

**CBSE Board  
Class XI Mathematics  
Sample Paper – 5**

Q1. The sum of the first  $n$  terms of the series  $1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+\dots$  is  $\frac{n(n+1)^2}{2}$ , when  $n$  is even. When  $n$  is odd, the sum is

- (A)  $\frac{n^2(n+1)}{2}$   
(B)  $\frac{n(n+1)(2n+1)}{6}$   
(C)  $\frac{n(n+1)^2}{2}$   
(D)  $\frac{n^2(n+1)^2}{2}$

Sol. (A) If  $n$  is odd,  $n-1$  is even. Sum of  $(n-1)$  terms will be  $\frac{(n-1)(n-1+1)^2}{2} = \frac{n^2(n-1)}{2}$ .

The  $n$ th term will be  $n^2$ . Hence the required sum

$$= \frac{n^2(n-1)}{2} + n^2 = \frac{n^2(n+1)}{2}$$

Q2. The sixth term in the expansion of  $\left(\frac{1}{x^{8/3}} + x^2 \log_{10} x\right)^8$  is 5600 when  $x$  is equal to

- (A) 10  
(B)  $\log_e 10$   
(C) 1  
(D) none of these

Sol. (A)

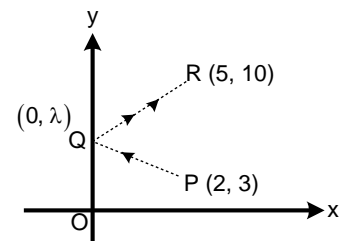
$$T_6 = {}^8C_5 \left(\frac{1}{x^{8/3}}\right)^3 (x^2 \log_{10} x)^5 \Rightarrow 56x^2(\log_{10} x)^5 = 5600$$

$\Rightarrow x^2 (\log_{10} x)^5 = 100$ , obviously  $x = 10$  satisfies the above equation.

Q3. A light ray emerging from the point source placed at  $P(2, 3)$  is reflected at a point 'Q' on the  $y$ -axis and then passes through the point  $R(5, 10)$ . Coordinates of 'Q' is

- (A) (0, 3)  
(B) (0, 2)  
(C) (0, 5)  
(D) (0, 1)

Sol. (C)



If  $P_1$  be the reflection of P in y-axis then  $P_1 = (-2, 3)$

$$\text{Equation of line } P_1R \text{ is } (y - 3) = \frac{10 - 3}{5 + 2}(x + 2) \Rightarrow y = x + 5. \text{ It meets } y\text{-axis at } (0, 5)$$

$$\Rightarrow Q \in (0, 5)$$

Q4. Length of the common chord of circles  $x^2 + y^2 - 2x - 4y + 1 = 0$  and  $x^2 + y^2 - 4x - 2y + 1 = 0$  is

- (A)  $\sqrt{14}$   
 (B)  $\sqrt{15}$   
 (C) 4  
 (D)  $\sqrt{17}$

Sol. (A)

Equation of common chord is

$$s_1 - s_2 = 0$$

$$2x - 2y = 0 \Rightarrow x - y = 0$$

$$PP' = \frac{\sqrt{P'C^2 - PC^2}}{4 - \frac{1}{2}}$$

$$\text{Length of chord} = \frac{2\sqrt{7}}{\sqrt{2}} = \sqrt{14}$$

Q5. Vertex of the parabola whose parametric equation is  $x = t^2 - t + 1, y = t^2 + t + 1; t \in \mathbb{R}$ , is

- (A) (1, 1)  
 (B) (2, 2)  
 (C) (1/2, 1/2)  
 (D) (3, 3)

Sol. (A)

$$x = t^2 - t + 1, y = t^2 + t + 1$$

$$\Rightarrow x + y = 2(t^2 + 1) \text{ and } y - x = 2t$$

$$\Rightarrow \frac{x + y}{2} = 1 + \left(\frac{y - x}{2}\right)^2$$

$$\Rightarrow (y - x)^2 = 2(x + y) - 4$$

$$\Rightarrow (y - x)^2 = 2(x + y - 2).$$

Vertex will be the point where lines  $y - x = 0$  and  $x + y - 2 = 0$  meet, i.e., the point (1, 1).

