Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Class: XII
Subject: Mathematics
Topic: Applications of Derivatives
No. of Questions: 20
Duration: 60 Min
Maximum Marks: 60

1. At what $X \in \left(0, \frac{\pi}{2}\right)$,f(x)= $\sin\left(X + \frac{\pi}{6}\right) + \cos(X + \frac{\pi}{6})$has a maximum value?

A. π/4
B. π/6
C. π/12
D. π/8

Ans. C

Solution: Here (X)= $\sin(X + 75)$

We get maximum value of sine function in (0, π/2

When$X$ =π/12

2. The gaussian curve $y = \frac{1}{\sqrt{2\pi}} e^{-X^2/2}$ has points of inflection at

A. X= ±1
B. X= 1 only
C. X =1 and X =0
D. X=-1and X=0

Ans. C

Solution:

Putting y``=0
We get point of inflection at x = ± 1

3. The maximum area of a rectangle that can be inscribed in a circle of radius 2 units is (in square units)

A. 4
B. 8π
C. 8
D. None of these

Ans. C

Solution:

If r units be the radius of the circle then the side of the max rectangle= $r\sqrt{2}$

Area= 8

4. If the subnormal of the curve $xy^n = a^{n+1}$ is of constant then the calue of n is

A. 1
B. 2
C. -2
D. None of these

Ans. C

Solution:

$$y^n + xny^{n-1}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{y^n}{nxy^{n-1}} = -\frac{y}{nx}$$

$$subnormal = -\frac{y^2}{nx}$$

$$to..get..a...cons\tan t..value..n..should..be..-2$$

to. get. a…cons tan t.. value..n.should..be..-2

Check it by substituting it back in original equation

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

5. Let $f(x)=(1+b^2)x^2+2bx+1$ and m (b) be the minimum value of f (x) for a given b. As b varies, the range of m (b) is

A. (1,2)
B. (2,3)
C. (3,4)
D. (0,1]

Ans. D

Solution:

$$f(x)=(1+b^2)\left[x^2+\frac{2b}{1+b^2}x\right]+1=(1+b^2)\left[x^2+\frac{2b}{1+b^2}x+\frac{b^2}{(1+b^2)^2}\right]-\frac{b^2}{1+b^2}+1$$

$$=(1+b^2)\left[x+\frac{b}{1+b^2}\right]^2+\frac{1}{1+b^2}\geq\sqrt{3}$$

$$\therefore \min .f(x)=m(b)=\frac{1}{1+b^2}\Rightarrow \textit{The range of } m(b) \textit{ is } (0,1]$$

6. The surface area of a sphere, when its volume is increasing at the same rate as its radiurs, is

A. 1
B. $\frac{1}{\sqrt[2]{\pi}}$
C. 2
D. None othe these

Ans. A

Solution:

S be the surface area of the sphere and V be the volume r=radius

$$S=4\pi r^2$$

$$V=\frac{4}{3}\pi r^3$$

$$\frac{dv}{dt}=4\pi r^2\frac{dr}{dt}=S\frac{dr}{dt}$$

$$given..that...\frac{dv}{dt}=\frac{dr}{dt}$$

$$S=1$$

7. The slope of the normal to the curve $x = 3t^2 + 1$, $y = t^3 - 1$ at $x = 1$ is

A. 0
B. $\frac{1}{2}$
C. ∞
D. -2

Ans. C

Solution:

$$x = 3t^2 + 1$$
$$dx/dt = 6t$$
$$y = t^3 - 1$$
$$dy/dt = 3t^2$$
$$dy/dx = \frac{3t^2}{6t} = \frac{t}{2}$$
$$when\ x = 1$$
$$t = 0$$
$$dy/dx = 0$$

