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CBSE Board
Class XI Mathematics
Sample Paper – 4

Q1. Let $f(x) = x^2 4x + 1$ and $g(x) = |x|$

If $h(x) = f(g(x)) + 10$. Find rangle of $h(x)$.

(a) $[8, \infty)$
(b) $[9, \infty)$
(c) $[7, \infty)$
(d) $[6, \infty)$

Sol. (b)

$h(x) = f\big(g(x)\big) + 10 = |x|^2 + 4|x| - 1 + 10$

$h(x) = |x|^2 + 4\,|x| + 9 = (|x| + 2)^2 + 5$

Hence range of $h(x)$ is $[9, \infty)$.

Q2. Find the domain of the function $f(x) = \sqrt{(2 - 2x - x^2)}$.

(a) $[1, 6]$
(b) $[1, 5]$
(c) $[1, 4]$
(d) $[1, 3]$

Sol. (a)

$f\ (x) = \sqrt{x-1} + \sqrt{6-x}$

$x - 1 \geq 0$ and $6 - x \geq 0$

$x \geq 1$ and $x \leq 6$

$\Rightarrow x \in [1, 6]$

Q3. If a $\sin^2\theta + b\cos^2\theta = m,\ b\sin^2\phi = n$ and $a\tan\theta = b\tan\phi$, find the value of $\frac{1}{n} + \frac{1}{m}$

(a) $\frac{1}{a} - \frac{1}{b}$
(b) $\frac{1}{a} + \frac{1}{b}$
(c) $\frac{2}{a} - \frac{2}{b}$
(d) $\frac{2}{a} + \frac{2}{b}$

Sol. (b)

$$a\sin^2\theta + b\cos^2\theta = m$$

$b\sin^2\phi + a\cos^2\gamma = n$

$a\tan\theta = b\tan\phi \Longrightarrow \frac{\tan\theta}{\tan\phi} = \frac{b}{a}$ ..... (1)

$a\tan^2\theta + \mathrm{b} = \mathrm{m}\sec^2\theta$

$\Longrightarrow \quad \tan^2\theta + b = m\,(1+\tan^2\theta)$

$\Longrightarrow \quad \tan^2\theta = \frac{m-b}{a-m}$ ..... (2)

Similarly $\tan^2\phi = \frac{n-a}{b-a}$ .....(3)

Using (2) and (3) we get

$$\frac{\tan^2\theta}{\tan^2\phi} = \frac{(m-b)\,(b-n)}{(a-m)\,(n-a)}$$ ...... (4)

Using (1) and (4) we get

$$\frac{(m-b)\,(b-\ n)}{(a-m)\,(n-a)} = \frac{b^2}{a^2}$$

$$a^2\,(mb - mn - b^2 + bm) + b^2\,(an = a^2 - mn + am)$$

$$a^2\,(mb - mm + bn) = b\left(m + n - \frac{mn}{a}\right) \Longrightarrow (m+n)\,(n-b) = mn\left(\frac{a}{b} - \frac{b}{a}\right)$$

$\Longrightarrow \quad \frac{m+n}{mn} = \frac{a+b}{ab} \Longrightarrow \frac{1}{m} + \frac{1}{n} = \frac{1}{a} + \frac{1}{b}.$

Q4. If L denotes the least value of the expression $y = 9\sec^2\alpha + 16\,cosec^2\alpha\ and\ M$ denotes then maximum value of the expression $\gamma = \sin^2\alpha + 8\,cos\alpha - 7,$ find the value of $(L+M)$.

(a) 40
(b) 30
(c) 20
(d) 50

Sol. (d)

$y = 9\sec^2\alpha + 16\,cosec^2\alpha$

$y = 25 + 9\tan^2\alpha + 16\cot^2\alpha$

$L = y_{min} = 25 + 2\times 3\times 4 = 49$

$\gamma = \sin^2 \alpha + 8 \cos \alpha - 7$

$= 1 \cos^2 \alpha + 8 \cos \alpha - 7$

$= -6 - (\cos^2 \alpha - 8 \cos \alpha)$

$= -6 - [(\cos \alpha - 4)^2 - 16]$

$\gamma = 10 - (\cos \alpha - 4)^2$

$m = \gamma_{max} = 10 - 9 = 1$

$L + M = 49 + 1 = 50$

Q5. Consider an isosceles triangle with base a, vertical angle 20° and lateral side each being b. Find the value of $\left(\frac{a^3+b^3}{ab^2}\right)$.

