

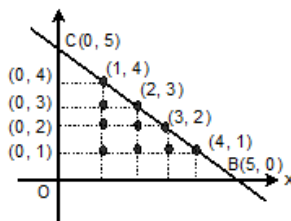
CBSE Board
Class XI Mathematics
Sample Paper – 8

- Q1. If the quadratic equations $3x^2 + ax + 1 = 0$ and $2x^2 + bx + 1 = 0$ have a common root then the value of $5ab - 2a^2 - 3b^2$, where $a, b \in \mathbb{R}$, is equal to
(A) Zero
(B) 1
(C) -1
(D) none of these

Sol. (B)
 $3x^2 + ax + 1 = 0$, $2x^2 + bx + 1 = 0$ have a common root.
Subtracting these equations, we get
 $x^2 + (a - b)x = 0$
 $\Rightarrow x = 0, x = (b - a)$
Clearly the common root is $(b - a)$
 $\Rightarrow 3(b - a)^2 + a(b - a) + 1 = 0$
 $\Rightarrow 3b^2 + 3a^2 - 6ab + ab - a^2 + 1 = 0$
 $\Rightarrow 5ab - 2a^2 - 3b^2 = 1$

- Q2. $P(x, y)$ is called a good point if $x, y \in \mathbb{N}$. Total number of good points lying inside the quadrilateral formed by the line $2x + y = 2$, $x = 0$, $y = 0$ and $x + y = 5$, is equal to
(A) 4
(B) 2
(C) 6
(D) 1

Sol. (c)
Adjacent figure indicates that there are exactly six good points inside the quadrilateral ABCD.



- Q3. The equation of the ellipse with $e = \frac{3}{4}$, foci on y -axis, centre at the origin and passing through the point $(6, 4)$ is
(A) $x^2 + 2y^2 = 16$

- (B) $16x^2 + 7y^2 = 688$
(C) $16x^2 + 7y^2 = 344$
(D) none of these

Sol. (B)

$$b^2 = a^2 \left(1 - \frac{9}{16}\right) = \frac{7}{16}a^2$$

Then equation is $\frac{16x^2}{7a^2} + \frac{y^2}{a^2} = 1$, it passes through (6, 4)

$$\Rightarrow \frac{16 \times 6^2}{7a^2} + \frac{16}{a^2} = 1 \Rightarrow a^2 = \frac{688}{7}$$

$$\Rightarrow 16x^2 + 7y^2 = 688 \text{ is ellipse.}$$

Q4.

$\{a_i\}, i = 1, 2, \dots, n$ is arithmetic sequence. If $a_7 = 9$ then value of common difference of the A.P. such that a_1, a_2, a_7 is minimum, is equal to

(A) $11/20$

(B) $\frac{11}{10}$

(C) $\frac{33}{20}$

(D) None of these

Sol. (C)

$$a_7 = a_1 + 6d$$

$$\Rightarrow a_2 = a_1 + d$$

$$a_1 a_2 a_7 = a_1(a_1 + d)^2$$

$$= 9a_1 \left(a_1 + \frac{9 - a_1}{6}\right)$$

$$= \frac{3}{2}a_1(5a_1 + 9)$$

$$= f(a_1) \quad (\text{say})$$

$$\Rightarrow f(a_1) = \frac{3}{2}(a_1 \cdot 5 + 5a_1 + 9)$$

$$= 15 \left(a_1 + \frac{9}{10}\right)$$

Clearly, $a_1 = -\frac{9}{10}$ is the point of minima for $f(a_1)$

$$\Rightarrow d = \frac{9 + \frac{9}{10}}{6} = \frac{33}{20}$$

- Q5. The sum of infinitely many terms of the series $\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \dots$ is:
- (A) 0
(B) 4
(C) 6
(D) Can't be determined

Sol. (C)

$$S_n = \frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \dots = \sum_{r=1}^n \frac{2r+1}{1^2+2^2+\dots+r^2} = \sum_{r=1}^n \frac{6(2r+1)}{r(r+1)(2r+1)}$$

$$= \sum_{r=1}^n \frac{6}{r(r+1)} = 6 \left[\sum_{r=1}^n \frac{1}{r} - \frac{1}{r+1} \right] = 6 \left[1 - \frac{1}{n+1} \right] = \frac{6n}{n+1}$$

$$\text{Now, } S_\infty = \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \frac{6n}{n+1} = \lim_{n \rightarrow \infty} \frac{6}{1+1/n} = 6$$

