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Tel: 1800-2000-838

Class: XI
Subject: Physics
Topic: OASK1511SA102
No. of Questions: 26

Time allowed: 3 hours **Maximum Marks: 70**

General Instructions:

a) All the questions are compulsory.
b) There are **26** questions in total.
c) Questions **1** to **5** are very short answer type questions and carry **one** mark each.
d) Questions **6** to **10** carry **two** marks each.
e) Questions **11** to **22** carry **three** marks each.
f) Questions **23** is value based questions carry **four** marks.
g) Questions **24** to **26** carry **five** marks each.
h) There is no overall choice. However, an internal choice has been provided in one question of two marks, one question of three marks and all three questions in five marks each. You have to attempt only one of the choices in such questions.
i) Use of calculators is **not** permitted. However, you may use log tables if necessary.
j) You may use the following values of physical constants wherever necessary:

Q1. What are the characteristics of elastic collision?

Sol. (i) Kinetic energy of the system remains conserved.
(ii) Linear momentum of the system remains conserved

Q2. What do you understand by the term conservative force?

Sol. Any force is called conservation force if,
(a) Work done against is independent of path.

Q3. If one mass of one electron is 9.11 x 10-31 kg, then how many electrons would weigh in 1 kg?

Sol. $9.11 \times 10^{-31} \times n = 1\, kg$
Therefore, n = 1.1×10^{30}

Q4. Give reason: “Though earth received solar energy, it is not warmed up continuously”.

Sol. This is because
a) The energy received is less due to large distance.

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b) Loss of energy takes place due to radiation, absorption and convection currents.

Q5. Justify: "When several passengers are standing in a moving bus, it is said to be dangerous."

Sol. Here, the centre of gravity of the system is raised and as such the whole system is in an unstable Equilibrium. When the running bus suddenly stops due to inertia of motion, the passengers fall Forward on each other and cause stampede.

Q6. State the laws of limiting friction.

Sol. The laws of limiting friction are as follows:
(a) The value of limiting friction depends on the nature of the two surfaces in contact and on the state of their smoothness.
(b) The force of friction acts tangential to the surfaces in contact in a direction opposite to the direction of relative motion.
(c) The value of limiting friction is directly proportional to the normal reaction between the two given surfaces.
(d) For any two given surface and for a given value of normal reaction the force of limiting friction is independent of the shape and surface area of surfaces in contact. Coefficient of limiting friction for two given surfaces in contact is defined as the ratio of the force of normal reaction N.

$$\mu = \frac{f_1}{N}$$

Q7. If 1A0 = 10-10 m and the size of a hydrogen atom is about 0.5 A0, then what is the total atomic volume in m3 of a mole of hydrogen atoms?

Sol. Volume of the one hydrogen atom = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times 3.14\ (0.5 \times 10^{-10})m^3$

$$= 5.23 \times 10^{-31}\ m^3$$

According to Avogadro's hypothesis, one mole of hydrogen contains 6.023 x 10^{23} atoms.
Atomic volume of 1 mole of hydrogen atom = 6.023 ×10^{23} ×5.23 ×10^{-31}

$$= 3.15 \times 10^{-7}\ m^3$$

Q8. What is the angle of projection at which the H_{max} and range are equal?.

Sol. $\frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin 2\theta}{g}$

$Sin^2\ \theta = 2 \times 2 \sin\theta\ cos\theta$
$Sin\theta = 4\cos\theta$
$Tan\ \theta = 4$
$Q = \tan^{-1}(4)$

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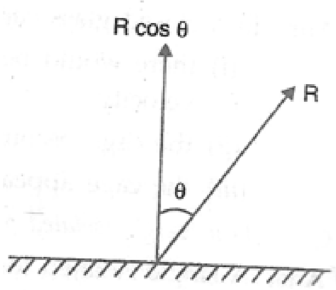
OR

Give reason: "One should take short steps rather than long steps when walking on ice".

Sol.



Let R represents the reaction offered by the ground. The vertical component R cos θ will balance the weight of the person and the horizontal component R sin Θ will help the person to walk forward.

Normal reaction = R cos θ
Friction force = R sin θ
Coefficient of friction

$$\mu = \frac{R\, sin\theta}{R\cos\theta} = \tan\theta$$

In along step, θ is more and tan θ is more. But μ has a fixed value. So, there is danger of shipping in along step.

Q9. The measured quantities a, b, c and x is calculated by using the relation x = $\frac{ab^2}{c^3}$. If the percentage errors in measurement of a, b and c are $\pm 1\%$, $\pm 2\%$ $and \pm$ 1.5%, then calculate the maximum percentage error in value of x obtained.