(a) 12
(b) 15
(c) 3
(d) 9

Sol. (c)

$B = 2A, C = 4A$

$A + B + C = \pi$

$\therefore \quad A = \frac{\pi}{7}, B = \frac{2\pi}{7}, C = \frac{4\pi}{7}$

$\frac{1}{b} + \frac{1}{c} - \frac{1}{a} = \frac{1}{2R} \left[\frac{1}{\sin B} + \frac{1}{\sin C} - \frac{1}{\sin A}\right] =$

$= \frac{1}{2R \sin A \sin B \sin C} [\sin A \sin C + \sin A \sin B - \sin B \sin C]$

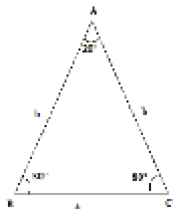
$= \frac{1}{2R \sin A \sin B \sin C} \left[\sin\left(\frac{\pi}{7}\right) \sin\left(\frac{4\pi}{7}\right) + \sin\left(\frac{2\pi}{7}\right) \sin\left(\frac{\pi}{7}\right) - \sin\left(\frac{2\pi}{7}\right) \, sin \left(\frac{4\pi}{7}\right)\right]$

$= \frac{1}{4R \sin A \sin B \sin C} \left[\left(\cos\frac{3\pi}{7} - \cos\frac{5\pi}{7}\right) + \left(\cos\frac{\pi}{7} - \cos\frac{3\pi}{7}\right) - \left(\cos\frac{2\pi}{7} - \cos\frac{6\pi}{7}\right)\right]$

Q6. In a triangle ABC, let angles A, B, C are in G.P. with common ratio 2. If circumradius of triangle ABC is 2, then find the value of $(b^{-1} + c^{-1} - a^{-1})$.

(a) 0
(b) 1
(c) 2
(d) 3

Sol. (a)



$\frac{a^3+b^3}{ab^2} = \frac{\sin^3 20°+\sin^3 80°}{\sin 20° \sin^2 80°} = \frac{8\sin^3 10° \cos^3 10°+co^3 10°}{(2\sin 10° \cos 10°)\cos^2 10°}$

$= \frac{8\sin^3 10°+1}{2\sin 10°} = \frac{6\sin 10°-2\sin 30°+1}{2\sin 10°} = 3.$

Q7. Solve the equation $3^{\sin 2x+2\cos^2 x} + 3^{1-\sin 2x+2\sin^2 x} = 28.$

(a) $(2n+1)\frac{\pi}{2}, m\pi - \frac{\pi}{4}$
(b) $(2n+1)\frac{\pi}{2}, m\pi - \frac{\pi}{5}$
(c) $(3n+1)\frac{\pi}{2}, m\pi - \frac{\pi}{4}$
(d) $(2n+1)\frac{\pi}{4}, m\pi - \frac{\pi}{4}$

Sol. (a)

Given equation is $3^{\sin 2x+2\cos^2 x} + 3^{1-\sin 2x+2\sin^2 x} = 28$

Or $3^{\sin 2x+2\cos^2 x} + 3^{3-(\sin 2x+2\cos^2 x)} = 28$

Let $3^{\sin 2x+2\cos^2 x} = y$

Then $y + \frac{3^3}{y} = 28$

Or $(y-1)(y-27) = 0$

$\Longrightarrow$ either $y - 1 = 0$

$\therefore$ $y = 1$

Or $3^{\sin 2x+2\cos^2 x} = 1$

$\Longrightarrow$ $\sin 2x + 2\cos^2 x = 0$

Or $2\cos x(\sin x + \cos x) = 0$

$\Longrightarrow$ either $\cos x = 0$

i.e. $x = (2n+1)\frac{\pi}{2}$

or $\tan x = -1 = \tan\left(-\frac{\pi}{4}\right)$

i.e. $x = n\pi - \frac{\pi}{4}$

Hence $x = (2n+1)\frac{\pi}{2}$

Or $3^{\sin 2x + 2\cos^2 x} + 3^{3}.3^{-(\sin 2x + 2\cos^2 x)} = 28$

Or $y - 28y + 27 = 0$

∴ $y = 27$

Or $3^{\sin 2x + 2\cos^2 x} = 3$

$\Longrightarrow$ $\sin 2x + 2\cos^2 x = 3$

$\Longrightarrow$ $\sin 2x + \cos 2x = 2$

$\Longrightarrow$ $\sin 2x = 1 \; and \; \cos 2x = 1$

(both satisfied simultaneously)

