Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

CBSE Board
Class XI Mathematics
Sample Paper – 7

Q1. Statement – 1: $f : R \to R$ $defined\ by\ f(x) = 7x - [7x]\ where\ [.]\ denotes\ greatest$ integer $\leq x\ \forall\ x\ \in R, f\ is\ not\ one - one\ function.$

Statement – 2: Periodic functions are always many one.

(a) If both statement -1 and statement -2 are true and statement -2 is the correct explanation of statement -1
(b) If both statement -1 and statement -2 are true but statement -2 is not the correct explanation of statement -1
(c) If statement -1 is true but statement -2 is false
(d) If statement -1 is false and statement -2 is true.

Sol. (a)

Let λ be the period of $f(x) = 7x - [7x]$

$\therefore f(x+\lambda) = f(x)$ for $\lambda = \frac{1}{7}$

$\Rightarrow f\left(x + \frac{1}{7}\right) = f(x)$

$\Rightarrow f(x)$ is periodic with period $\frac{1}{7}$

$\Rightarrow$ Periodic functions are many one functions

$\Rightarrow$ Statement - 2 is true & many one function cannot be one - one function so Statement - 1 is true.

Q2. $\begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix}$

(a) 0
(b) abc
(c) abc (a+b+c)
(d) ab+bc+ca

Sol. (a)

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

$$\begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix} = ca^2b(a+b) - a^2bc(c+a) - bc[ab(a+b) - ca(c+a)]$$

$$+ bc(b+c)(ab-ac)$$

$$= abc[a^2 + ab - ac - a^2 - ab - b^2 + c^2 + ac + b^2 - c^2] = 0$$

Q3. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 7$ then the angle between $\vec{a}$ & $\vec{b}$ is

(a) π/4
(b) π/6
(c) π/3
(d) 2π/3

Sol. (c)

Q4. How many different arrangements can be made out of the letters in the expansion $A^2B^3C^4$. When written in full?

(a) $\frac{9!}{2!3!4!}$
(b) $2! + 3! + 4!\,(2!\,3!\,4!)$
(c) $2!\,3! - 4$
(d) $\frac{9!}{2!+3!+4!}$

Sol. (a)

Q5. P. Q, R, S are to give lectures to an audience. The organizer can arrange the order of their presentation in

(a) 4 ways
(b) 12 ways
(c) 256 ways
(d) 24 ways

Sol. (d)

Q6. If $\frac{a+b}{1-ab}, b, \frac{b+c}{1-bc}$ $are\ in\ A.P.then$ $\frac{1}{a}, b, \frac{1}{c}$ $are\ in$

(a) A.P.

(b) G.P.

(c) A.G.P.

(d) None of these

Sol. (a)

$$\frac{a+b}{1-ab}+\frac{b+c}{1-bc}=2b$$

$$(a+b)(1-bc)+(b+c)(1-ab)=2b(1-ab)(1-bc)$$

$$\Rightarrow (b^2+1)(2abc-(a+c))=0(\text{on simplifica tion}).$$

$$\therefore \frac{1}{\text{b}}=\frac{2ac}{a+c}(\text{on simplifica tion}).$$

Q7. The point (-4, -1) , (-2, -4), (4, 0) and (2,3) are the vertices of a

(a) Rhombus

(b) Rectangle

(c) Square

(d) Trapezium

Sol. (b)

Q8. If the angle between the lines $ax^2-2hxy+by^2=45°, then$

(a) $a^2+6ab+b^2=4h^2$

(b) $a^2-ab+b^2=h^2$

(c) $a^2+3ab+b^2=h^2$

(d) $(a+b)^2=4h^2$

Sol. (a)

$$ax^2-2hxy+by^2=0$$

$$\tan 45=\frac{2\sqrt{h^2-ab}}{a+b}$$

$$a+b=2\sqrt{h^2-ab}$$

$$(a+b)^2=4(h^2-ab)$$

$$a^2+6ab+b^2=4h^2$$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Q9. The distance between the centres of the circles $x^2 + y^2 + 4x + 6y - 8 = 0$ and $x^2 + y^2 + 6x - 4y + 12 = 0$ is
(a) $5\sqrt{2}$
(b) 5
(c) $\sqrt{26}$
(d) None of these

Sol. (c)

circle $x^2 + y^2 + 4x + 6y - 8 = 0$
centre(-2,-3)
and $x^2 + y^2 + 6x - 4y + 12 = 0$ centre(-3,2)
distance

$$\sqrt{(-2+3)^2 + (2+3)^2} = \sqrt{26}$$

Q10. The length of a latus rectum of the hyperbola $12x^2 - 4y^2 = 27$ *is*
(a) 9
(b) 51/4
(c) 9/2
(d) None of these

