$\Rightarrow \sqrt{(x+1)^2+y^2} = (x+2) + (y+2)i$

$\Rightarrow \sqrt{(x+1)^2+y^2} = x + and\ y+2=0$

$\Rightarrow (x+1)^2+4=(x+2)^2 \;\Rightarrow\; (x+2)^2-(x+1)^2=4 \qquad \{y=-2\}$

$x=\frac{1}{2} \quad \therefore \qquad z=\frac{1}{2}-2i$

Q21. 2 American couple & 3 Indian couple are to be seated on a round table. Let m denotes the number of ways when the Indian couple are together and n denotes the number of ways when both American and Indian couples are together. Find (m – n).

(a) 4990
(b) 4991
(c) 4992
(d) 4993

Sol. (c)

$m = 6!\,2^3$

$m = 6!\,2^3 \qquad \boxed{H_1W_1}\ \boxed{H_2W_2}\ \boxed{H_3W_3}\ M_1F_1\ \ M_2F_2$

$n = 4!\cdot 2^5 \qquad \boxed{M_1F_1}\ \boxed{M_2F_2}\ \boxed{H_1W_1}\ \boxed{H_2W_2}\ \boxed{H_3W_3}$

$m-n = 2^3\cdot 4![30-4]$

$= 8\cdot 24\cdot 26 = 4992$

Q22. A swimming team has eight members, only two of which are boys. The coach wants to take a delegation from the team to a special swimming camp. If the delegation must have either five or six members and must include atleast on boy, then the number of ways to select the delegation, is

(a) 74
(b) 75
(c) 76
(d) 77

Sol. (d)

Case -1

Deligation consists of 5 members.

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Total number of ways of selecting 5 member team – number of ways when no boy is selected

$= {}^{8}C_5 - {}^{6}C_5 = 50$

Case – 2

Deligation consists of members

$\therefore$ Total – no boy is selected

$${}^{8}C_6 - {}^{6}C_5 = 27$$

$$\text{Total} = 50 + 27 = 77$$

Q23. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$$

(a) $\frac{1}{2}$
(b) $\frac{3}{2}$
(c) $\frac{2}{1}$
(d) None of these

Sol. (a)

$$\lim_{x \to 0} \frac{\tan x - \sin x}{x^3} = \lim_{x \to 0} \frac{\left(x + \frac{x^3}{3} + \frac{2}{15}x^5 + \cdots\right) - \left(x + \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)}{x^3} = \frac{1}{3} + \frac{1}{31} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

Alternative solution:

$$\lim_{x \to 0} \frac{\tan x\,(1 - \cos x)}{x^3} = \lim_{x \to 0} \frac{\tan x}{x}\left(\frac{1 - \cos x}{x^2}\right) = 1 \times \frac{1}{2}$$

Q24. Find the value of a & b

$$\lim_{x \to -\infty} \left[\sqrt{x^2 - x + 1} - ax - b\right] = 0$$

(a) $a = -1, b - \frac{1}{2}$
(b) $a = -1, b + \frac{1}{2}$
(c) $a = +1, b - \frac{1}{2}$
(d) $a = +1, b + \frac{1}{2}$

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Sol. (a)

$\lim_{x \to \infty} \sqrt{x^2 - x + 1} - ax - b = 0$ (Must be $(\infty, -\infty)$) $\therefore$ a < 0

Now $\lim_{x \to \infty} \frac{\left(\sqrt{x^2-x+1} - (ax+b)\right)}{\left(\sqrt{x^2-x+1+(ax+b)}\right)} \left(\sqrt{x^2 - x + 1} + (ax + b)\right)$

$= \lim_{x \to \infty} \frac{x^2-x+1-(ax+b)^2}{\sqrt{x^2-x+1}+ax+b} = 0$

$= \lim_{x \to \infty} \frac{x^2(1-a^2)-x(2b+1)+1-b^2}{|x|\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+ax+b}$ $\qquad$ $Must\ be\ \left(\frac{\infty}{\infty}\right)$ from

& degree of $N_r < degree\ of\ D_r$. [For Limit = zero]

$\therefore$ $1 - a^2 = 0$ & $2ab + 1 = 0$

$a = -1$ & $B = \frac{1}{2}$

Q25. For all $n \in N, 10^n + 3.4^{n+2} + 5$ is divisible by

(a) 2
(b) 3
(c) 4
(d) 5

Sol. (b)

$f(n) = 10^n + 3\,.\,4^{n+2} + 5$

$f(1) = 10 + 48 + 5 = 63$, which is divisible b 7 and 3

$f(2) = 100 + 3(256) + 5 = 105 + 768 = 873$, Which is divisible by 3.