Which is not possible for any value of x.

Or $n\pi - \frac{\pi}{4}$

Q8. $\sin 6x = \sin 4x - \sin 2x$

(a) $(2m+1)\frac{\pi}{6}, n\pi, (2k+1)\frac{\pi}{4}$
(b) $(1m+1)\frac{\pi}{6}, n\pi, (2k+1)\frac{\pi}{4}$
(c) $(2m+1)\frac{\pi}{6}, n\pi, (2k+1)\frac{\pi}{5}$
(d) $(2m+1)\frac{\pi}{7}, n\pi, (2k+1)\frac{\pi}{4}$

Sol. (a)

$\sin 6 = \sin 4x - \sin 2x$

$2\sin 3x \cos 3x = 2\sin x \cos 3x$

$\Longrightarrow$ $2\cos 3x\,(\sin 3x - \sin x) = 0$

$\Longrightarrow$ $\cos 3x = 0$

$3x = (2n+1)\frac{\pi}{2}$

$x = (2n+1)\frac{\pi}{6}, n \in I$

Or $\sin 3x = \sin x$

$3x = 4\pi + (-1)\, x$

When n is even, let n = 2m

$3x = 2m\pi + x$

$x = m\pi.$

When n is odd, let n =2m + 1

$4x = (2m+1)\,\pi$

$x = (2m+1)\frac{\pi}{4}, n \in I$

Q9. If $a_1, a_2, a_3, \dots \dots \dots .. a_{4001}$ are terms of an A. P. such that

$\frac{I}{a_1 a_2} + \frac{I}{a_2 a_3} + \cdots + \frac{1}{a_{4000} a_{4001}} = 10 \; and \; a_2 + a_{4000} = 50$ then find the value of $|a_1 - a_{4000}|$.

(a) 30
(b) 40
(c) 50
(d) 60

Sol. (a)

$\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \cdots + \frac{1}{a_{4000} \cdot a_{4001}} = 10$

Let the common difference of A.P. is d then

$\Rightarrow \quad \frac{1}{d}\left[\frac{d}{a_1 a_2} + \frac{d}{a_2 a_3} + \cdots + \frac{d}{a_{4000} \cdot a_{4001}}\right] = 10$

$\Rightarrow \quad \frac{1}{d}\left[\frac{a_2 - a_1}{a_1 a_2} + \frac{a_3 - a_2}{a_2 a_3} + \cdots + \frac{a_{4001} - a_{4000}}{a_{4000} . a_{4001}}\right] = 10$

$\Rightarrow \quad \frac{1}{d}\left[\left(\frac{1}{a_1} - \frac{1}{a_2}\right) + \left(\frac{1}{a_2} - \frac{1}{a_3}\right) + \cdots \left(\frac{1}{a_{4000}} - \frac{1}{a_{4001}}\right)\right] = \frac{1}{d}\left[\frac{1}{a_1} - \frac{1}{a_{4001}}\right]$

$= \frac{1}{d}\left[\frac{a_{4001} - a_1}{a_1 a_{4001}}\right] = 10$

$\Rightarrow \quad a_1 a_{4001} = 400$ ......(1)

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Given $a_2 + a_{400} = a_1 + a_{4001} = 50$ .....(2)

$\therefore \quad |(a_1 - a_{4000})| = \sqrt{(a_1 - a_{4001})^2 - 4\, a_1\, a_{4001}} = \sqrt{2500 - 4 \times 400} = 30.$

Q10. Let x and y be positive integers such that $\prod_{i=1}^{59} \left(24^{\left(\frac{1}{4+1}+\frac{1}{1+2}+\cdots\frac{1}{60}\right)}\right) = 2^x.3^y$ then find the value of $x + 7$.