- Q6. The value of ${}^{2n}C_n - {}^nC_1 \cdot {}^{2n-2}C_n + {}^nC_2 \cdot {}^{2n-4}C_n - \dots$ is equal to
- (A) 3^n
(B) 4^n
(C) 5^n
(D) none of these

Sol. (D)

$${}^{2n}C_n - {}^nC_1 \cdot {}^{2n-2}C_n + {}^nC_2 \cdot {}^{2n-4}C_n - \dots$$

$$= \text{coefficient of } x^n \text{ in } [{}^nC_0 (1+x)^{2n} - {}^nC_1 (1+x)^{2n-2} + {}^nC_2 (1+x)^{2n-4} - \dots]$$

$$= \text{coefficient of } x^n \text{ in } [1 - (1+x)^2]^n = 2^n$$

- Q7. The length of the latus rectum of the parabola whose focus is (3, 3) and directrix is $3x - 4y - 2 = 0$ is
- (A) 2
(B) 3
(C) 1
(D) 4

Sol. (A)

$$\text{Semilatus rectum} = \frac{|3 \times 3 - 4 \times 3 - 2|}{5} = 1, \text{ so latus rectum} = 2.$$

- Q8. If a, b, c, d are four positive real numbers such that $abcd = 1$, then minimum value of $(1+a)(1+b)(1+c)(1+d)$ is
 (A) 8
 (B) 12
 (C) 16
 (D) 20

Sol. (C)
 Applying AM \geq GM

$$\frac{1+a}{2} \geq (a)^{1/2}, \quad \frac{1+b}{2} \geq (b)^{1/2}, \quad \frac{1+c}{2} \geq (c)^{1/2}, \quad \frac{1+d}{2} \geq (d)^{1/2}$$

Multiplying all focus $(1+a)(1+b)(1+c)(1+d) \geq 16$

- Q9. If the normal at the end of a latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through one extremity of the minor axis, then the eccentricity of the ellipse is given by the equation
 (A) $e^2 + e - 1 = 0$
 (B) $e^2 + e + 1 = 0$
 (C) $e^4 + e^2 + 1 = 0$
 (D) $e^4 + e^2 - 1 = 0$

Sol. (D)
 Normal at $\left(ae, \frac{b^2}{a} \right)$ is $\frac{ax}{e} - ay = a^2 - b^2$

It passes through $(0, -b)$

$$\Rightarrow ab = a^2 - b^2 = a^2 e^2 \Rightarrow b = ae^2 \Rightarrow b^2 = a^2 e^4$$

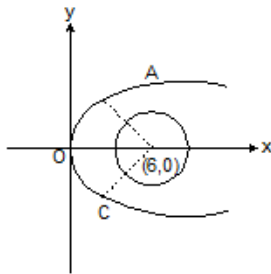
$$\Rightarrow a^2(1 - e^2) = a^2 e^4 \Rightarrow e^4 + e^2 - 1 = 0.$$

- Q10. Minimum distance between the curve $y^2 = 4x$ and $x^2 + y^2 - 12x + 31 = 0$ is equal to

- (A) $\sqrt{21}$
 (B) $\sqrt{26} - \sqrt{5}$
 (C) $\sqrt{21} - \sqrt{5}$
 (D) $\sqrt{28} - \sqrt{5}$

Sol. (C)

Shortest distance will take place along the common normal.



Equation of normal for $y^2 = 4x$ at $(at^2, 2t)$ is

$$y = -tx + t + t^3$$

If it passes through $(6, 0)$, then

$$t^3 - 5t = 0$$

$$\Rightarrow t = 0, t = \pm\sqrt{5}$$

$$\Rightarrow A \equiv (5, 2\sqrt{5}), C \equiv (5, -2\sqrt{5})$$

$$\text{Now, } PA = PC = \sqrt{1+20} = \sqrt{21}, OP = 6$$

$$\therefore \text{Minimum distance} = (\sqrt{21} - \sqrt{5}) \text{ units.}$$

- Q11. If $\frac{1-i\alpha}{1+i\alpha} = A + iB$, then $A^2 + B^2$ equals to
- (A) 1
 (B) α^2
 (C) -1
 (D) $-\alpha^2$

Sol. (A)

$$A + iB = \frac{1-i\alpha}{1+i\alpha} \Rightarrow A - iB = \frac{1+i\alpha}{1-i\alpha} \Rightarrow (A + iB)(A - iB) = \frac{(1-i\alpha)(1+i\alpha)}{(1+i\alpha)(1-i\alpha)} = 1$$

$$\Rightarrow A^2 + B^2 = 1.$$

- Q12. if the normal at the point $P(\theta)$ to the ellipse $\frac{x^2}{14} + \frac{y^2}{5} = 1$ intersects it again at the point $Q(2\theta)$, then $\cos\theta$ is equal to
- (A) $2/3$
 (B) $-2/3$
 (C) $3/4$
 (D) none of these.

