

CBSE Board
Class XI Mathematics
Sample Paper – 9

- Q1. The greatest value of positive integer which divides $(r + 2)(r + 3)(r + 5) \forall r \in N$ is
- (a) 4!
 - (b) 2!
 - (c) 5!
 - (d) 3!

Sol. (a)
Let $P(r) = (r + 2)(r + 3)(r + 4)(r + 5) \forall r \in N$ as $r \in N$ means $P(r)$ is the product of four consecutive positive integers which means $P(r)$ is the multiple of 4!.
 \therefore Greatest positive integer which divides $(r + 1)(r + 2)(r + 3)(r + 4)$ is 4!

- Q2. The sum of the coefficients in the expansion of $(1 - x)^{10}$ is
- (a) 0
 - (b) 1
 - (c) 10^2
 - (d) 1024

Sol. (a)
 $(1 - x)^{10} = {}^{10}C_0 + {}^{10}C_1(-x) + {}^{10}C_2(-x)^2 + {}^{10}C_3(-x)^3 + \dots$
put $x = 1$ we get
 ${}^{10}C_0 - {}^{10}C_1 + {}^{10}C_2 - {}^{10}C_3 + \dots + {}^{10}C_{10} = 0$

- Q3. Statement - 1 : Let the function $f(x) = x^2 - x + 1$
 $\forall x \geq \frac{1}{2}$ & $g(x) = \frac{1}{2} + \sqrt{x - \frac{3}{4}}$, then $f(x) = g(x)$ have two solutions.

Statement - 2 : $f(x)$ & $g(x)$ are inverse of each other.

- (a) If both statement -1 and statement -2 are true and statement -2 is the correct explanation of statement -1
- (b) If both statement -1 and statement -2 are true but statement -2 is not the correct explanation of statement -1
- (c) If statement -1 is true but statement -2 is false

(d) If statement -1 is false and statement -2 is true.

Sol.

(d)

$$y = f(x) = x^2 - x + 1, f^{-1}(y) = x \quad \dots(A)$$

$$\Rightarrow x^2 - x + 1 - y = 0$$

$$\Rightarrow x = \frac{1}{2} \pm \sqrt{y - \frac{3}{4}}$$

$$\Rightarrow x = \frac{1}{2} + \sqrt{y - \frac{3}{4}} \quad \left(\because x \geq \frac{1}{2} \right)$$

$$\Rightarrow y = g^{-1}(x) = f(x) \quad \dots\text{using(A)}$$

$\Rightarrow f(x)$ & $g(x)$ are inverse of each other.

$$\Rightarrow f(x)g(x) \text{ when } f(x) = x \quad \dots\text{using(B)}$$

By (A) & (B) we have

$$x^2 - x + 1 - x$$

$$\Rightarrow (x - 1)^2 = 0$$

$\Rightarrow x = 1$, only one solution exist which contradict s the Statement -1

so Statement -1 is false and Statement -2 is true.

Q4. Which one of the following statement is not a false statement?

- (a) P : Each radius of a circle is a chord of the circle.
- (b) q : Circle is a particular case of an ellipse
- (c) r : $\sqrt{13}$ is a rational number
- (d) s : The centre of a circle bisect each chord of the circle.

Sol.

(b)

We know that equation of an ellipse is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if we take $a = b$ then we get

$x^2 + y^2 = a^2$ which satisfies all the conditions of circle

\therefore Circle is the particular cases of an ellipse.

Q5. Six identical coins are arranged in row. The number of ways in which the number of tails is equal to number of heads is

- (a) 20
- (b) 120
- (c) 9
- (d) 40

Sol.

(a)

- Q6. The number of distinct positive $2^2 3^2 5^3$ is
- (a) 25
 - (b) 36
 - (c) 72
 - (d) 144

Sol. (c)

- Q7. There are three piles of identical yellow, black and green balls and each pile contains at least 20 balls. The number of ways of selecting 20 balls if the number of black balls to be selected is twice the number of yellow balls, is
- (a) 6
 - (b) 7
 - (c) 8
 - (d) 9

Sol. (b)

Let the number of yellow balls = x .
The number of black balls = $2x$
The number of green balls = y .
 $x + 2x + y = 20$ & $3x + y = 20$
Since $0 \leq y \leq 20 \Rightarrow y = 20 - 3x$.
 $0 \leq 20 - 3x \leq 20$
 $\Rightarrow 0 \leq 3x \leq 20$ or $0 \leq x \leq 6$