Q6. If  $(x_1, y_1)$  and  $(x_2, y_2)$  are the ends of a focal chord of the parabola  $y^2 = 4ax$ , then

$$x_1x_2 + y_1y_2 =$$

- (A)  $-3a^2$   
 (B)  $3a^2$

- (C)  $-4a^2$   
 (D)  $4a^2$

Sol. (A)  
 $x_1x_2 + y_1y_2 = a^2 t_1^2 t_2^2 + 4a^2 t_1 t_2 = a^2 - 4a^2 = -3a^2$ .

Q7. A fair die is tossed eight times. Probability that on the eighth throw a third six is observed is,

- (A)  ${}^8C_3 \frac{5^5}{6^8}$   
 (B)  $\frac{{}^7C_2 \cdot 5^5}{6^8}$   
 (C)  $\frac{{}^7C_2 \cdot 5^5}{6^7}$   
 (D) none of these

Sol. (B)  
 Third six occurs on 8th trial. It means that in first 7 trials we must exactly 2 sixes and 8th trial must result in a six.

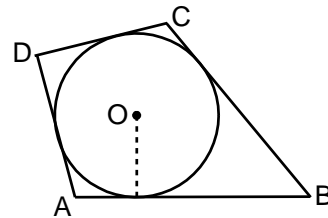
$$\Rightarrow \text{Required probability} = {}^7C_2 \cdot (1/6)^2 \cdot (5/6)^5 \cdot (1/6) = \frac{{}^7C_2 \cdot 5^5}{6^8}.$$

Q8. ABCD is a quadrilateral circumscribed about a circle of unit radius, then:

- (A)  $AB \sin \frac{C}{2} \cdot \sin \frac{A}{2} = CD \sin \frac{B}{2} \cdot \sin \frac{D}{2}$   
 (B)  $AB \sin \frac{A}{2} \cdot \sin \frac{B}{2} = CD \cdot \sin \frac{C}{2} \cdot \sin \frac{D}{2}$   
 (C)  $AB \sin \frac{A}{2} \cdot \sin \frac{D}{2} = CD \sin \frac{C}{2} \cdot \sin \frac{B}{2}$   
 (D)  $AB \sin \frac{A}{2} \cdot \cos \frac{B}{2} = CD \sin \frac{C}{2} \cdot \cos \frac{D}{2}$

Sol. (B)  
 Let 'O' be the centre of circle and 'D' be it's point of contact with side AB,

$$\begin{aligned} \text{Thus, } AD &= OD \cdot \cot \frac{A}{2} = \cot \frac{A}{2} \\ \text{and, } DB &= OC \cdot \cot \frac{B}{2} = \cot \frac{B}{2} \\ \Rightarrow AD + DB &= \cot \frac{A}{2} + \cot \frac{B}{2} \end{aligned}$$



$$= \frac{\sin\left(\frac{A+B}{2}\right)}{\sin\frac{A}{2} \cdot \sin\frac{B}{2}} = AB$$

$$\text{Similarly, } CD = \frac{\sin\left(\frac{C+D}{2}\right)}{\sin\frac{C}{2} \cdot \sin\frac{D}{2}}$$

$$\text{Since } A + B + C = 2\pi$$

$$\therefore \frac{A+B}{2} = \pi - \frac{C+D}{2}$$

$$\Rightarrow \sin\left(\frac{A+B}{2}\right) = \sin\left(\frac{C+D}{2}\right)$$

$$\Rightarrow AB \cdot \sin\frac{A}{2} \cdot \sin\frac{B}{2} = CD \sin\frac{C}{2} \cdot \sin\frac{D}{2}$$

Q9. The sum of all the value of  $m$  for which the roots  $x_1$  and  $x_2$  of the quadratic equation  $x^2 - 2mx + m = 0$  satisfy the condition  $x_1^3 + x_2^3 = x_1^2 + x_2^2$ , is :