Sol. $x = \frac{ab^2}{c^3}$

$$\left(\frac{\Delta x}{x}\right)_{max} = \frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c}$$

$$\frac{\Delta a}{a} = \pm 1\%, \frac{\Delta b}{b} = \pm 2\%, \frac{\Delta c}{c} = \pm 3\%$$

$$\left(\frac{\Delta x}{x}\right)_{max} = 1\% + 2\% \times 2\% + 3 \times 1.5\%$$

= (1+4+4.5) %
9.5 %

Q10. What would be the velocity of the top end at the time of touching the ground if a rod of length and mass M held vertically is let go down, without slipping at the point of contact?

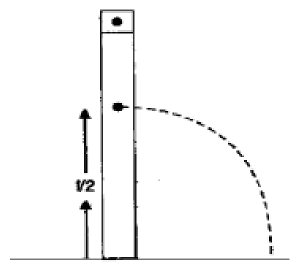
Sol. Loss in potential energy = Gain in rotational Kinetic energy

$$mg\ \frac{1}{2} = \frac{1}{2}\ \frac{M1^2}{3}\ .\omega^2$$

$$\omega = \sqrt{\frac{3g1}{1^2}} - \sqrt{\frac{3g}{1}}$$

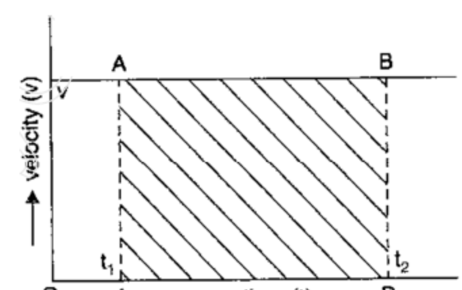
$$v = lm$$

$$= \sqrt{3gl}$$



Q11. How does the velocity- time graph for uniform motion helps to calculate the displacement covered during a given time t?

Sol. Consider the velocity- time graph for uniform motion along a straight path. The graph is a straight line parallel tot eh time axis Ref below figure



Let A and B be two points on velocity – time graph corresponding to the instants t_1 and t_2. As the motion is uniform hence $AA_1 = BB_1 = v$
Area under v-t graph between t_1 and t_2 = area ABB_1A_1
$= AA_1 \times A_1B_1 = v(t_2 - t_1)$
Velocity is defined as $v = \frac{dispacement}{Time} = \frac{X_2 - X_1}{t_2 - t_1}$

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$$v(t_2 - t_1) = x_2 - x_1$$

Area $ABB_1A_1 = (x_2 - x_1)$
Hence displacement of a particle in time interval (t_2-t_1) is numerically equal to the area under velocity- time graph between the instant t_1 and t_2.

Q12. If two bodies of different masses m_1 and m_2 are dropped from two different heights a and b, give the ratio of time taken by the two bodies to drop through these distance?

Sol. Let t_1 and t_2 are the time taken by two bodies of masses m1 and m2 to drop from heights 'a' and 'b'.

Using equation of motion h = $ut + \frac{1}{2}at^2$

$$u = 0 \;\; and \; a = g$$

$$a = \frac{1}{2}gt_1^2 \quad \Rightarrow t_1 = \sqrt{\frac{2a}{g}}$$

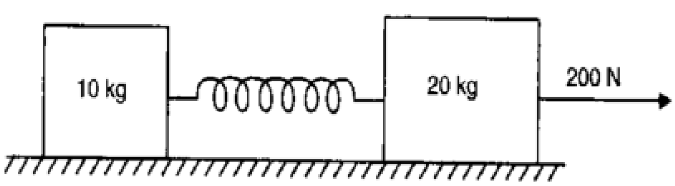
$$b = \frac{1}{2}gt_2^2 \quad \Rightarrow t_2 = \sqrt{\frac{2b}{g}}$$

$$\frac{t_1}{t_2} = \sqrt{\frac{2a/g}{2b/g}}$$

$$\frac{t_1}{t_2} = \sqrt{\frac{2a}{2b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$t_1 : t_2 = \sqrt{a} : \sqrt{b}$$

Q13. What will be the energy shared in the spring at the instant when the 10 kg mass has acceleration 12 m/s^2 if two masses 10 kg and 20 kg are connected by a massless spring. A force of 200 N acts a 20 kg mass?



Sol. Since F = ma

$$F = 10 \times 12 = 120\ N$$

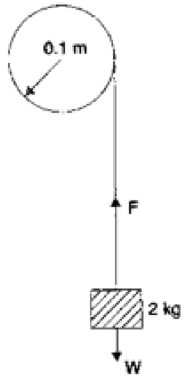
$$F = kx = 2400\ x$$

$$x = \frac{1}{20}$$

Energy stored in the springing E = ½ kx^2

$$E = \frac{1}{2} \times 2400 \times \left(\frac{1}{20}\right)^2$$
$$= 3\,J$$

Q14. In the diagram given below, a tangential force of 2 kg wt is applied round the circumference of the flywheel with the help of a string and mass arrangement. Now, if the radius of the wheel is 0.1 m, find the acceleration of the mass. Assume that the moment of inertia of a solid fly wheel about its axis is 0.1 kg m^{-2}.