- Q8. We are required to form different words with the help of letters of the word INTEGER. Let m_1 be the number of words in which I and N are never together and m_2 be the number of words which begin with I and end with R. Then $\frac{m_1}{m_2}$ is given by
- (a) 30
 - (b) 1/30
 - (c) 6
 - (d) 42

Sol. (a)

- Q9. The A.M. of the roots of a quadratic equation is A and G.M. of its roots is G. The quadratic equation is
- (a) $x^2 + Ax + G^2 = 0$

- (b) $x^2 + 2Ax + G^2 = 0$
(c) $x^2 - Ax + G^2 = 0$
(d) $x^2 - 2Ax + G^2 = 0$

Sol. (d)

The equation will be

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$ has roots α, β

$$A = \frac{\alpha + \beta}{2}, G = \sqrt{\alpha\beta}$$

$$\therefore \alpha + \beta = 2A, \alpha\beta = G^2$$

The equation is $x^2 - 2Ax + G^2 = 0$.

- Q10. If $21(x^2 + y^2 + z^2) = (x + 2y + 4z)^2$, then x, y, z are in
(a) A.P.
(b) G.P.
(c) A.G.P.
(d) None of these

Sol. (b)

$21 = 1^2 + 2^2 + 4^2$ so that

$$(1^2 + 2^2 + 4^2)(x^2 + y^2 + z^2) = (x + 2y + 4z)^2$$

$$\text{i.e., } (2x - y)^2 + (4y - 2z)^2 + (z - 4x)^2 = 0$$

$$\text{or } x = \frac{y}{2} = \frac{z}{4}, \text{ A.G.P. with C.R.} = \frac{1}{2}.$$

- Q11. The area of the quadrilateral formed by the points $(1, -2), (2, 3), (-4, 1)$ and $(-5, 2)$, is
(a) 20 sq. units
(b) 21. Sq. units
(c) 18 sq. units
(d) 10 sq. units

Sol. (c)

area..of..the..quadrilateral..formed...by..the..given..point s

$$= [\text{area..of..the..triangle..formed..by}(1,-2), (2,3), (-4,1)] + [\text{area..formed..}$$

$$\text{by..the..triangle } (2,3), (-4,1), (-5,2)]$$

$$= |-14| + 4 = 18 \text{sq. units}$$

Q12. Which of the following is false?

- (a) $3(2x - y)^2 - 2(2x - y) - 1 = 0$ represents a pair of parallel lines
- (b) $x^2 + 2xy \tan \frac{\pi}{5} - y^2 = 0$ represents a pair of perpendicular lines
- (c) $9x^2 - 24xy + 16y^2 = 1$ represents a pair of coincident lines
- (d) $xy = x$ represent a pair of perpendicular lines

Sol. (c)

Q13. The length of the tangent to the circle $x^2 + y^2 - 3x + 4y + \lambda = 0$ from $(-2, 1)$ is 4, then λ is equal to

- (a) ± 1
- (b) 1
- (c) -1
- (d) None of these

Sol. (b)

$$\text{length of the tangent} = \sqrt{(-2)^2 + (1)^2 - 3(-2) + 4(1) + \lambda} = 4$$

$$\Rightarrow 15 - \lambda = 16$$

$$\lambda = -1$$

Q14. A woman running round a race course notes that the sum of the distance of two flag – posts from her is always 10 meters and the distance between the flag – posts is 8 metres. Then the area of the Race course she encloses in square metres is

- (a) 15π
- (b) 12π
- (c) 18π
- (d) 8π

Sol. (a)

A race course notes that the sum of the distance of two flag – posts from her is always 10 metres = constant this race course is an ellipse with length of the major axis = $2a = 10$

$$\text{Then } a = 5$$

$$2ae = 8$$

$$\text{Then } e = 4/5$$

$$\text{Now } b = 3$$

$$\text{Area} = \pi ab = 15\pi \text{ s. metres.}$$

Q15. The equation of the normal to the hyperbola $7x^2 - 5y^2 = 232$ at $(6, -2)$ is

- (a) $5x - 21y = 72$
- (b) $21x + 5y = 116$
- (c) $5x + 21y + 12 = 0$
- (d) None of these