- (A)  $\frac{3}{4}$   
(B) 1  
(C)  $\frac{9}{4}$   
(D)  $\frac{5}{4}$

Sol. (D)

$$\text{Given } x_1 + x_2 = 2m \quad x_1 x_2 = m$$

$$\text{According to given condition } x_1^3 + x_2^3 = x_1^2 + x_2^2$$

$$(x_1 + x_2) (x_1^2 + x_2^2 - x_1 x_2) = x_1^2 + x_2^2$$

$$(x_1 + x_2) ((x_1 + x_2)^2 - 3x_1 x_2) = (x_1 + x_2)^2 - 2x_1 x_2$$

$$2m (4m^2 - 3m) = 4m^2 - 2m$$

$$\text{Clearly since is } \frac{5}{4}$$

Q10. The sum of  $n$  terms of the series  $\frac{3}{1^2} + \frac{5}{1^2 + 2^2} + \frac{7}{1^2 + 2^2 + 3^2} + \dots$  is:

- (A)  $\frac{6n}{n+1}$   
(B)  $\frac{n}{n+1}$

- (C)  $\frac{6}{n+1}$   
 (D) None of these

Sol. (A)

$$S_n = \frac{3}{12} + \frac{5}{1^2 + 2^2} + \frac{7}{1^2 + 2^2 + 3^2} + \dots = \sum_{r=1}^n \frac{2r+1}{1^2 + 2^2 + \dots + r^2} = \sum_{r=1}^n \frac{6(2r+1)}{r(r+1)(2r+1)}$$

$$= \sum_{r=1}^n \frac{6}{r(r+1)} = 6 \left[ \sum_{r=1}^n \frac{1}{r} - \frac{1}{r+1} \right] = 6 \left[ 1 - \frac{1}{n+1} \right] = \frac{6n}{n+1}$$

Q11. If  $C_0, C_1, C_2, \dots, C_n$  are binomial coefficients then

$$\lim_{n \rightarrow \infty} \left[ C_n - \left(\frac{2}{3}\right)C_{n-1} + \left(\frac{2}{3}\right)^2 C_{n-2} + \dots + (-1)^n \left(\frac{2}{3}\right)^n C_0 \right] \text{ is}$$

- (A) 0  
 (B) 1  
 (C) -1  
 (D) 2

Sol. (A)

Take  $x = \frac{2}{3}$

$$\lim_{n \rightarrow \infty} [C_n - C_{n-1}x + C_{n-2}x^2 + \dots + (-1)^n C_0 x^n]$$

$$= \lim_{n \rightarrow \infty} [C_0 - C_1x + C_2x^2 + \dots + (-1)^n C_n x^n] = \lim_{n \rightarrow \infty} [1 - x]^n$$

$$= \lim_{n \rightarrow \infty} \left(1 - \frac{2}{3}\right)^n = \lim_{n \rightarrow \infty} \frac{1}{3^n} = 0$$

Q12. If  $a, b, c$  are in H.P. then the straight line  $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$  always passes through a fixed point, that point is

- (A) (-1, -2)  
 (B) (-1, 2)  
 (C) (1, -2)  
 (D) (1, -1/2)

Sol. (C)

Since  $a, b, c$  are in H.P.

$$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

Hence  $\frac{x}{a} + \frac{y}{b} + \frac{2}{b} - \frac{1}{a} = 0$

$$\Rightarrow (x-1) + \frac{a}{b}(y+2) = 0$$

Hence the line always passes through (1, -2)

Q13. A common tangent of  $9x^2 - 16y^2 = 144$  and  $x^2 + y^2 = 9$  is

- (A)  $y\sqrt{7} = 3x\sqrt{2} + 15$
- (B)  $y\sqrt{7} = 3x\sqrt{2} - 15$
- (C)  $y\sqrt{2} = 3x\sqrt{7} + 15$
- (D) None of these

Sol. (D)

Any tangent to  $\frac{x^2}{16} - \frac{y^2}{9} = 1$  is  $y = mx + c$  where,  $c^2 = 16m^2 - 9$ .