Sol. Let 'a' be the linear acceleration of the mass and 'T' the tension in the spring. It is clear that

mg – T = ma ---------------------- (i)

Let the angular acceleration of the flywheel be 'α'. The couple applied to the flywheel is

I α = TR ------------------------ (ii)

The linear acceleration α and angular acceleration are related to each other as

a = Rα ------------------------- (iii)

From equation (i), (ii) and (iii)

Mg – Iα/R = m R α

$$a = \frac{mgT}{(I+mR)^2}$$

It is given that m = 2kg R = 0.1 m and I = 0.1 kgm^2

Substituting these values we get,

$$a = \frac{2\times9.8\times0.1}{(0.1+2\times0.1)^2}\ rad\ s^{-2}$$

$$= 16.7\ rads^{-2}$$

Q15. If an artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth, then

(a) Determine the height of the satellite above the earth's surface.

(b) If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, find the speed with which it hits the surface of the earth

Sol. (a)

$$V_u = \sqrt{\frac{GM}{r}} = R\sqrt{\frac{g}{(R+h)}} \quad \left[aas\ g = \frac{GM}{R^2}\ and\ r = \ R+h\right]$$

In this problem

$V_u = \frac{1}{2}\, v_u = \frac{1}{2}\sqrt{2gR}$

$\frac{R^2 g}{R+h} = \frac{1}{2}\, gR$

$h = R = 6400\ km$

(b) By conservation of ME.

$$0 + \left(-\frac{GMm}{r}\right) = \frac{1}{2}\, mv^2 + \left(-\frac{GMm}{R}\right)$$

$$v^2 = 2GR\left[\frac{1}{R} - \frac{1}{2R}\right]$$

$$V = \sqrt{\frac{GM}{R}} - \sqrt{gR} = 8\ km/s$$

OR

Uniform spring whose upstretched length is *l* has a force constant k. the spring is cut into two pieces of upstretched lengths, l_1 and l_2, where $l\,1 = nl_2$ and n is an integer. What are the corresponding force constant k_1 and k_2 in terms of n and k?

Sol. $l = l_1 + l_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(i)$

$l_1 = \frac{n}{2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(ii)$

$K = \frac{Mg}{l}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(iii)$

$k_1 = \frac{Mg}{l_1}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(iv)$

$K_2 = \frac{Mg}{l_2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(v)$

Dividing the equation (iv) by (iii),

$\frac{K_1}{K} = \frac{l}{l_2} = 2\,\frac{l_1+l_2}{l_1} = 1 + \frac{l_2}{l_1}$

From the equation (ii) we find $\frac{l_1}{l_2} = n$

$$\frac{K_1}{k} = 1 + \frac{1}{n}$$

From equation (v) and (iii)

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$$\frac{K_2}{K} = \frac{l}{l_2} = \frac{l_1+l_2}{l_2} = \frac{l_1}{l_1} + 1$$

From equation (ii) we have $\frac{l_1}{l_2} = n$

$$\frac{K_2}{K} = (n+1)$$

$$K_2 = K(n+1)$$

Q16. What is the average speed during the whole journey, if a body covers half of its journey with a speed of 40 m/s and other half with a speed of 60 m/s.

Sol. Average speed = $\frac{Total\ Distance}{Time\ taken}$

Let x be the distance to be covered,

Average speed = $\frac{x}{\frac{x}{2v_1}+\frac{x}{2v_2}}$

Where $\frac{x}{2v_1} =$ time taken to cover first half of the distance, $\frac{x}{2v_2} =$ time taken to cover the second half of the distance,

Average speed = $\frac{x}{\frac{x(v_1+v_2)}{2v_1v_2}}$

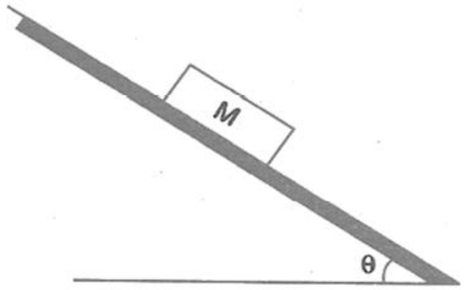
$$= \frac{x(x)2v_1v_2}{x(v_1v_2)} = \frac{2v_1v_2}{v_1v_2}$$

$$V_{av} = \frac{2x\ 40\ m/s\ \ x60m/s}{100\ m/s}$$

$$= 48\ ms^{-1}$$

Q17. If a block of mass M is placed on a frictionless, inclined plane of angle ϴ. Determine
a) The acceleration of the block after it is released
b) The force exerted by the incline on the block

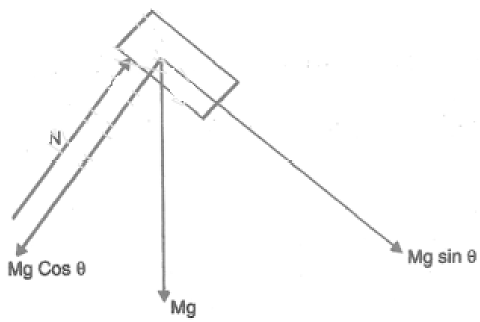
Sol. When the block is released, it will move down the incline.
Let its acceleration be a.
As the surface is frictionless, so the contact force will be normal to the plane. Let it be N.