(a) 3530
(b) 3540
(c) 3550
(d) 3570

Sol. (b)

$24^{\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots\frac{1}{60}\right)}.\ 24^{\left(\frac{2}{3}+\frac{2}{4}+\frac{2}{5}+\cdots\frac{2}{60}\right)}.\ 24^{\left(\frac{3}{4}+\frac{3}{5}+\cdots\frac{3}{60}\right)}.24^{\left(\frac{59}{60}\right)}$

$\Rightarrow \quad 24^{(59+58+57+\cdots+1)+\frac{1}{59}(58+57+\cdots+1)\ldots\frac{.1}{2}}$

$\Rightarrow \quad 24^{\frac{1}{60}\times\frac{60\times59}{2}+\frac{1}{59}\times\frac{58\times59}{2}+\cdots\frac{1}{2}}$

$\Rightarrow \quad 24^{\frac{1}{2}(59+58+57+\cdots1)}$

$\Rightarrow \quad 24^{\frac{1}{2}\times\frac{59\times60}{2}} \quad \Rightarrow \quad 24^{59\times15=(2^3.3)^{59\times15}}$

$\Rightarrow \quad (2^3)^{59\times15}.3^{59\ldots15} = 2^x.2^y$

$\therefore \quad x = 59 \times 45 \quad \Rightarrow \quad x + y = 59 \times 60 = 3540$

$y = 59 \times 15.$

Q11. Find n for which n! $(100 - n\,!)$ is minimum

(a) 0
(b) 50
(c) 100
(d) 150

Sol. (b)

$= n!\ (100 - n)! = \left(\frac{n!\,(100-n)!}{100!}\right).100! = \frac{100!}{{}^{100}C_n}$

${}^{100}C_n$ is max. when n = 50

$\Rightarrow \quad \frac{100!}{^{100}C_n}$ is min. when $n = 50$

$\Rightarrow \quad n = 50.$

Q12. Find the coefficient of x^{201} in the expansion of $1 + (1 + x) + (1 + x)^2 + \cdots + (1 + x)^{2011}$.

(a) $^{2012}C_{202}$
(b) $^{2012}C_{203}$

(c) $^{2010}C_{202}$

(d) $^{201}C_{202}$

Sol. (a)

$1 + (1 + x) + (1 + x)^2 + \cdots + (1 + x)^{2011}$

$= 1.\left(\frac{(1+x)^{2012}-1}{1+x-1}\right) = \frac{1}{x}\left((1 + x)^{2012} - 1\right) \Rightarrow$ coefficient of x^{201} in: $\frac{1}{x}.(1 + x)^{2012} - \frac{1}{x}$

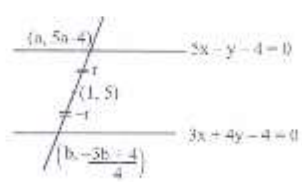
$\Rightarrow$ Coefficient of x^{202} in: $(1 + x)^{2012}$

$\Rightarrow \quad ^{2012}C_{202}.$

Q13. A line is such that its segment between the straight lines $5x - y - 4 = 0$ *and* $3x + 4y - 4 = 0$ is bisected at the point (1, 5). Obtain the equation.

(a) $83x - 35y + 92 = 0$
(b) $83x + 35y + 92 = 0$
(c) $83x - 35y - 92 = 0$
(d) $83x + 35y - 92 = 0$

Sol. (a)



$a + b = 2$ ...... (1)

and $5a - 4 + \frac{4-b}{4} = 10$

$20a - 3b - 12 = 40$

$20a - 3b = 52$ ...... (2)

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Method I: $+3a + 3b = +6$

$20a - 3b = 52$

$23a = 58$

$a = \frac{58}{23};$ $b = 2 - \frac{58}{23} = \frac{46-58}{23} = -\frac{12}{23}$

$\therefore$ $5a - 4 = 5 \times \frac{58}{23} - 4 = \frac{290-92}{23} = \frac{198}{23}$