If it touch the circle  $x^2 + y^2 = 9$ , then

$$\frac{|c|}{\sqrt{1+m^2}} = 3$$

$$\Rightarrow c^2 = 9(1+m^2) = 16m^2 - 9 \Rightarrow m^2 = \frac{18}{7}$$

$$\Rightarrow c^2 = 16 \cdot \frac{18}{7} - 9 = \frac{225}{7}$$

$\Rightarrow$  Hence, common tangents are

$$y \pm 3x\sqrt{\frac{2}{7}} \pm \frac{15}{\sqrt{7}}$$

$$\text{i.e. } y\sqrt{7} = \pm 3x\sqrt{2} \pm 15$$

Q14. The value of  $\sum_{n=1}^{13} (i^n + i^{n+1})$ , where  $i = \sqrt{-1}$  equals

- (A)  $i$
- (B)  $i-1$
- (C)  $-i$
- (D) none of these

Sol. (B)

$$\text{Given summation} = \sum_{n=1}^{13} i^n + \sum_{n=1}^{13} i^{n+1}$$

$$= i \left( \frac{1-i^{13}}{1-i} \right) + i^2 \left( \frac{1-i^{13}}{1-i} \right) = i \frac{(1-i)}{1-i} - \frac{(1-i)}{1-i} = i-1$$

Q15. If the graph of the continuous function  $y = f(x)$  passes through  $(a, 0)$ , then

$$\lim_{x \rightarrow a} \frac{\ln(1 + 6f^2(x)) - 3f(x)}{3f(x)}$$
 is equal to:

- (A) 1  
 (B) 0  
 (C) -1  
 (D) none of these

Sol. (C)

$$\text{Since } f(a) = 0 \Rightarrow \lim_{x \rightarrow a} (6f^2(x) - 3f(x)) = 0$$

$$\begin{aligned} \lim_{x \rightarrow a} \frac{\ln(1 + 6f^2(x)) - 3f(x)}{3f(x)} &= \lim_{x \rightarrow a} \frac{\ln(1 + 6f^2(x)) - 3f(x)}{(6f^2(x) - 3f(x))} \cdot \frac{(6f^2(x) - 3f(x))}{3f(x)} \\ &= \lim_{x \rightarrow a} \frac{6f(x) - 3}{3} = -1 \end{aligned}$$

Q16. Let  $A = \{(x, y) \mid y = e^x, x \in \mathbb{R}\}$  and  $B = \{(x, y) \mid y = e^{-x}, x \in \mathbb{R}\}$ . Then

- (A)  $A \cap B = \phi$   
 (B)  $A \cap B \neq \phi$   
 (C)  $A \cup B = \mathbb{R}^2$   
 (D) None of these

Sol. (B)

$y = e^x$  and  $y = e^{-x}$  will intersect at one point

$$A \cap B \neq \phi$$

Q17. The smallest positive value of  $x$  (in degrees) for which  $\tan(x + 100^\circ) = \tan(x + 50^\circ) \tan x \tan(x - 50^\circ)$  is :

- (A)  $30^\circ$   
 (B)  $45^\circ$   
 (C)  $60^\circ$   
 (D)  $90^\circ$

Sol. (A)

The relation may be written as  $\frac{\tan(x + 100^\circ)}{\tan(x - 50^\circ)} = \tan(x + 50^\circ) \tan x$

$$\begin{aligned} \Rightarrow \frac{\sin(x + 100^\circ) \cos(x - 50^\circ)}{\sin(x - 50^\circ) \cos(x + 100^\circ)} &= \frac{\sin(x + 50^\circ) \sin x}{\cos(x + 50^\circ) \cos x} \\ \Rightarrow \frac{\sin(2x + 50^\circ) + \sin(150^\circ)}{\sin(2x + 50^\circ) - \sin(150^\circ)} &= \frac{\cos(50^\circ) - \cos(2x + 50^\circ)}{\cos(50^\circ) + \cos(2x + 50^\circ)} \\ \Rightarrow \frac{\sin(2x + 50^\circ)}{\sin 150^\circ} &= \frac{-\cos 50^\circ}{\cos(2x + 50^\circ)} \Rightarrow \cos 50^\circ + 2 \sin(2x + 50^\circ) \cos(2x + 50^\circ) = 0 \end{aligned}$$

$$\begin{aligned} \Rightarrow \cos 50^\circ + \sin(4x + 100^\circ) &= 0 \Rightarrow \cos 50^\circ + \cos(4x + 10^\circ) = 0 \\ \Rightarrow \cos(2x + 30^\circ) \cos(2x - 20^\circ) &= 0 \Rightarrow x = 30^\circ, 55^\circ \\ \Rightarrow \text{The smallest value of } x &= 30^\circ \end{aligned}$$