Sol. (a)

$$\frac{x^2}{\frac{27}{12}} - \frac{y^2}{\frac{27}{4}} = 1$$

$$length..of..the..latus..rectum = \frac{2b^2}{a} = 9$$

where

$a^2 = 27/12$

$b^2 = 27/4$

Q11. A tetrahedron has vertices at 0 (0,0,0), A (1,2,1) B (2,1,3) and C (-1,1,2). Then the angle between the faces OAB and ABC will be
(a) $\cos^{-1}\left(\frac{19}{35}\right)$
(b) $\cos^{-1}\left(\frac{17}{31}\right)$
(c) 30°
(d) 90°

Sol. (a)

Q12. $\frac{1+\tan^2\left(\frac{\pi}{4}-A\right)}{1-\tan^2\left(\frac{\pi}{4}-A\right)} =$

(a) Sin2A
(b) Cosec 2A
(c) Cos 2A
(d) Sec 2A

Sol. (b)

$$\frac{1+\tan^2(\frac{\pi}{4}-\text{A})}{1-\tan^2(\frac{\pi}{4}-\text{A})} = \frac{1}{\cos(\frac{\pi}{2}-2A)} = \frac{1}{\sin 2A} = \cos ec 2A$$

Q13. In ΔABC, the angles A, B, C are in A.P. and the sides a, b, c are in G.P., then the triangle is

(a) Equilateral
(b) Isosceles right angled
(c) Right angled
(d) Obtuse angled

Sol. (a)

In a triangle the angle A, B, C are in A.P. and the sides, a, b, c are in G.P. , then the triangle is equilateral

Q14. If $\cot^{-1}\frac{1}{x}+\cot^{-1}\frac{1}{y}+\cot^{-1}\frac{1}{z}=\frac{\pi}{2}$ *, then* $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=$

(a) 0
(b) 1
(c) xyz
(d) $(xyz)^{-1}$

Sol. (d)

$$\cot^{-1}\frac{1}{x}+\cot^{-1}\frac{1}{y}+\cot^{-1}\frac{1}{z}=\frac{\pi}{2}$$

$$\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}$$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

$$\frac{x+y+z-xyz}{1-xy-yz-zx}=\infty$$

$$1-xy-yz-zx=0$$

$$\frac{1}{xyz}-\frac{1}{z}-\frac{1}{x}-\frac{1}{y}=0$$

$$(xyz)^{-1}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$$

$$\text{then } \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=(xyz)^{-1}$$

Q15. If $3^{50}(x+iy)=\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)^{100}$ $\forall\ x, y \epsilon R$, then ordered pair (x, y) is given by

(a) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

(b) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

(c) $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

(d) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

Sol. (a)

$$\text{Consider } \left|z^2+2z \quad \cos \quad \alpha\right| \leq |z|^2+2|z||\cos\alpha|$$

$$\leq |z|^2+2|z|$$

$$<(\sqrt{3}-1)^2+2(\sqrt{3}-1)$$

$$=3+1-2\sqrt{3}+2\sqrt{3}-2$$

$$=2$$

$$\therefore \left|z^2+2z\cos\alpha\right|<2$$

Q16. Which is not the factor of $x^4 + 4$?

(a) $x + 1 + i$

(b) $x + 1 - i$

(c) $x - 1 + I$

(d) $x - 1 + 2i$

Sol. (d)

$$x^4 + 4$$
$$= x^4 + 4x^2 + 4 - 4x^2$$
$$= (x^2 + 2)^2 - (2x)^2$$
$$= (x^2 - 2x + 2)(x^2 + 2x + 2)$$
$$= (x^2 + 2x + 1 + 1)(x^2 - 2x + 1 + 1)$$
$$= ((x+1)^2 - i^2)[(x-1)^2 - i^2]$$
$$= (x+1-i)(x+1+i)(x-1-i)(x-1+i)$$

Q17. If $(x^2 + x + 3)^2 - (p - 3)(x^2 + x + 3)(x^2 + x + 2) + (p - 1)(x^2 + x + 2)^2 = 0$ has at least one real root, then range of p is

(a) $\left[-5, \frac{39}{7}\right]$

(b) $\left(5, \frac{39}{7}\right]$

(c) $\left(5, -\frac{39}{7}\right)$

(d) None of these

Sol. (b)

The given equation can be written as

$$\left(\frac{x^2+x+3}{x^2+x+2}\right)^2 - (p-3)\left(\frac{x^2+x+3}{x^2+x+2}\right) + (p-4) = 0.$$

$$\Rightarrow R^2 - (p-3)R + (P-4) = 0 \qquad \text{...(*)}$$

where $R = \dfrac{x^2+x+3}{x^2+x+2}$

$$= 1 + \frac{1}{x^2+x+2} \geq 1$$

Now (R) will be maximum if (x^2+x+2) is minimum and minimum of x^2+x+2 is $\frac{7}{4}$

$\therefore$ Maximum of $R = 1 + \frac{1}{7/4} = \frac{11}{7}$

$\therefore R \in \left(1, \frac{11}{7}\right]$ i.e $1 < R \leq \frac{11}{7}$

Now at least one root of (*) must lies in

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

$1 < R \leq \frac{11}{7}$.