Sol. (B)

$$\frac{\sqrt{14}x}{\cos \theta} - \frac{\sqrt{5}y}{\sin \theta} = 14 - 5; \text{ as it passes through } (\sqrt{14} \cos 2\theta, \sqrt{5} \sin 2\theta)$$

$$\text{so, } \frac{14(2\cos^2 \theta - 1)}{\cos \theta} - \frac{5 \times 2 \sin \theta \cos \theta}{\sin \theta} = 9$$

$$\Rightarrow 28 \cos \theta - \frac{14}{\cos \theta} - 10 \cos \theta = 9$$

$$\Rightarrow 18 \cos^2 \theta - 9 \cos \theta - 14 = 0$$

$$\Rightarrow (3 \cos \theta + 2)(6 \cos \theta - 7) = 0 \Rightarrow \cos \theta = -\frac{2}{3}$$

Q13.

If $a^4b^4 - a^4 - b^4 = 2a^2b^2$ ($a, b \in \mathbb{R} - \{0\}$), then the line $\frac{x}{a^2} + \frac{y}{b^2} = 1$, will pass through

- (A) (2, 1)
- (B) (1, 1)
- (C) (2, 3)
- (D) (0, 0)

Sol. (B)

$$\text{Given } a^4b^4 - a^4 - b^4 = 2a^2b^2$$

$$\Rightarrow a^4b^4 = (a^2 + b^2)^2$$

$$\Rightarrow a^2 + b^2 = a^2b^2$$

$$\text{Given line } b^2x + a^2y = a^2b^2$$

$$\Rightarrow b^2x + a^2y = a^2 + b^2$$

$$\Rightarrow (x - 1)b^2 + a^2(y - 1)$$

clearly the line will be passing through (1, 1).

Q14. The coefficient of x^n in $\left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots + \frac{(-1)^n x^n}{n!}\right)^2$ is

(A) $\frac{(-n)^n}{n!}$

(B) $\frac{(-2)^n}{n!}$

- (C) $\frac{1}{(n!)^2}$
(D) $-\frac{1}{(n!)^2}$

Sol. (B)

$$\text{Coefficient of } x^n \text{ in } \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots + \frac{(-1)^n x^n}{n!}\right)^2$$

$$\text{Coefficient of } x^n \text{ in } \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots\right)^2$$

$$\text{Coefficient of } x^n \text{ in } (e^{-x})^2$$

$$\text{Coefficient of } x^n \text{ in } e^{-2x} = \frac{(-2)^n}{n!}$$

Q15.

Let $0 < A, B < \frac{\pi}{2}$ satisfying the equalities $3 \sin^2 A + 2 \sin^2 B = 1$ and $3 \sin 2A - 2 \sin 2B =$

0. Then $A + 2B =$:

- (A) $\pi/4$
(B) $\pi/3$
(C) $\pi/2$
(D) None of these

Sol. (C)

From the second equation, we have

$$\sin 2B = \frac{3}{2} \sin 2A \quad \dots(1)$$

and from the first equality

$$3 \sin^2 A = 1 - 2 \sin^2 B = \cos 2B \quad \dots(2)$$

$$\text{Now } \cos(A + 2B) = \cos A \cdot \cos 2B - \sin A \cdot \sin 2B$$

$$= 3 \cos A \cdot \sin^2 A - \frac{3}{2} \cdot \sin A \cdot \sin 2A$$

$$= 3 \cos A \cdot \sin^2 A - 3 \sin^2 A \cdot \cos A = 0$$

$$\Rightarrow A + 2B = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

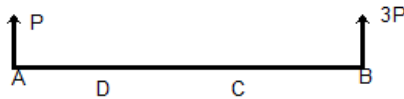
$$\text{Given that } 0 < A < \frac{\pi}{2} \text{ and } 0 < B < \frac{\pi}{2} \Rightarrow 0 < A + 2B < \pi + \frac{\pi}{2}$$

$$\text{Hence } A + 2B = \frac{\pi}{2}.$$

Q16. Two like parallel forces P and $3P$ act on a rigid body at point A and B respectively. If the forces are interchanged in position, the resultant will be displaced through the distance of