Here for the block we can apply equation for motion along the plane and equation for equilibrium perpendicular to the plane.

Mg sin θ = Ma

$$a = g\sin\theta$$
$$Mg\cos\theta - N = 0$$
$$N = Mg\cos\theta$$

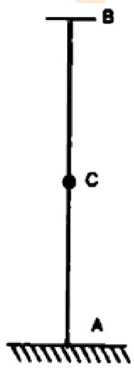


Q.18 A body of mass 2 kg is at rest at a height of 10 m above the ground.

(a) Calculate its potential energy and kinetic energy after it has fallen through half the height.

(b) Find the velocity at this instant.

Sol.



Total energy at B = kinetic energy + potential energy

= 0 + mgh

$= 2 \times 9.8 \times 10$
= 196 J

It descends half the height, it loses potential, energy which is given by

$$= mg\frac{h}{2}$$
$$= \frac{1}{2}\ mgh = 98\ J$$

Its potential energy at C = (196 – 98) = 98 J

The loss of potential energy = gain in kinetic energy
= 196 – 98
= 98 J

Kinetic energy = ½ mv^2

$$\frac{1}{2} \times 2 \times v^2 = 98$$
$$v^2 = 98 = 7\sqrt{2}\ m/s$$

Q19. The planet Mars has two moons, A and B
(i) How would you calculate the mass of Mars, A has a period 7 hours, 39 minutes and an orbital radius of 9.4 x 103 km
(ii) Assuming that Earth and Mars move in circular orbits around the sun with the Martian orbit being 1.52 times the orbital radius of the earth, then what is the length of the Martian year in days?

Sol. (i) The Sun's Mass replaced by the Martian mass Mm

$$T^2 = \frac{4\pi^2}{GMm}\ R^3$$

$$Mm = \frac{4\pi^2}{G} \times \frac{R^3}{T^2}$$

$$Mm = \frac{4\times(3.14)^2\times(9.4)^3\times(10)^{18}}{6.67\times10^{-11}\times(459\times60)^2}$$

$$= \frac{4\times(3.14)^2\times(9.4)^3\times10^{18}}{6.67\times(4.59\times6)^2\times10^{-5}}$$

$= 6.48 \times 10^{23}$ kg

(ii) Using Kepler's third law

$$\frac{T_M^2}{T_E^2} = \frac{R_{MS}^3}{R_{ES}^3}$$

Where R_{MS} is the Mars (Earth) - Sun distance

$$T_M = \left(\frac{R_{MS}}{R_{ES}}\right)^{3/2} - T_E^2$$

$$= (1.52)^{3/2} \times 365 = 684\ days$$

OR

Deduce the height at which the value of g is the same as at a depth of $\frac{R}{2}$?

Sol. At depth = $\frac{R}{2}$, value of acceleration due to gravity

$$g' = g\left(1 - \frac{R}{2R}\right) = \frac{g}{2}$$

At height x,

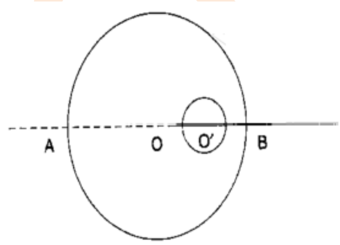
$$g' = g\left(1 - \frac{2x}{g}\right)$$

$$g\left(1 - \frac{2x}{R}\right) = \frac{g}{2}$$

$$\frac{1}{2} = \frac{2x}{R} \Rightarrow \frac{R}{4}$$

Q20. Find the centre of mass of the remaining disc, if a circular hole of radius 1 m is cut off from a disc of radius 6 m and the centre of the hole is 3 m from the centre of the disc.