Now roots of the equation (*) are p - 4 and 1.

We must have $1 < p - 4 \leq \frac{11}{7}$

$\Rightarrow 5 < p \leq \frac{11}{7} + 4 \Rightarrow 5 < p \leq \frac{39}{7}$

$\Rightarrow$ Minimum value of p is greater th an 5 and Maximum value of p is $\frac{39}{7}$.

$\therefore p \in \left(5, \frac{39}{7}\right]$

2nd solution :

Let $x^2 + x + 2 = t$

$\therefore (x^2 + x + 3)^2 - (p - 3)(x^2 + x + 3)$

$(x^2 + x + 2) + (p - 4)(x^2 + x + 2)^2 = 0$

$\Rightarrow (t + 1)^2 - (p - 3) \;\; t \;\; (t + 1) + (p - 4)t^2 = 0$

$\Rightarrow 2t + 1 - pt + 3t = 0$

$\Rightarrow (5 - p)t + 1 = 0$

$t = \frac{1}{p - 5}$

$\Rightarrow x^2 + x + 2 = \frac{1}{p - 5}$

$\Rightarrow \left(x + \frac{1}{2}\right)^2 + \frac{7}{4} = \frac{1}{p - 5}$

$\Rightarrow \frac{1}{p - 5} \geq \frac{7}{4}\left(\because \left(x + \frac{1}{2}\right)^2 \geq \frac{7}{4}\right)$

$\Rightarrow p - 5 > 0 \;\; and \;\; p - 5 \leq \frac{4}{7}$

$\Rightarrow p > 5 \;\; and \;\; p \leq \frac{39}{7}$

$\therefore 5 < p \leq \frac{39}{7} \;\; \therefore p \in \left(5, \frac{39}{7}\right]$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Q18. $\underset{x \to 0}{Lt} \quad \frac{\tan^2 \frac{9x}{2} cosec \frac{2x}{9}}{\sin \frac{81x}{4}} =$

(a) 1

(b) 9/2

(c) 2/9

(d) None of these

Sol. (b)

$$\underset{x \to 0}{Lt} \frac{\tan^2 \frac{9x}{2} \operatorname{cosec} \frac{2x}{9}}{\sin \frac{81x}{4}}$$

$$= \underset{x \to 0}{Lt} \frac{(\frac{9x}{2})^2 (\frac{2x}{9})(\frac{81x}{4}) \sin^2 \frac{9x}{2} \operatorname{cosec} \frac{2x}{9}}{(\frac{9x}{2})^2 (\frac{2x}{9})(\frac{81x}{4}) \sin \frac{81x}{4}} = \frac{9}{2}$$

Q19. The area of the triangle formed by the coordinate axes and the tangent to the curve $x = 2t^2 + 1, \; y = t^3 + 1 \; at \; t = -1 \; is$

(a) 27/4

(b) 27/8

(c) 6

(d) 5

Sol. (a)

$\frac{dx}{dt} = 4t$

$\frac{dy}{dt} = 3t^2$

$\frac{dy}{dx} = \frac{3t^2}{4t} = -\frac{3}{4} at..t = -1$

$then...x = 3, y = 0$

$\tan gent$

$y = -1(x-3) = -x + 3$

$y = 0 then...x = 3$

$x = 0..then..y = 3$

$area = \frac{1}{2} 3.3 = \frac{9}{2}$

Q20. $\int \frac{\sec^5 x}{cosec\, x}\, dx\ =$

(a) $\frac{1}{4}\sec^4\ x$

(b) $\frac{1}{3}\sec^3 x$

(c) $\sec^2 x\ \ cosec\, x$

(d) None of these

Sol. (a)

$$\int \frac{\sec^5\ x}{\text{cosec}\ x}\,dx = \int \sec^4 x \tan x dx$$

$$= \frac{1}{4}\sec^4\ x + c$$

Q21. The area of the region bounded by the curve y = log x, the x – axis and the line x = 2 is

(a) Log_2

(b) 2

(c) 3/2

(d) 2 $\log_2$ – 1

Sol. (d)

$$\int_1^2 \log x dx = [x \log x - x]_1^2 = 2\log 2 - 2 + 1 = 2\log 2 - 1$$

Q22. The integrating factor of the differential equation $(y \log y)dx = (\log y - x)\ dy\ \ is$