- (A) $\frac{1}{2} AB$
(B) $\frac{1}{3} AB$
(C) $\frac{1}{4} AB$
(D) $\frac{3}{4} AB$

Sol. (A)



Case I: If the resultant act at C , then

$$AC = \left(\frac{AB}{P+3P} \right) 3P = \frac{3}{4} AB$$

Case II: If the resultant acts at D , then

$$AD = \left(\frac{AB}{3P+P} \right) P = \frac{1}{4} AB \Rightarrow CD = AC - AD = \frac{1}{2} AB.$$

Q17. The number of solutions of the equation $x^3 + 2x^2 + 5x + 2\cos x = 0$ in $[0, 2\pi]$ is:

- (A) 0
(B) 1
(C) 2
(D) 3

Sol. (A)

$$\text{Let } f(x) = x^3 + 2x^2 + 5x + 2 \cos x$$

$$\Rightarrow f'(x) = 3x^2 + 4x + 5 - 2 \sin x$$

$$= 3 \left(x + \frac{2}{3} \right)^2 + \frac{11}{3} - 2 \sin x$$

$$\text{Now } \frac{11}{3} - 2 \sin x > 0 \quad \forall x \quad (\text{as } -1 \leq \sin x \leq 1)$$

$$\Rightarrow f'(x) > 0 \quad \forall x$$

$\Rightarrow f(x)$ is an increasing function.

$$\text{Now } f(0) = 2$$

$$\Rightarrow f(x) = 0 \text{ has no solution in } [0, 2\pi].$$

Q18. If $\tan x = n \tan y$, $n \in \mathbb{R}^+$ then maximum value of $\sec^2(x - y)$ is equal to:

(A) $\frac{(n+1)^2}{2n}$

(B) $\frac{(n+1)^2}{n}$

(C) $\frac{2}{(n+1)^2}$

(D) $\frac{(n+1)^2}{4n}$

Sol. (D)

$$\begin{aligned} \tan x &= n \tan y, \cos(x - y) \\ &= \cos x \cos y + \sin x \sin y \\ &\Rightarrow \cos(x - y) = \cos x \cos y (1 + \tan x \tan y) \\ &= \cos x \cos y (1 + n \tan^2 y) \\ \Rightarrow \sec^2(x - y) &= \frac{\sec^2 x \sec^2 y}{(1 + n \tan^2 y)^2} \\ &= \frac{(1 + \tan^2 x)(1 + \tan^2 y)}{(1 + n \tan^2 y)^2} \\ &= \frac{(1 + n^2 \tan^2 y)(1 + \tan^2 y)}{(1 + n \tan^2 y)^2} \\ &= 1 + \frac{(n-1)^2 \tan^2 y}{(1 + n \tan^2 y)^2} \end{aligned}$$

$$\begin{aligned} \text{Now, } \left(\frac{1+n \tan^2 y}{2} \right)^2 &\geq n \tan^2 y. \\ \Rightarrow \frac{\tan^2 y}{(1+n \tan^2 y)^2} &\leq \frac{1}{4n} \\ \Rightarrow \sec^2(x-y) &\leq 1 + \frac{(n-1)^2}{4n} = \frac{(n+1)^2}{4n} \end{aligned}$$

Q19. Equation of a circle that cuts the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, lines $x = -g$ and $y = -f$ orthogonally, is;

- (A) $x^2 + y^2 + 2gx + 2fy + g^2 + f^2 - c = 0$
 (B) $x^2 + y^2 + 2gx + 2fy + g^2 + f^2 + c = 0$
 (C) $x^2 + y^2 + 2gx + 2fy - g^2 - f^2 - c = 0$
 (D) none of these

Sol. (D)

Lines $x = -g$ and $y = -f$ will be diameter for the circle. Equation of circle will be

$(x+g)^2 + (y+f)^2 = r^2$. Now this intersect $x^2 + y^2 + 2gx + 2fy + c = 0$ orthogonally

Hence $2g.g + 2f.f = c + k$

$$\Rightarrow k = 2g^2 + 2f^2 - c$$

Q20. If $i = \sqrt{-1}$, then $4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{365}$ equals