$\therefore$ $m_{line} = \frac{\frac{198}{23}-5}{\frac{58}{23}-1} = \frac{198-115}{58-23} = \frac{83}{35}$

$\therefore$ Equation of line is

$y - 5 = \frac{83}{35}(x-1) \implies 35y - 175 = 83x - 83 \implies 83x - 35y + 175 - 83 = 0$

$\implies$ $83x + 35y + 92 = 0$

Q14. Consider 3 lines

L_1: $5x - y + 4 = 0$

L_2: $3x - y + 5 = 0$

L_3: $x + y + 8 = 0$

If these lines enclosed a triangle ABC and sum of the squares of the tangents of the interior angles can be expressed in the form p/q where p and q are relatively prime number, compute the value of $(p+q)$.

(a) 465
(b) 466
(c) 467
(d) 468

Sol. (a)

Arranging the lines in descending order

$m_1 = 5;$ $m_2 = 3$ and $m_3 = -1$

$\therefore$ $\tan A = \frac{m_1 - m_2}{1 + m_1 m_3} = \frac{2}{1+15} = \frac{1}{8}$

$\tan B = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{3+1}{1-3} = -2$

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$$\text{Tan } C = \frac{m_3 - m_1}{1 + m_3 m_1} = \frac{-1-5}{1-5} = \frac{3}{2}$$

$$\sum \tan^2 A = \frac{1}{64} + 4 + \frac{9}{4} = \frac{1+256+144}{64} = \frac{401}{64} \quad \Rightarrow \quad p + q = 465$$

Q15. Find the length of the common chord of the circles $(x-a)^2 + (y-b)^2 = c^2$ *and* $(x-b)^2 + (y-a)^2 = c^2$

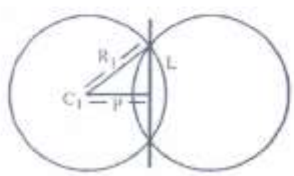
(a) $2\sqrt{c^2 + \frac{(a-b)^2}{2}}$

(b) $2\sqrt{c^2 - \frac{(a+b)^2}{2}}$

(c) $2\sqrt{c^2 - \frac{(a-b)^2}{2}}$

(d) $2\sqrt{c^2 + \frac{(a+b)^2}{2}}$

Sol. (c)



Equation of common chord is

$x^2 + y^2 - 2ax - 2by + a^2 + b^2 - c^2 = 0$

$x^2 + y^2 - 2bx - 2ay + a^2 + b^2 - c^2 = 0$

Or $y = x$

$\therefore \quad L = \sqrt{R_1^2 - P^2}; R_1 = C, P = \left|\frac{a-b}{\sqrt{2}}\right|$

$\therefore \quad \text{Length of common chord} = 2L = 2\sqrt{c^2 - \frac{(a-b)^2}{2}}.$

Q16. Find the equation of a circle passing through the origin if the line pair, $x^2 - y^2 - 8x - 6y + 7 = 0$ is orthogonal it. If this circle is orthogonal to circle $x^2 + y^2 - kx + 2ky - 8 = 0$ then find the value of k.

(a) $K = -\frac{4}{5}$

(b) $K = -\frac{5}{5}$

(c) $K = +\frac{4}{5}$

(d) $K = +\frac{5}{5}$

Sol. (a)

Line pair $x^2 - y^2 - 8x - 6y + 7 = 0$

Or $(x + y - 1)\,(x - y - 7) = 0$

Or $(4, -3) =$ centre of circle and radius $= 5$

$\therefore$ Equation of circle will be $(x-4)^2 + (y+3)^2 = 5$

If circle cut the another circle $x^2 + y^2 - kx + 2ky - 8 = 0$ orthgonality then

$\Rightarrow$ $2(-4)\,(-k_2) + 2(3)\,(k) = -8 + 0$

$\Rightarrow$ $4k + 6k = -8$

$\Rightarrow$ $10k = -8$

$\Rightarrow$ $k = -\frac{8}{10} = -\frac{4}{5}$

Q17. Find the length of the chord of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ whose middle point is $\left(\frac{1}{2}, \frac{2}{5}\right)$.