Sol. Let O be the centre of the disc and O' that of the hole. To find the centre of mass, we use the fact that a body balances at this point. The algebraic sum of the moments of the weights about the centre of gravity is zero. The weight W_1 of the disc acts at point O. The hold can be regarded as a negative weight W_2acting at O'. If X is distance of the centre of gravity of the combination from point O then



$$x = \frac{W_1 x\ O + (-W_2) x^3}{W_1 + (-W_2)}$$

$$W_1 = \rho\pi x (6)^2 = 36\rho\pi$$

$$W_2 = \rho\pi x\ (1)^2 = \rho\pi$$

Where r is the mass per unit area of the disc.by passing the value of W1 and W2 we get,

$$x = \frac{-\rho\pi x^3}{36\ \rho\pi - \rho\pi}\ m = \frac{-3}{35}\ m$$

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Q21. Tunnel is dug through the earth from one side to the other side along with a diameter. The motion of a particle into the tunnel is simple harmonic motion. Find the time period, neglect all the frictional forces and assume that the earth has a uniform density. Assume that $G = 6.67 \times 10^{-11}$ $Nm^2 kg^{-2}$; density of the earth = 5.51×10^3 kg m^{-3}.

Sol. The tunnel is dug along the diameter of the earth. Consider the case of a particle of mass m at a distance y from the Centre of the earth. There will be a gravitational attraction of the earth on particle due to the portion of matter contained in a sphere of radius y. the mass of the sphere of radius y is given by

$$M = volume \times density$$

$$M = \frac{4}{3}\pi y^3 x\, d$$

This mass can be regarded as concentrated at the centre of the earth. The force F between this mass and the particle of mass m is given by

$$F = -\frac{GMm}{y^2}$$

Negative sign shows that the force is of attraction

$$F = -G\left(\frac{4}{3}\pi y^3 d\right)\frac{m}{y^2} = -Gx\left(\frac{4}{3}\pi y d\right)y$$

$$F \infty\, y$$

The force is directly proportional to the displacement hence the motion is simple harmonic motion.

The constant k = $\frac{4}{3}\pi md G$

The time period T $= 2\pi\sqrt{(m/k)}$

$$T = 2\pi\sqrt{\left(\frac{3m}{4\pi mdG}\right)} = 2\pi\sqrt{\left(\frac{3}{4\pi dG}\right)}$$

$$T = \sqrt{\left(\frac{3\pi}{dG}\right)} = \sqrt{\left(\frac{3\times 3.14}{5.51\times 10^5\times 6.67\times 10^{-11}}\right)}$$

$$T = 42.2\ minutes$$

OR

A body of mass m is suspended by two strings making angles α and β with the horizontal. Calculate the tensions in the two strings.

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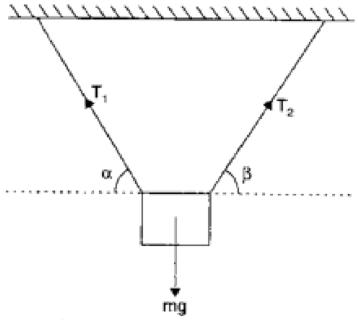

Sol. Consider components of tensions T1 and T2 along the horizontal and vertical directions

$T_1 \cos \alpha + T_2 \cos \beta = 0$ ---------------------- (i)

$T_1 \cos \alpha = T_2 \cos \beta$ ------------------------- (ii)

$T_1 \sin \alpha + T_2 \sin \beta = mg$

From (i)

$T_2 = \frac{T_1 cos\alpha}{cosß}$ And substituting if in (ii) we get

$$T_1\ sin\alpha + \left(\frac{T_1 cos\alpha}{cosß}\right) sinß = mg$$

$$T_1 \frac{\sin(\alpha+ß)}{cosß} = mg$$

$$T_1 = \frac{mg\ cosß}{\sin(\alpha+ß)}$$

Hence

$$T_2 = \frac{T_1 cos\alpha}{cosß} = \frac{mgcosß}{\sin(\alpha+ß)} \cdot \frac{cos\alpha}{cosß}$$

$$T_2 = \frac{mgcos\alpha}{\sin(\alpha+ß)}$$

Q22. (i) Calculate the angular momentum and rotational kinetic energy of earth about its own axis.
(ii)How long could this amount of energy supply one KW power to each of the 3.5 x 109 persons on earth?

Sol. Assume that earth to be a solid sphere. We know that the moment of inertia of a solid sphere about its axis is

$$I = \frac{2}{5} MR^2 = \frac{2}{5} \times (6.0 \times 10^{24}\ kg) \times (6.4 \times 10^6 m)^2$$
$$= 9.8 \times 10^{37}\ kg\ m^2$$

In one day the earth completes one revolution. Hence the angular velocity is given by

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$$\omega = \frac{2\pi}{24\times60\times60} = 7.27 \times 10^{-5}\ rad/sec$$