- (A) $1 - i\sqrt{3}$
 (B) $-1 + i\sqrt{3}$
 (C) $i\sqrt{3}$
 (D) $-i\sqrt{3}$

Sol. (C)

$$\text{We have } 4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{365} = 4 + 5\omega^{334} + 3\omega^{365}$$

$$= 4 + 5\omega + 3\omega^2 = 4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) + 3\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) = i\sqrt{3}$$

Q21. Total number of 3 letters words that can be formed from the letters of the word 'SAHARANPUR', is equal to:

- (A) 210
 (B) 237
 (C) 247
 (D) 227

Sol. (C)
1S, 3A, 1H, 2R, 1N, 1P, 1U when all letters are different.

$$\begin{aligned}\text{Corresponding ways} &= {}^7C_3 \cdot 3! \\ &= {}^7P_3 = 210\end{aligned}$$

When two letters are of one kind and other is different.

$$\text{Corresponding ways} = {}^2C_1 \cdot {}^6C_1 \cdot \frac{3!}{2!} = 36$$

When all letters are alike, corresponding ways = 1.

$$\begin{aligned}\text{Thus total words that can be formed} \\ &= 210 + 36 + 1, \text{ i.e. } 247\end{aligned}$$

- Q22. If $A = \{\phi, \{\phi\}\}$, then the power set of a is
- (A) A
 - (B) $\{\phi, \{\phi\}, A\}$
 - (C) $\{\phi, \{\phi\}, \{\{\phi\}\}, A\}$
 - (D) none of these

Sol. (C)
Power set means set of all the subjects
 \therefore power set of $A = \{\phi, \{\phi\}, \{\{\phi\}\}, A\}$

- Q23. The value of $\lim_{x \rightarrow \infty} \frac{x^n + nx^{n-1} + 1}{e^{[x]}}$, when n is integer is:
- (A) 1
 - (B) 0
 - (C) n
 - (D) $n(n-1)$

Sol. (B)

$$\begin{aligned} \lim_{x \rightarrow \infty} [x] &= \lim_{x \rightarrow \infty} \frac{[x]x}{x} \\ &= \lim_{x \rightarrow \infty} \frac{[x]}{x} \cdot \lim_{x \rightarrow \infty} x = \lim_{x \rightarrow \infty} x \left(\text{as } \lim_{x \rightarrow \infty} \frac{[x]}{x} = 1 \right) \\ \lim_{x \rightarrow \infty} \frac{x^n + nx^{n-1} + 1}{e^{[x]}} \\ &= \lim_{x \rightarrow \infty} \frac{x^n + nx^{n-1} + 1}{e^x} \\ &= \lim_{x \rightarrow \infty} \frac{x^n + nx^{n-1} + 1}{1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \frac{x^{n+1}}{(n+1)!} + \dots} \\ &= \lim_{x \rightarrow \infty} \frac{1 + \frac{n}{x} + \frac{1}{x^n}}{\frac{1}{x^n} + \frac{1}{x^{n-1}} + \frac{1}{2x^{n-2}} + \dots + \frac{1}{n!} + \frac{x}{(n+1)!} + \frac{x^2}{(n+2)!} + \dots} \\ &= \frac{1+0+0}{0+0+\dots+0+\frac{1}{n!}+\infty} = \frac{1}{\infty} = 0. \end{aligned}$$

- Q24. Maximum value of $\log x/x$ in interval $[2, \infty]$ is
(A) $\log 2/2$
(B) 0
(C) $1/e$
(D) 1

Sol. (D)

$$f'(x) = \frac{1}{x^2}(1 - \ln x)$$

Clearly $f(x)$ increasing for $x < e$ and decreases for $x > e$

$\Rightarrow x = e$ is the point of local maxima

$$\therefore \text{maximum } f(x) = \frac{1}{e}.$$

- Q25. Total number of four digit numbers having all different digits, is equal to
(A) 4536
(B) 504
(C) 5040
(D) 72

Sol. (A)
Let the number be $x_1 x_2 x_3 x_4$.

Then x_1 can be chosen in 9 ways. x_2 can be chosen in 9 ways. Similarly x_3 and x_4 can be chosen in 8 and 7 ways respectively.