Q18. The value of is  $\sum_{r=1}^5 \cos(2r-1)\frac{\pi}{11}$ :

- (A)  $\frac{1}{2}$   
(B)  $\frac{1}{3}$   
(C)  $\frac{1}{4}$   
(D)  $\frac{1}{6}$

Sol. (A)

$$\begin{aligned} &\sum_{r=1}^5 \cos(2r-1)\frac{\pi}{11} \\ &\cos \frac{\pi}{11} + \cos \frac{3\pi}{11} + \cos \frac{5\pi}{11} + \cos \frac{7\pi}{11} + \cos \frac{9\pi}{11} \\ &= \frac{2\sin \frac{\pi}{11}}{2\sin \frac{\pi}{11}} \left( \cos \frac{\pi}{11} + \cos \frac{3\pi}{11} + \cos \frac{5\pi}{11} + \cos \frac{7\pi}{11} + \cos \frac{9\pi}{11} \right) \\ &= \frac{\left( \sin \frac{2\pi}{11} + \sin \frac{4\pi}{11} - \sin \frac{2\pi}{11} + \sin \frac{6\pi}{11} - \sin \frac{4\pi}{11} + \sin \frac{8\pi}{11} - \sin \frac{6\pi}{11} + \sin \frac{10\pi}{11} - \sin \frac{8\pi}{11} \right)}{2\sin \frac{\pi}{11}} \\ &= \frac{\sin \frac{10\pi}{11}}{2\sin \frac{\pi}{11}} = \frac{\sin \left( \pi - \frac{\pi}{11} \right)}{2\sin \frac{\pi}{11}} = \frac{1}{2} \end{aligned}$$

Q19. The foci of the ellipse  $25(x+1)^2 + 9(y+2)^2 = 225$  are at  
(A) (-1, 2) and (-1, -6)  
(B) (-2, 1) and (-2, 6)  
(C) (-1, -2) and (-2, -1)  
(D) (-1, -2) and (-1, -6).

Sol. (A)



The given equation is  $25(x+1)^2 + 9(y+2)^2 = 225$

$$\text{Or, } \frac{(x+1)^2}{9} + \frac{(y+2)^2}{25} = 1$$

Centre of the ellipse  $\equiv (-1, -2)$

and  $a = 3$ ,  $b = 5$ , so that  $a < b \Rightarrow 3 = 5\sqrt{1-e^2}$

or  $e^2 = 1 - 9/25 = 16/25 \Rightarrow e = 4/5$

Hence foci are  $(-1, -2 - 5 \times 4/5)$  and  $(-1, -2 + 5 \times 4/5)$

i.e. foci are  $(-1, -6)$  and  $(-1, 2)$ .

Q20. If  $\left| \frac{z_1}{z_2} \right| = 1$  and  $\arg(z_1 z_2) = 0$ , then

(A)  $z_1 = z_2$

(B)  $|z_2|^2 = z_1 z_2$

(C)  $z_1 z_2 = 1$

(D) none of these.

Sol. (B)

Let  $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$  then  $\left| \frac{z_1}{z_2} \right| = 1$

$$\Rightarrow |z_1| = |z_2| \Rightarrow |z_1| = |z_2| = r_1.$$

$$\text{Now } \arg(z_1 z_2) = 0 \Rightarrow \arg(z_1) + \arg(z_2) = 0 \Rightarrow \arg(z_2) = -\theta_1$$

$$\text{Therefore, } z_2 = r_1(\cos(-\theta_1) + i\sin(-\theta_1)) = r_1(\cos\theta_1 - i\sin\theta_1) = \bar{z}_1$$

$$\Rightarrow \bar{z}_2 = (\bar{\bar{z}_1}) = z_1 \Rightarrow |z_2|^2 = z_1 z_2.$$