Angular momentum $I_\omega = (98 \times 10^{37}\ kg\ m^2)(7.27 \times 10^{-5} s^{-1})$

$$= 7.1 \times 10^{33}\ kg\ m^2/sec$$

The rotational energy

$$\frac{1}{2}\ I\omega^2 = \frac{1}{2}\ (9.8 \times 10^{37}\ kg\ m^2)(7.27 \times 10^{-5} s^{-1})^2$$

$$= 2.6 \times 10^{29}\ J$$

Power supplied by this energy

$$p = \frac{Energy}{Time} = \frac{2.6\times10^{29}}{t}\ watt$$

$$= \frac{2.6\times10^{29}}{10^{39} t}\ KW$$

Power required by 3.5×10^9 persons = $3.5\times10^9 \times 1$ kilowatt

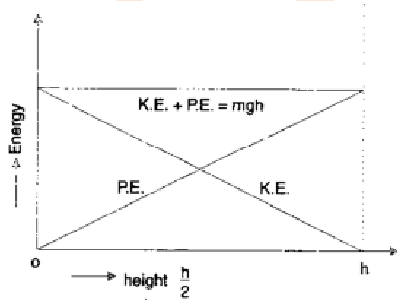
$$\frac{2.6\times10^{29}}{10^3 t} = 3.5 \times 10^9$$

$$t = \frac{2.6\times10^{29}}{10^3\times3.5\times10^9}\ sec$$

$$= 2.35 \times 10^9\ years$$

Q23. Show the variation of potential energy, K.E and the total energy of a body freely on earth from a height 'h' by using a graph.

Sol.



Graphs depicting variations of (i) gravitational potential energy (P.E) (ii) K.E and (iii) the total sum of potential and Kinetic energies for a freely falling body are shown in the diagram.

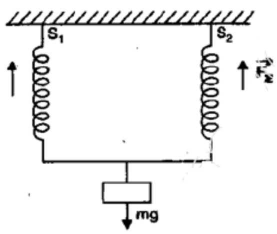
(i) Gravitational potential energy decease as the body falls downwards and is zero at earth.
(ii) Kinetic energy increase as the body falls downwards and will be at maximum when the

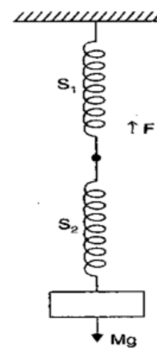
body just strikes the ground.

(iii) The sum of kinetic and potential energy remains constant at all during its free fall.

Q24. Find the expression for time period of motion of a body suspended by two springs connected in parallel and series.

Sol.





Consider the body of mass M suspended by two springs connected in parallel. Let K1 and K2 be the spring constants of two springs.

Let the body be pulled down so that each spring is stretched through a distance y. Restoring F1 and F_2 will be developed in the springs S_1 and S_2.

According to Hooke's law $F_1 = -K_1y$ and $F_2 = -K_2y$

Since both the forces acting in the same direction, total restoring force acting on the body is given by

$F = F_1 + F_2 = -K_1 y - K_2 y = -(K_1 + K_2)\, y$

Acceleration produced in the body is given by

$$a = \frac{F}{M} = -\frac{(K_1+K_2)Y}{M} \quad \text{……………………………….(i)}$$

Since $\frac{(K_1+K_2)}{M}$ $is\ constnat\ a = -y$

Hence motion of the body is SHM

The time period of body is given by

$$T = 2\pi\sqrt{\frac{y}{|a|}} = 2\pi\sqrt{\frac{M}{K_1+K_2}} \quad \text{………..(ii)}$$

$$K_1 = K_2 = K$$

$$T = 2\pi\sqrt{\frac{M}{2K}}$$

For series:

Consider the body of mass M suspended by two springs S1 and S2 which are connected in series.

Let k_1 and k_2 be the spring constants of spring $S_1$and S_2.

At any instant the displacement of the body from equilibrium position is y in the downward direction. If y_1 and y_2 be the extension produced in the springs S_1 and S_2.

$y = y_1 + y_2$ ---------------------------- (i)

Restoring the forces developed in S1 and S2 are given by,

$F1 = -k_1 + y_1$ ---------------------------------- (ii)

$F2 = -k_2 + y_2$ ---------------------------------- (iii)

Multiplying the equation (ii) by k_2 and equation (iii) by k_1 and adding we get,

$K_2F_1+k_1F_2 = -k_1k_2 (y_1+y_2) = -k_1k_2 y$

Since both the springs are connected in series.

$F_1 = F_2 =F$

$$F\,(k_1 + K_2) = -K_1K_2y$$

$$F = \frac{K_1K_2}{(K_1+K_2)y}$$

If 'a' be the acceleration produced in the body of mass 'M' then,

$$a = \frac{F}{M} = \frac{K_1K_2y}{(K_1+K_2)} \text{....................(iv)}$$

$$T = 2\pi\sqrt{\frac{y}{|a|}} = 2\pi\sqrt{\frac{(K_1+K_2)M}{K_1K_2}}$$

$$T = 2\pi\sqrt{\left(\frac{1}{K_1}+\frac{1}{K_2}\right)\,M}$$

OR

a) What causes variation in velocity of a particle?

b) A car travels first half of a length S with velocity v_1. The second half is covered with velocities v_2 and v_3 for equal intervals. Find the average velocity of the motion.