Sol. (a)

$$\text{slope of the tangent at } (6, -2) = \frac{dy}{dx} \text{ at } (6, -2) = -\frac{42}{10}$$

$$\text{slope of the normal at } (6, -2) = \frac{10}{42}$$

$$\text{equation } y + 2 = \frac{10}{42}(x - 6)$$

$$5x - 21y = 72$$

Q16. If $\sin A = -\frac{12}{13}$ and A lies in fourth quadrant, $\frac{\sin A + \cos A}{\sin A - \cos A} =$

- (a) 1
 (b) 7/17
 (c) -7/17
 (d) -17/7

Sol. (b)

In 4th quadrant $\cos A$ is positive

$$\cos A = 5/13$$

$$\frac{\sin A + \cos A}{\sin A - \cos A} = \frac{-\frac{12}{13} + \frac{5}{13}}{-\frac{12}{13} - \frac{5}{13}} = \frac{7}{17}$$

Q17. In a ΔABC if $b = \sqrt{3} + 1$, $c = \sqrt{3} - 1$ and $A = 60^\circ$, then $(B - C) =$

- (a) 45°
 (b) 90°
 (c) 120°
 (d) 150°

Sol. (b)

$$\cos A = \cos 60 = \frac{b^2 + c^2 - a^2}{2bc}$$

$$[b = \sqrt{3} + 1, c = \sqrt{3} - 1]$$

$$a = \sqrt{6}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\sin B = \frac{\sqrt{3} + 1}{\sqrt{6}} \sin 60 = \frac{\sqrt{3} + 1}{\sqrt{2} \cdot \sqrt{3}} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

$$B = 105$$

$$C = 15$$

$$B - C = 90$$

- Q18. If $z = \frac{2+3i}{5-i}$, then $z \bar{z} =$
- (a) 2
 (b) $1/\sqrt{2}$
 (c) $1/2$
 (d) None of these

Sol. (c)

$$z = \frac{2+3i}{5-i} = \frac{(2+3i)(5+i)}{(5-i)(5+i)} = \frac{7+17i}{26}$$

$$\bar{z} = \frac{7-17i}{26}$$

$$\text{then } z \bar{z} = \frac{7+17i}{26} \cdot \frac{7-17i}{26} = 1/2$$

- Q19. If $x = \cos \theta + i \sin \theta$, then $\frac{x^2-1}{x^2+1} =$
- (a) $-i \tan \alpha$
 (b) $\tan \alpha$
 (c) $\cot \alpha$
 (d) $i \tan \alpha$

Sol. (d)

$$\frac{x^2-1}{x^2+1} = \frac{\cos 2\alpha + i \sin 2\alpha - 1}{\cos 2\alpha + i \sin 2\alpha + 1} = \frac{(\cos 2\alpha - 1 + i \sin 2\alpha)(\cos 2\alpha + 1 - i \sin 2\alpha)}{(\cos 2\alpha + 1) - (i \sin 2\alpha)^2}$$

$$= \frac{\cos^2 2\alpha - 1 + 2i \sin 2\alpha + \sin^2 2\alpha}{4 \cos^2 \alpha} = i \tan \alpha$$

- Q20. If ω is an imaginary cube root of 1, then $\omega + i^2 + \omega^3 + i^4 + \omega^5 + i^6 + \dots$ upto 200 terms =
- (a) -1
 (b) $-\omega$
 (c) $1 + \omega$
 (d) ω

Sol. (d)

$$\omega + i^2 + \omega^3 + i^4 + \omega^5 + i^6 + \dots \text{ upto 200 terms}$$
$$= (w + 1 + w^2) + (w + 1 + w^2) + \dots + w + (-1) + (+1) + (-1) + \dots$$
$$= w$$

$$[1 + w + w^2 = 0$$

and

$$w^3 = 1]$$

Q21. If $x = 6 + 5i$ and $y = 1 - 3i$, then imaginary part of $x\bar{y}$ is

- (a) -9
- (b) -13
- (c) 13
- (d) 23

Sol. (d)

$$\bar{y} = 1 + 3i$$

$$x\bar{y} = 6 + 5i + 18i - 15$$

$$\text{imaginary part} = 23$$

Q22. If $0 < \alpha < \pi$, then the quadratic equation $(\cos(\alpha) - 1)x^2 + x \cos \alpha + \sin \alpha = 0$ has

- (a) Both roots imaginary
- (b) Only one roots imaginary
- (c) Only one root irrational
- (d) None of these

Sol. (d)

$$D = B^2 - 4AC$$

$$= \cos^2 \alpha + 4 \sin \alpha (1 - \cos \alpha) > 0 \forall 0 < \alpha < \pi$$

\therefore Roots are real and distinct.