(a) $6\sqrt{\frac{41}{25}}$

(b) $7\sqrt{\frac{41}{25}}$

(c) $8\sqrt{\frac{41}{25}}$

(d) $9\sqrt{\frac{41}{25}}$

Sol. (b)

Equation of the chord having $\left(\frac{1}{2}, \frac{2}{5}\right)$ as mid-point is

$$\frac{1/4}{25} + \frac{4/25}{16} - 1 = \frac{\left(\frac{1}{2}\right)x}{25} + \frac{\left(\frac{2}{5}\right)y}{16} - 1 \qquad \{S_1 = T\}$$

$\Rightarrow 4x + 5y = 4 \;\; \Rightarrow 5y = 4\,(1 - x)$

Solving with ellipse, we get $16x^2 + 16\,(1-x)^2 = 400 \implies x^2 - x - 12 = 0 \implies x = 4, -3$

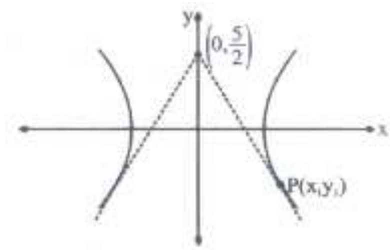
For $x = 4, y = \frac{-12}{5}, for\ x = -3, y = \frac{16}{5}$

Therefore, length of the chord $= \sqrt{\left\{7^2 + \left(\frac{28}{5}\right)^2\right\}} = 7\sqrt{\frac{41}{25}}.$

Q18. Tangents are drawn to the hyperbola $3x^2 - 2y^2 = 25$ from the point $(0, 5/2)$. Find their equations.

(a) $3x + 2y - 5 = 0 : 3x - 2y + 5 = 0$
(b) $3x + 2y - 5 = 0 : 3x - 2y - 5 = 0$
(c) $3x - 2y - 5 = 0 : 3x - 2y - 5 = 0$
(d) $3x - 2y - 5 = 0 : 3x - 2y + 5 = 0$

Sol. (a)



$$\frac{x^2}{\frac{25}{3}} - \frac{y^2}{\frac{25}{2}} = 1$$

Tangent at $P(x_1\ y_1)$ passes through $\left(0, \frac{5}{2}\right)$

$\therefore \quad 3x_1^2 - 2y_1^2 = 25$ .....(1)

And equation of tangent at $P(x_1\ y_1)\ is\ 3xx_1 - 2yy_1 = 25$

Therefore slope of tangent at $P = \frac{3x_1}{2y_1} = \frac{y_1 - \frac{5}{2}}{x_1 - 0}$ .... (2)

From (1) and (2) get the $(x_1\ y_1)$

Hence equation of tangent will be obtain

$3x + 2y - 5 = 0\,;\,3x - 2y + 5 = 0$

Q19. Find the solution(s) of the following equations

$\bar{z} = iz^2$

(a) $(0,0)\ (0,\ 1).\left(\frac{\sqrt{3}}{2},\frac{-1}{2}\right)\ \left(\frac{-\sqrt{3}}{2},\frac{-1}{2}\right)$
(b) $(0,0)\ (0,\ 2).\left(\frac{\sqrt{3}}{2},\frac{-1}{2}\right)\ \left(\frac{-\sqrt{3}}{2},\frac{-1}{2}\right)$
(c) $(0,0)\ (0,\ 3).\left(\frac{\sqrt{3}}{2},\frac{-1}{2}\right)\ \left(\frac{-\sqrt{3}}{2},\frac{-1}{2}\right)$
(d) $(0,0)\ (0,\ 4).\left(\frac{\sqrt{3}}{2},\frac{-1}{2}\right)\ \left(\frac{-\sqrt{3}}{2},\frac{-1}{2}\right)$

Sol. (a)

$\bar{z} = iz^2$

$\Rightarrow x - iy = i(x^2 - y^2 + 2ixy)$

$\Rightarrow x - iy = -2xy + i(x^2 - y^2)$

$\Rightarrow x + 2xy = and\ x^2 - y^2 + y = 0$

$\Rightarrow x(1 + 2y) = 0\ and\ x^2 - y^2 + y = 0$

$\Rightarrow x = 0\ or\ y = -\frac{1}{2}$

If $x = 0\,, y\,(1 - y) = 0\ \Rightarrow y = 0\ or\ 1$

If $y = -\frac{1}{2}, x^2 - \frac{1}{4} - \frac{1}{2} = 0, x^2 - \frac{1}{4} -, \frac{1}{2} = 0$