Sol. (a) Change in magnitude of velocity. Change in direction of motion of the motion and change in magnitude as well as direction of the motion.

(b) Average velocity,

$$v = \frac{Total\ Displacement}{Total\ time\ taken}$$

Time taken to cover the first half of the length = $\frac{S}{2V_1}$

Time taken to cover the second half of the length = 2t

$$v = \frac{S}{\frac{S}{2v_1} + 2t}$$

Second half is divided equally into two parts with equal time

$$\frac{S}{2} = v_2 t + V_3 t$$
$$= (v_2 + v_3)t$$

$$2t = \frac{S}{(v_2 + v_3)}$$

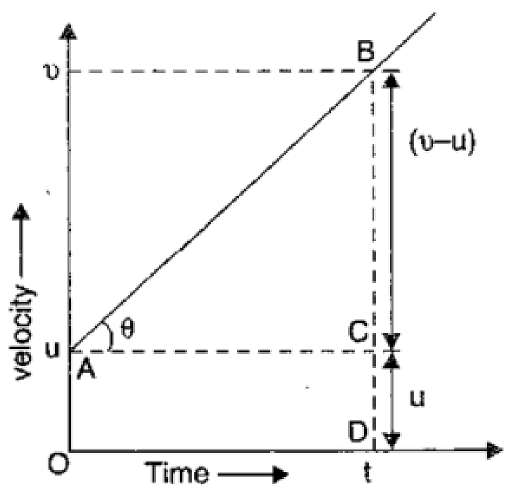
$$V = \frac{S}{\frac{S}{2v} + \frac{S}{(v_2 + v_3)}}$$

$$v = \frac{2v_1(v_2 + v_1)}{(v_2 + v_3 + 2v_1)}$$

Q25. Explain the kinematic equation for uniformly accelerated motion.

Sol. Uniformly accelerated motion, we can drive some simple equation that relate displacements (x), time taken (f), initial velocity (u), final velocity (v) and acceleration (a).

(i) Velocity attends after time t: the velocity-time graph for positive constant acceleration of a Particle.



Let u be the initial velocity of the particle at l =0 and v is the final velocity of the particle after time t. consider two points A and B on the curve corresponding to t=0 and t =t. Draw ZBD perpendicular to time axis. Also draw AC perpendicular to BD.

OA = CD = u

BC = $(v - u)$ $and\ OD = t$

Now,

Slope of $v - t\ graph = acceleartion\ (a)$

$a = slope\ of\ v - t\ graph$

$\tan\theta = \frac{BC}{AC} = \frac{BC}{OD}$
$a = \frac{v-u}{t}$
$v - u = at$
$v = u + at$

(ii) Distance travelled in time t will be,

x_0 = position of the particle at t =0 from the origin
x = position of the particle at t =t from the origin
$(x – x_0)$ = S = distance travelled by the particle in the time interval (t -0) =t
Distance travelled by a particle in the given time
Interval = area under velocity-time graph
$(x – x_0)$ = area OABD
= area of Trapezium OABD
= ½ [Sum of parallel sides **x** perpendicular distance between parallel sides]
= ½ (OA + BZD) x AC
= ½ (u + v) x t
v =u +at

$(x – x_0)$ = ½ (u + u + at) x t
= ½ (2u + at) x t
= ut + ½ at^2
x – x0 = S
S = ut + ½ at^2

(i) Velocity attained after travelling a distance S:

Distance travelled by a particle in time t is equal to the area under velocity-time graph.
The distance (s) travelled by a particle during time interval t is given by
S = area under v – t graph
S = area of Trapezium OABD
= ½ (sum of parallel sides) x perpendicular distance between these parallel sides

$S = \frac{1}{2}(OA + OD) \times AC$ ……………………(i)

Acceleration a = slope of v – t graph

$A = \frac{BC}{AC} = \frac{BD-CD}{AC} = \frac{v-u}{AC}$

$AC = \left(\frac{v-u}{a}\right)$ ……………………..(ii)

OA = u and BD = v ………………(iii)

From equation (i), (ii) and (iii) we get

$S = \frac{1}{2}(v+u)\frac{(v-u)}{a}$
$S = \frac{v^2-u^2}{2a}$

$v^2 - u^2 = 2aS$

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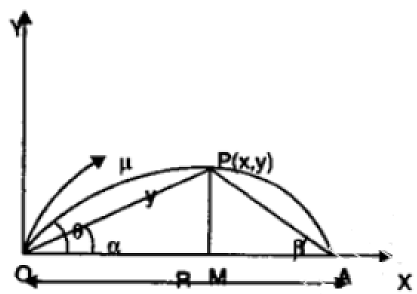
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

OR

A particle is thrown over a triangle from one end of a horizontal base that grazing h vertex fall on the other end of the base. If α and β be the same angles and Θ the angle of projection, prove that tan Θ = tan α + tan β.