Q21.  $\lim_{x \rightarrow \infty} \frac{\ln[x]}{x} =$

(A) 1

(B) 0

(C) -1

(D) does not exist

Sol. (B)

$$x-1 \leq [x] \leq x$$

$$\Rightarrow \frac{\ln(x-1)}{x} \leq \frac{\ln[x]}{x} \leq \frac{\ln x}{x} \Rightarrow \lim_{x \rightarrow \infty} \frac{\ln[x]}{x} = 0$$

Q22. A spherical balloon is pumped at the constant rate of  $3 \text{ m}^3/\text{min}$ . The rate of increase of its surface area at certain instant is found to be  $5 \text{ m}^2/\text{min}$ . At this instant, its radius is equal to

- (A)  $\frac{1}{5} \text{ m}$
- (B)  $\frac{3}{5} \text{ m}$
- (C)  $\frac{6}{5} \text{ m}$
- (D)  $\frac{2}{3} \text{ m}$

Sol. (C)

$$\frac{dv}{dt} = 3 \text{ m}^3 / \text{m m}$$

$$\frac{dA}{dt} = 5 \text{ m}^2 / \text{m m}$$

We know  $V = \frac{4}{3} \pi r^3$  and  $A = 4\pi r^2$

$$\Rightarrow \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt} \quad \text{and} \quad \frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$\Rightarrow r = 6/5 \text{ m}$$

Q23. Three natural numbers are taken at random from the set  $A = \{x : 1 \leq x \leq 100, x \in \mathbb{N}\}$ . then the probability of that AM of the number taken is 25

(A)  $\frac{{}^{77}C_2}{{}^{100}C_3}$

(B)  $\frac{{}^{25}C_2}{{}^{100}C_3}$

(C)  $\frac{{}^{74}C_2}{{}^{100}C_3}$

(D) none of these

Sol. (C)

$$x_1 + x_2 + x_3 = 75$$

$$\text{coefficient } x^{75} \text{ in } (x^1 + x^2 + \dots + x^{100})^3 = {}^{74}C_2$$

$$\text{Hence probability} = \frac{{}^{74}C_2}{{}^{100}C_3}.$$

Q24. A teacher takes 3 children from her class to the zoo at a time as often as she can, but she doesn't take the same set of three children more than once. She finds out that she goes to the zoo 84 times more than a particular child goes to the zoo, Total number of students in her class in equal to:

- (A) 12
- (B) 14
- (C) 10
- (D) 11

Sol.

(C)  
 Let the number of students be  $n$ . Then total number times the teacher goes to zoo is equal to  ${}^n C_3$  and total number of times a particular student goes to the zoo is equal to  ${}^{n-1} C_2$

Thus  ${}^n C_3 - {}^{n-1} C_2 = 84$

$$\Rightarrow \frac{n(n-1)(n-2)}{3!} - \frac{n(n-1)(n-2)}{2} = 84$$

$$\Rightarrow n(n-1)(n-2) - 3(n-1)(n-2) = 504$$

$$\Rightarrow (n-1)(n-2)(n-3) = 504$$

$$\Rightarrow (n-1)(n-2)(n-3) = 9 \cdot 8 \cdot 7$$

$$\Rightarrow n = 10$$

Q25. The equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0) and (0, 0, 1) and has its radius as small as possible is :

- (A)  $3(x^2 + y^2 + z^2) + 2(x + y + z) - 1 = 0$
- (B)  $3(x^2 + y^2 + z^2) - 2(x + y + z) - 1 = 0$
- (C)  $(x^2 + y^2 + z^2) - 2(x + y + z) - 1 = 0$
- (D)  $(x^2 + y^2 + z^2) + 2(x + y + z) - 1 = 0$

Sol.