$x^2 - \frac{3}{4} = 0, x = \pm\frac{\sqrt{3}}{2}$

$(0,0)(0,1).\left(\frac{\sqrt{3}}{2},\frac{-1}{2}\right)\ \left(\frac{-\sqrt{3}}{2},\frac{-1}{2}\right)$

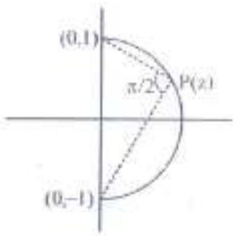
Q20. Represent the locus of z which satisfies the following equations or inequations on argand plane.

$amp\ (z + i) - amp\ (z - i) = \frac{\pi}{2}$

(a) Z lies on semi-circle whose diametric end points are (0, 1) and (0, +2)
(b) Z lies on semi-circle whose diametric end points are (0, 1) and (0, + 1)
(c) Z lies on semi-circle whose diametric end points are (0, 1) and (0, –2)
(d) Z lies on semi-circle whose diametric end points are (0, 1) and (0, –1)

Sol. (d)

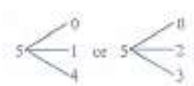
Z lies on semi-circle whose diametric end points are (0, 1) and (0, –1)



Q21. 5 different objects are to be distributed among 3 persons such that no two persons get the same number of objects. Find the number of ways in which it can be done.

(a) 70
(b) 80
(c) 90
(d) None of these

Sol. (c)



$= \frac{5!}{0!.1!.4!} \times 3! + \frac{5!}{0!.2!.3!} \times 3! \;\; = 30 + 60 = 90$ ways

Q22. 12 persons are to be seated at a square table, three on each side. 2 persons with to sit on north side and two wish to sit on the east side. One other person insists on occupying the middle seat (which may be on any side). Find the number of ways they can be seated.

(a) 2! 3! 8!
(b) 2! 3! 9!
(c) 2! 4! 8!
(d) 3! 3! 8!

Sol. (a)

When person who sits on the middle seat sit on the north or east side then number of ways $=$ ${}^{2}C_1 \times \; 2! \times 3! \times 7!$

When the person who sits on the middle seat on the west or south side then number of ways $=$ ${}^{2}C_1 \times \; {}^{3}C_2 \times 2! \times \; {}^{3}C_2 \times 2! \times 7!$

Number of ways$= 7! \times 8 \times 12 = 12 \times 8!$

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Q23. Evaluate the following limits:

$$\lim_{x \to 0} \left(\frac{a^x+b^x+c^x}{3}\right)^{a/x}, (a,b,c>0)$$

(a) $(abc)^{\frac{2}{3}}$
(b) $(abc)^{\frac{2}{4}}$
(c) $(abc)^{\frac{2}{5}}$
(d) $(abc)^{\frac{2}{6}}$

Sol. (a)

$$e^{\lim_{x\to 0}\frac{2}{3}\left(\frac{a^x-1}{x}+\frac{b^x-1}{x}\right)+\frac{c^x-1}{x}} = e^{\frac{2}{3}(lna+lnb+\ln c)} = e^{\frac{2}{3}\ln(abc)} = (abc)^{2/3}$$

Q24. Find the values of a & b

$f(x)$ is the function such that $\lim_{x \to 0} \frac{f(x)}{x} = 1$ *and* $\lim_{x \to 0} \frac{x\,(1+a\cos x)-b\sin x}{(f(x))^3} = 1$, then find the value of a and b.