Alternative Solution : (Short Cut Method)

(a) False, a quadratic equation can not have only one root imaginary as imaginary roots always exist in pair and conjugate of each other.

(b) False, can not have only one root irrational as irrational roots always occurs in pair.

(c) False, for imaginary roots $D < 0$

None of (a), (b), (c) exist we left with only the choice (d).

- Q23. Statement -1 : $\forall x \in R, 2x^2 - x + 5 > 0$.
 Statement -2 : If $D = b^2 - 4ac < 0$, then $ax^2 + bx + c$ & 'a' have opposite sign $\forall x \in R$.
 (a) If both statement -1 and statement -2 are true and statement -2 is the correct explanation of statement -1
 (b) If both statement -1 and statement -2 are true but statement -2 is not the correct explanation of the statement -1
 (c) If statement -1 is true but statement -2 is false
 (d) If statement -1 is false and statement -2 is true.

Sol. (c)

Let $f(x) = 2x^2 - x + 5$
 $\therefore D = 1 - 40 = -39 < 0$
 $a = 2 > 0, D < 0$ then $ax^2 + bx + c$ & 'a' have same sign so Statement -2 is false and
 $f(x) = 2x^2 - x + 5$
 $= 2\left(x^2 - \frac{x}{2}\right) + 5 = 2\left(x - \frac{1}{4}\right)^2 + 5 - \frac{1}{8}$
 $= 2\left(x - \frac{1}{4}\right)^2 + \frac{39}{8} > 0$
 \therefore Statement -1 is true.

- Q24. If $f: R \rightarrow R$ satisfies $f(x + y) = f(x) + f(y), \forall x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^n f(r) =$
 (a) $7/2 n$
 (b) $7/2 (n + 1)$
 (c) $7n (n + 1)$
 (d) $7/2n (n + 1)$

Sol. (d)

$$\sum_{r=1}^n f(r) = f(1) + f(2) + f(3) + \dots + f(n)$$

$$= 7 + 2.7 + 3.7 + \dots + n.7 = \frac{7}{2}n(n+1)$$

- Q25. $x \xrightarrow{Lt} -1 \frac{\sqrt{x^2+8} - \sqrt{5-4x}}{x^3+1}$
 (a) $1/3$
 (b) $2/3$
 (c) 4
 (d) $1/9$

Sol. (d)

$$\lim_{x \rightarrow -1} \frac{\sqrt{x^2 + 8} - \sqrt{5 - 4x}}{x^3 + 1} \left[\frac{0}{0} \text{ form} \right]$$
$$= \lim_{x \rightarrow -1} \frac{\frac{2x}{2\sqrt{x^2 + 8}} - \frac{-4}{2\sqrt{5 - 4x}}}{3x^2} = 1/9$$

Q26. The probability of not getting a sum 7 in a single throw with a pair of dice is

- (a) 1/6
- (b) 2/3
- (c) 1/3
- (d) 5/6

Sol. (d)

Q27. If the probability that A and B will die within a year are p and q respectively, then the probability that only one of them will be alive at the end of the year is

- (a) p + q
- (b) p + q - 2 pq
- (c) p + q - pq
- (d) p + q + pq

Sol. (b)

Q28. The probability for a randomly chosen month to have its 10th days as Sunday is

- (a) 1/84
- (b) 10/12
- (c) 10/84
- (d) 1/7

Sol. (d)

- Q29. Two persons each makes a single throw with a pair of dice. The probability that the throws are unequal is given by
- (a) $1/63$
 - (b) $73/63$
 - (c) $51/63$
 - (d) None of these

Sol. (d)

- Q30. It is known that the probability of a man aged 25 years to survive one more year is 0.992 and that he will die within a year is 0.008. An insurance company offers to sell such a man Rs. 1000 one year life insurance policy for a premium of Rs. 10. What is the Company's expected gain?
- (a) 2
 - (b) 200
 - (c) 2000
 - (d) 500

Sol. (a)