(a) $\frac{1}{\log y}$

(b) $\log(\log y)$

(c) $1 +\ \log y$

(d) $\log y$

Sol. (d)

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

$$y\log y \quad dx = (\log y - x)dy$$

$$\Rightarrow \frac{dy}{dx} = \frac{y\log y}{\log y - x} \Rightarrow \frac{dx}{dy} = \frac{\log y - x}{y\log y}$$

$$\Rightarrow \frac{dx}{dy} = \frac{1}{y} - \frac{x}{y\log y} \Rightarrow \frac{dx}{dy} + \frac{1}{y\log y}x = \frac{1}{y}$$

$$I.F. = e^{\int \frac{1}{y\log y}dy} = e^{\int \frac{1}{t}dt}$$

$$= e^{\log_e t} \qquad (where \quad t = \log y)$$

$$= t = \log y$$

$$I.F. = \log y$$

Q23. A team of 4 musicians is to be chosen at random out of 3 boys. 2 girls and 4 children. The chance that exactly two of them will be children is

(a) 10/12
(b) 4/9
(c) 3/43
(d) None of these

Sol. (a)

Q24. The probability that the birth days of six different persons will fall in exactly two months is

(a) 1/6
(b) $^{12}C_2 \times \frac{2^6}{12^6}$
(c) $^{12}C_2 \times \frac{2^6-1}{126}$
(d) $\frac{341}{12^5}$

Sol. (d)

Q25. The median from the following data:

Wages/week (Rs.)	No. of workers
50-59	15
60-69	40
70-79	50
80-89	60
90-99	45
100-109	40
110-119	15

is

(a) 83.17
(b) 84.08
(c) 82.17
(d) 85.67

Sol. (b)

Q26. If A is an 3 ×3 non – singular matrix such that AA' = A'A and B = A^{-1} A', then BB' equals
(a) $I + B$
(b) I
(c) B^{-1}
(d) $(B^{-1})'$

Sol. (b)

$$B = A^{-1}A' \Rightarrow AB = A'$$

$$ABB' = A'B' = (BA)' = (A^{-1}A'A)' = (A^{-1}AA')' = A.$$

$$\Rightarrow BB' = I.$$

Q27. A ray of light along $x + \sqrt{3y} = \sqrt{3}$ gets reflected upon reaching x – axis, the equation of the reflected ray is
(a) $y = \sqrt{3x} - \sqrt{3}$
(b) $\sqrt{3}\, y = x - 1$
(c) $y = x + \sqrt{3}$
(d) $\sqrt{3}\, y = x - \sqrt{3}$

Sol. (d)
Slope of the incident ray is - 1/√3.
So, the slope of the reflected ray must be 1/√3.
The point of incidence is $(\sqrt{3}, 0)$. So, the equation of reflected ray is $Y = \frac{1}{\sqrt{3}}\ (x - \sqrt{3})$.

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Q28. An ellipse is drawn by taking a diameter of the circle $(x-1)^2 + y^2 = 1$ as its semiminor axis and a diameter of the circle $x^2 + (y-2)^2 = 4$ as its semi – major axis. If the centre of the ellipse is the origin and its axes are the coordinate axes, then the equation of the ellipse is

(a) $4x^2 + y^2 = 4$
(b) $x^2 + 4y^2 = 8$
(c) $4x^2 + y^2 = 8$
(d) $x^2 + 4y^2 = 16$

Sol. (d)

Semi minor axis b = 2

Semi major axis a = 4

Equation of ellipse $= \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \Rightarrow \frac{x^2}{16} + \frac{y^2}{4} = 1$

$\Rightarrow x^2 + 4y^2 = 16.$

Q29. $\frac{d^2x}{dy^2}$ *equals*

(a) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$
(b) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$
(c) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$
(d) $\left(\frac{d^2y}{dx^2}\right) - 1$

Sol. (c)

$$\frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left(\frac{1}{\left(\frac{dy}{dx}\right)}\right) = -\frac{1}{\left(\frac{dy}{dx}\right)^2}\frac{d}{dy}\left(\frac{dy}{dx}\right)$$

$$= -\left(\frac{dy}{dx}\right)^{-2}\frac{1}{\left(\frac{dy}{dx}\right)}\frac{d}{dx}\left(\frac{dy}{dx}\right) = -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Q30. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

(a) $2x + 1 = 0$

(b) $x = -1$

(c) $2x - 1 = 0$

(d) $x = 1$

Sol. (b)

The locus of perpendicular tangents is directrix, i.e,

$x = -a$; $x = -1$