(a) $a = \frac{+5}{2}, b = \frac{-3}{2}$
(b) $a = \frac{-5}{2}, b = \frac{+3}{2}$
(c) $a = \frac{+5}{2}, b = \frac{+3}{2}$
(d) $a = \frac{-5}{2}, b = \frac{-3}{2}$

Sol. (d)

Use expansion of function

$$\lim_{x \to 0} \frac{x\,(1+a\cos x)-b\sin x}{x^3} = \lim_{x \to 0} \frac{1+a\cos x-b\frac{\sin x}{x}}{x^3}$$

$1+a-b=0$ ……(1)

$$\lim_{x \to 0} \frac{x\,(1+a\cos x)-(1+a)\sin x}{x^3} = \lim_{x \to 0} \frac{(x-\sin x)}{x^3} + \frac{a(x\cos x-\sin x)}{x^3} = 1$$

$\frac{1}{6} + \cos x \cdot a \cdot \frac{(x-\tan x)}{x^3} \quad \Rightarrow \quad \frac{1}{6} - \frac{a}{3} = 1$

$\frac{-a}{3} = \frac{5}{6} \quad \Rightarrow \quad a = \frac{5}{2};$ Now from equation (1), $b = -\frac{3}{2}$

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Q25. The remainder when $8^{2n} - (62)^{2n+1}$ is divided by 9 is

(a) 2
(b) 7
(c) 8
(d) 0

Sol. (a)

Using the modulo arithmetic, we have

$$8 - 1\ (modulo\ 9)$$

Also $62 = -1\ (modulo\ 9)$

$\Rightarrow$ $8^{2n} - (62)^{2n+1} = [(1-)^{2n} - (-1)^{2n+1}]\ mod\ 9$

$\Rightarrow$ Remainder = 2

Q26. $\sim(p \vee q)\ \vee (\sim p\ \wedge q)$ is logically equivalent to

(a) $\sim p$
(b) p
(c) q
(d) $\sim q$

Sol. (a)

$\sim (p \vee q) \vee (\sim p \wedge q) \equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$\equiv \sim p \wedge (\sim q \vee q) \equiv \sim p \wedge t \equiv \sim p$

Q27. The probability that in a family of four children, there will be atleast one boy, is
(a) 1/16
(b) 3/16
(c) 13/16
(d) 15/16

Sol. (d)
The total no. of sample points n(S) =2^4 = 16
Reqd. prob. = 1- probability of having no boy child = 1-(1/16) = 15/16

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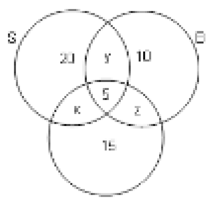
Q28. In a survey of 70 businessmen, it is found that 20 of them own only scooters, 10 own only bikes and 15 own only cars. 5 businessmen own all the three. Find the probability that a businessmen selected at random possess only two items.

(a) 1/7
(b) 2/7
(c) 3/7
(d) 4/7

Sol. (b)

From the Venn diagram, 20 + 10 + 15 + 5 + (x + y + z) = 70.



Therefore, x + y + z = 20.

So, the required probability = 20/70 = 2/7

Q29. YZ-plane divides the line joining the points (3, 5, – 7) and (– 2, 1, 8) in the ratio

(a) 1 : 2
(b) 2 : 3
(c) 3 : 2
(d) 2 : 1

Sol. (c)

Let the ratio be $m : n$, then x-coordinate of the point of division is zero

$$\Rightarrow \quad \frac{-2m+3n}{m+n}=0 \Rightarrow \frac{m}{n}=\frac{3}{2}.$$

Q30. A point R with xco-ordinates 4 lies on the line segment joining the points P(2,-3, 4) and Q (8, 0, 10) find the co-ordinates of the point R

(a) $(4,-2,6)$
(b) $(4,+2,6)$
(c) $(3,-2,6)$
(d) $(3,+2,6)$

Sol. (a)

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Let the point. R divides the line segment joining the point P and Q in the ration λ : 1, Then co – ordinates of Point R

$$\left[\frac{8\lambda+2}{\lambda+1},\frac{-3}{\lambda+1},\frac{10\lambda+4}{\lambda+1}\right]$$

The x co-ordinates of point R is 4

$$\Rightarrow \frac{8\lambda+2}{\lambda+1}=4 \quad , \quad \lambda=\frac{1}{2}$$

$\therefore$ co-ordinates of point R

$$\left[4,\frac{-3}{\frac{1}{2}+1},\frac{10\times\frac{1}{2}+4}{\frac{1}{2}+1}\right] \quad \text{i.e.}(4,-2,6)$$