\therefore Total number of such numbers

$$= 9 \cdot 9 \cdot 8 \cdot 7 = 4536$$

- Q26. A and B each throw a dice. The probability that A's throw is not greater than B's is
(A) $1/6$
(B) $5/12$
(C) $1/2$
(D) $7/12$

Sol. (D)

$$A \leq B$$

$$P(A > B)$$

B draw	A draw	Total numbers
1	2, 3, ..., 6	5
2	3, 4, ..., 6	4
3	4, 5, 6	3
4	5, 6	2
5	6	1
		Total = 15

$$P(A > B) = \frac{15}{30}$$

$$P(A \leq B) = 1 - \frac{15}{30} = \frac{7}{12}$$

Q27. The distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line

$$\frac{x}{2} = \frac{y}{3} = \frac{z}{-6} \text{ is:}$$

- (A) 7 unit
(B) 4 unit
(C) 1 unit
(D) 2 unit

Sol. (A)

Here we are not to find perpendicular distance of the point from the plane but distance measured along with the given line. The method is as follow:

The equation of the line through the point $(1, -2, 3)$ and parallel to given line is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = r \text{ (say)}$$

The coordinate of any point on it is $(2r+1, 3r-2, -6r+3)$.

If this point lies in the given plane then

$$2r+1 - (3r-2) + (-6r+3) = 5 \Rightarrow -7r = -1 \text{ or } r = \frac{1}{7}$$

$$\therefore \text{ point of intersection is } \left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7} \right)$$

\therefore The required distance

$$= \text{the distance between the points } (1, -2, 3) \text{ and } \left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7} \right)$$

$$= \sqrt{\left(1 - \frac{9}{7}\right)^2 + \left(-2 + \frac{11}{7}\right)^2 + \left(3 - \frac{15}{7}\right)^2} = \frac{1}{7} \sqrt{49} = 1 \text{ unit.}$$

Q28. In a triangle ABC if $\cos A + 2\cos B + \cos C = 2$. The sides of the triangle are in:

- (A) H.P.
(B) G.P.
(C) A.P.
(D) None of these

Sol. (C)

$$\cos A + 2\cos B + \cos C = 2 \text{ or, } \cos A + \cos C = 2(1 - \cos B)$$

$$\Rightarrow 2 \cos\left(\frac{A+C}{2}\right) \cos\left(\frac{A-C}{2}\right) = 4 \sin^2 \frac{B}{2}$$

$$\Rightarrow \cos \frac{A-C}{2} = 2 \sin \frac{B}{2} \Rightarrow \cos \frac{A-C}{2} = 2 \cos \frac{A+C}{2}$$

$$\Rightarrow \cos \frac{A}{2} \cos \frac{C}{2} + \sin \frac{A}{2} \sin \frac{C}{2} = 2 \cos \frac{A}{2} \cos \frac{C}{2} - 2 \sin \frac{A}{2} \sin \frac{C}{2}$$

$$\Rightarrow \cot \frac{A}{2} \cot \frac{C}{2} = 3 \Rightarrow \sqrt{\frac{s(s-a)}{(s-b)(s-c)}} \sqrt{\frac{s(s-c)}{(s-a)(s-b)}} = 3$$

$$\Rightarrow \frac{s}{s-b} = 3 \Rightarrow s = 3s - 3b \Rightarrow 2s = 3b$$

$$\Rightarrow a + c = 2b \Rightarrow a, b, c \text{ are in A.P.}$$

Q29.

If A and B are independent events such that $P(A) = 0.3$, $P(A \cup \bar{B}) = 0.8$, then $P(B)$ equals to

- (A) $\frac{1}{7}$
- (B) $\frac{2}{7}$
- (C) $\frac{3}{4}$
- (D) $\frac{1}{4}$

Sol.

(B)

$$P(A) + P(\bar{B}) - P(A)P(\bar{B}) = 0.8$$

$$0.3 + (1-x) - 0.3(1-x) = 0.8$$

$$(1-x)(0.7) = 0.5$$

$$1-x = \frac{5}{7} \Rightarrow x = 1 - \frac{5}{7} = \frac{2}{7}$$

Q30. The solution set of the inequality $\log_{10}(x^2 - 16) \leq \log_{10}(4x - 11)$

- (A) (3, 5]
- (B) (4, 5]
- (C) (6, 5]
- (D) n

Sol.

(B)

Since base of log is same both the sides and greater than 1, hence inequality will remain same.

$$\Rightarrow x^2 - 16 \leq 4x - 11 \Rightarrow x^2 - 4x - 5 \leq 0 \Rightarrow -1 \leq x \leq 5 \dots\dots\dots(1)$$

Also $x^2 - 16 > 0$ and $4x - 11 > 0 \Rightarrow$ either $x < -4$ or $x > 4 \dots\dots\dots(2)$

Taking intersection of (1) and (2) ...

$$x \in (4, 5]$$