So, $f(n) = 10^n + 3\,.\,4^{n+2} + 5$ is divisible by 3.

Q26. If a variable takes the discrete values

$\alpha + 4, \alpha - \frac{7}{2}, \alpha - \frac{5}{2}, \alpha - 3, \alpha - 2, \alpha + \frac{1}{2}, \alpha - \frac{1}{2}, \alpha + 5(\alpha >)$, then the median is

(a) $\alpha - \frac{5}{4}$
(b) $\alpha - \frac{1}{2}$
(c) $\alpha - 2$
(d) $\alpha + \frac{5}{4}$

Sol. (a)

Arrange is ascending order

$\Rightarrow \alpha - \frac{7}{2}, \alpha - 3, \alpha - \frac{5}{2}, \alpha - 2, \alpha - \frac{1}{2}, \alpha + \frac{1}{2}, \alpha + 4, \alpha + 5$

$\Rightarrow \frac{1}{2}[4^{th} + 5^{th}\ value]$

$\Rightarrow \frac{1}{2}\left[2\alpha - \frac{5}{2}\right]$ $\Rightarrow \alpha - \frac{5}{4}$

Q27. A speaks the truth in 60 percent cases and B speaks the truth in 70 percent cases. The probability that they will say the same thing while describing a single event is

(a) 0.56
(b) 0.54
(c) 0.38
(d) 0.94

Sol. (b)

For the required event, either both of them should speak the truth or both of them should tell a lie.
Probability of the required event
= (0.6)(0.7) + (0.4)(0.3)
= 0.42 + 0.12 = 0.54

Q28. Three squares of a chess board are selected at random. The probability of getting 2 squares of one colour and other of a different colour is
(a) 16/21
(b) 8/21
(c) 3/32
(d) 3/8

Sol. (a)
Option (1) is the correct answer. In a chess board, there are 64 squares of which 32 are white and 32 are black. Since 2 of one colour and 1 of other can be 2W, 1B, or 1W, 2B, the number of ways is $({}^{32}C_2 \times 32C_1) \times 2$ and also, the number of ways of choosing any 3 boxes is ${}^{66}C_3$.

Hence, the required probability = $\frac{{}^{32}C_2 \times {}^{32}C_1 \times 2}{{}^{64}C_3} = \frac{16}{21}$

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Q29 Let P (3, 2, 6) be a point in space and Q be a point on the line
r = (i – j + 2k) + μ (– 3i + j + 5k)
Then the value of μ for which the vector PQ is parallel to the plane x –4y +3z – 1 is
(a) 1/4
(b) -1/4
(c) 1/8
(d) -1/8

Sol. (a)

As Q lies on the given line

Let $\mathbf{OQ} = (\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + \mu(-3\mathbf{i} + \mathbf{j} + 5\mathbf{k})$

Also $\mathbf{OP} = 3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

As $\mathbf{PQ}$ is parallel to the plane $x - 4y + 3z = 1$

$\mathbf{PQ} \cdot (\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}) = 0$

$\Rightarrow (\mathbf{OQ} - \mathbf{OP}) \cdot (\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}) = 0$

$\Rightarrow (1 + 4 + 6) + \mu(-3 - 4 + 15) - (3 - 8 + 18) = 0$

$\Rightarrow 11 + 8\mu - 13 = 0 \Rightarrow \mu = 1/4$

Q30. If the points P(1, 0, -6), Q (-3, P,q) and R(-5, 9,6) are collinear, find the values of P and q
(a) 6 and 2
(b) 6 and 3
(c) 6 and 4
(d) 6 and 5

Sol. (a)

Given points

$P(1, 0, -6)$, $Q(-3, P, q)$ and $R(-5, 9, 6)$ are collinear

Let point Q divider PR in the ratio K:1

$\therefore$ co-ordinates of point $P\left(\frac{1-5K}{K+1}, \frac{0+9K}{K+1}, \frac{-6+6K}{K+1}\right)$

$Q(-3, P, q)$

$\frac{1-5K}{K+1} = -3$

$1 - 5K = -3K - 3$

$-2K = -4$

$K = \frac{-4}{-2}$

$K = 2$

$\therefore$ the value of P and q are 6 and 2.