(B)

Let equation of sphere be given by

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0 \quad \dots(1)$$

As sphere passes through points (1, 0, 0), (0, 1, 0) and (0, 0, 1). So we have

$$1 + 2u + d = 0, 1 + 2v + d = 0, 1 + 2w + d = 0$$

$$\text{On solving } u = v = w = -\frac{1}{2}(d + 1)$$

If  $r$  is the radius of the sphere, then

$$r = \sqrt{u^2 + v^2 + w^2 - d}$$

$$r^2 = \frac{3}{4}(d + 1)^2 - d = \mu \text{ (say)}$$

for  $r$  to be minimum

$$\frac{d\mu}{dd} = 0 \Rightarrow \frac{3}{4} \cdot 2(d + 1) - 1 = 0 \text{ or } d = -\frac{1}{3}$$

$$\text{Also, } \frac{d^2\mu}{dd^2} = \frac{3}{2} = \text{positive at } d = -\frac{1}{3}$$

$$\text{Hence } \mu \text{ is minimum at } d = -\frac{1}{3}$$

So, substituting value of d we have  $u = v = w = -\frac{1}{3}$

∴ Equation of the sphere

$$x^2 + y^2 + z^2 - \frac{2}{3}(x + y + z) - \frac{1}{3} = 0 \Rightarrow 3(x^2 + y^2 + z^2) - 2(x + y + z) - 1 = 0.$$

Q26. There are 20 persons among whom are two brothers. The number of ways in which we can arrange them around a circle so that there is exactly one person between the brothers is .

- (A) 19!
- (B)  $2 \times 18!$
- (C)  $2! 17!$
- (D) None of these

Sol. (B)

We can arrange 18 persons around a circle in  $(18-1)! = 17!$  Ways. Now, there are exactly 18 places where we can arrange the two brothers. Also, the two brothers can be arranged in  $2!$  Ways. Thus, the number of ways of arranging the persons subject to the given condition is  $(17!)(18)(2!) = 2(18!)$ .

Q27. If the area of the triangle on the complex plane formed by the points  $z$ ,  $iz$  and  $z + iz$  is 50 square units, then  $|z|$  is

- (A) 5
- (B) 10
- (C) 15
- (D) none of these

Sol. (B)

$$\frac{1}{2} |z|^2 = 50 \Rightarrow |z|^2 = 100 \Rightarrow |z| = 10$$

Q28. In a triangle ABC, the value of

$$\frac{\cos^2 B - \cos^2 C}{b + c} + \frac{\cos^2 C - \cos^2 A}{c + a} + \frac{\cos^2 A - \cos^2 B}{a + b} \text{ is :}$$

- (A) 0
- (B) 1
- (C) 2
- (D) 3

Sol. (A)

$$\frac{\cos^2 B - \cos^2 C}{b + c} = \frac{\sin^2 C - \sin^2 B}{k(\sin B + \sin C)} = \frac{\sin C - \sin B}{k}$$

$$\sum \frac{\cos^2 B - \cos^2 C}{b + c} = \frac{1}{k} \sum (\sin C - \sin B) = 0$$

Q29. When a particle be kept at rest under the action of the following force

- (A) - 8N, - 5N, 13N<sup>-</sup>
- (B) - 7N, - 4N, - 12N
- (C) - 5N, - 8N, - 10N
- (D) 14N, - 2√5N, - 6N

Sol. (A)

Since 8 N + 5 N = 13 N

∴ 13 N ↓ is equal and opposite to the resultant of 8 N ↑ and 5 N ↑.

Q30. The line  $y = x + p$  ( $p$  is parameter) cuts the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  at P and Q, then midpoint of PQ

lies on

- (A)  $a^2y + b^2x$
- (B)  $a^2y + b^2x = 0$
- (C)  $ay + bx = 0$
- (D) none of these

Sol. (B)

Solving the line and ellipse, we get

$$\frac{x^2}{a^2} + \frac{(x+p)^2}{b^2} = 1$$

$$\Rightarrow (a^2 + b^2)x^2 + 2a^2px + a^2p^2 - a^2b^2 = 0$$

$$\Rightarrow \bar{x} \frac{x_1 + x_2}{2} = \frac{-a^2p}{a^2 + b^2} \text{ and } \bar{y} \frac{y_1 + y_2}{2} = \frac{b^2p}{a^2 + b^2}$$

$$\Rightarrow \bar{y} = \frac{-b^2}{a^2} \bar{x} \Rightarrow \text{mid point lies on } y = \frac{-b^2}{a^2} x$$