Sol.



$\tan\alpha = \frac{y}{x}$

From the where TR is horizontal range.

$tanß = \frac{Y}{MA} = \frac{Y}{R-x}$

$tan\alpha + tanß = \frac{y}{x} + \frac{y}{R-x}$

$= \frac{(R-x+x)y}{x(R-x)} = \frac{yR}{x(R-x)}$

$\tan\alpha + tanß = \frac{yR}{x(R-x)}$ ……………………………….(i)

$x = (u\cos\theta)t$ ……………………………..(ii)

$y = (u\sin\theta)t - \frac{1}{2}gt^2$ ……………………. (iii)

From equation (ii) and (iii),

$$y = x\,tan\theta\ \left[1 - \frac{xg}{2u^2\cos^2\theta\,tan\theta}\right]$$

Substituting, $R = \frac{2u^2 sin\theta\, cos\theta}{g}$

$$y = x\ tan\theta\ \left[1 - \frac{xy}{2u^2 cos\theta\ sin\theta}\right]$$

$$y = x \tan\theta\ \left[1 - \frac{x}{R}\right]$$

$$\frac{y}{x} = \tan\theta\ \left(\frac{R-x}{R}\right)\text{.........................(iv)}$$

Putting (iv) in (i), we get,

$$tan\alpha + tanß = \frac{yR}{x(R-X)} = \ tan\theta$$

$$tan\alpha + tanß = tan\theta$$

Q26. If a stone is dropped from the top of a mountain and n second later another stone is thrown vertically downwards with a velocity of u m/s, then how far below the top of the mountain will be the second stone overtake the first?

Sol. The second stone will catch up with the first stone when the distance covered by it in (t – n) second will equal the distance covered by the first stone in t second.
The distance covered by the first stone in t second = ½ gt^2 and distance covered by the second stone in (1 –n) second.

$$u\ (t-n) + \frac{1}{2}\ g(t-n)^2$$
$$\frac{1}{2}\ gt^2 = u\ (t-n) + \frac{1}{2} g(t-n)^2$$
$$\frac{1}{2}\ g[t^2 - (t-n)^2] = u\ (t-n)$$
$$\frac{1}{2}\ g[(2t-n)n] = u\ (t-n)$$

$$gnt - \frac{1}{2} gn^2 = ut - un$$
$$t(gn-u) = (\frac{1}{2} gn - u)n$$

$$t = \frac{n\left(\frac{1}{2}\,gn-u\right)}{(gn-u)}$$

The distance covered by the first stone in this time

$$h = \frac{1}{2} gt^2 = \frac{1}{2}\ g\ \left[\frac{n\left(\frac{1}{2}gn-u\right)}{(gn-u)}\right]$$

Thus the second stone will overtake the first at distance

$$\frac{1}{2} g\ \left[\frac{n\left(\frac{gn}{2}-u\right)}{(gn-u)}\right]^2$$

OR

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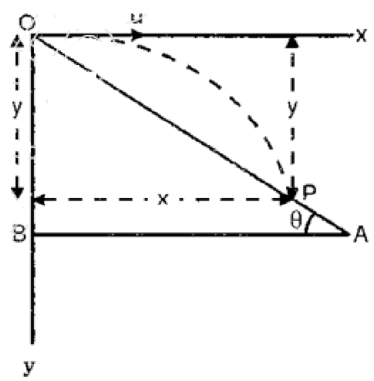
A particle is projected horizontally with a speed u from top of a plane inclined at an angle Θ with the horizontal direction. How far from the point of projection will the particle strike the plane?

Sol. Let the particle projected from O strike the inclined plane OA at P after time t and coordinates of P be (x,y).



Taking motion of projectile from O to P along x-axis we have
$x_0 = 0$, x = x, ux = u, ax =0, t=t
Using the relation x = x0 + uxt + ½ ax t^2
We get x = ut or i =x/u
Taking motion of projectile along y – axis
$y_0 = y$, y = y, u = 0, $a_y = g$, t = t

$$Using\ the\ relation\ y = y_0 + u_y t + \frac{1}{2}\ a_y t^2$$

$$y = 0 + 0 + \frac{1}{2}\ gt^2 = \frac{1}{2}gt^2 = \frac{1}{2}g\frac{X^2}{u^2}$$

$$y = x\ tan\theta\ , so\ gx^2\ /2u^2 = xtan\theta$$

$$x = \frac{2u^2tan\theta}{g}$$

And $y = x\tan\theta = \frac{2u^2\tan^2\theta}{g}$

Distance OP = $\sqrt{x^2 + y^2}$

$$= \frac{2u^2tan\theta}{g}\ \sqrt{1 + \tan^2\theta}$$

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$$= \frac{2u^2 tan\theta \, sec\theta}{g}$$