Slope of the normal =∞

8. The tangent at P on the curve $y^2 = 2X^3$ is ^ to the line 4 x- 3y= 0. Then the abscissa of the point P is

A. 0 or 1/8
B. 0 or -1/8
C. 1/8
D. -1/4

Ans. C

Solution:

$$2y\frac{dy}{dx} = 6x^2$$
$$slope..of..the..given..line = 4/3$$
$$\frac{-3}{4} = \frac{6x^2}{2y} \Rightarrow y^2 = 16x$$
$$x = 1/8$$

9. Let f(X) $xe^{ax}, x \geq 0$ $x + ax^2 - x^3$, x > 0 where a is positive constant then the inverval in which f (x) is increasing is

A. (-2/a,a/3)
B. (-2/a,a/2)
C. (2/a,-a/3)
D. None of these

Ans. C

Solution:

$$f'(x) = \begin{cases} e^{ax} + ax\, e^{ax}, x < 0 \\ 1 + 2ax - 3x^2, x > 0 \end{cases} and\ \ f''(x) = \begin{cases} e^{ax}(2a + a^2 x) x < 0 \\ 2a - 6x, x > 0 \end{cases}$$

$$When\ ever\ \ f''\ \ x > 0,\ \ f'\ (x)\ \ increases$$

$$\therefore ax + 2 > 0 (and\ \ x < 0)\ \ and\ \ 2a - 6x > 0\ \ (and\ \ x > 0) \Rightarrow x(-2/a, a/3)$$

10. The stationary point of the function X^x at

A. X=e
B. X=1/e
C. X=1
D. X=$\sqrt{e}$

Ans. B

Solution:

Take y = X^x

Here dy/dx=0 give log x+1 =0

Then x=1/e is the stationary point

11. The median of an equilatera 1 triangle is increasing at the rate of $\sqrt[2]{3}$ cm sec. What is the rate at which its sides are increasing

A. 4cm/sec
B. 2cm/sec
C. 4/3cm/sec
D. None of these

Ans. A

Solution:

Let the median be h cm of an equilateral triangle , sides=a cm

$$h = \frac{\sqrt{3}}{2}a$$

$$\frac{dh}{dt} = \frac{\sqrt{3}}{2}\frac{da}{dt}$$

$$\frac{da}{dt} = 4cm/\sec$$

12. The point at which the tangent to the curve $y = 2x^2 - x + 1$ is parallel to the line $y = 3x + 9$ is

A. (2,1)
B. (1,2)
C. (3,9)
D. (-2,1)

Ans. B

Solution:

Point is (1,2) which lies on the curve

Find dy/dx and equate it equal to the slope of given line

13. The equation of the tangent to the curve $y = \sin 2x + \cos x$ at(0,1) is

A. $y = 2x - 1$
B. $y = 2x + 2$
C. $y = 1$
D. $y = 2x + 1$

Ans. D

Solution:

$$y = \sin 2x + \cos x$$

$$\frac{dy}{dx} = 2\cos 2x - \sin x.[.at(0,1)] = 2$$

$$reqd..equation$$

$$y - 1 = 2(x - 0) = 2x$$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

14. If $s = a\cos\frac{bt}{c}$, then the acceleration is

A. $-\frac{b^2}{c^2}s$

B. $-\frac{a^2}{c^2}s$

C. $\frac{b^2}{c^2}s$

D. $-\frac{ab^2}{c^2}s$

Ans. A

Solution:

$$s = a\cos\frac{bt}{c}$$

$$\frac{ds}{dt} = a\left(-\sin\frac{bt}{c}\right)\frac{b}{c}$$

$$acceleration = \frac{d^2s}{dt^2} = -\frac{ab^2}{c^2}\cos\frac{bt}{c} = -\frac{b^2}{c^2}s$$

15. The slope of the normal to the curve $27y = x^3 - 3x + 2$, where abscissa of the point is -2, is

A. 1

B. -1

C. -3

D. None of these

Ans. C

Given that$x = -2$

Then $27y = -8 + 6 + 2 = 0$

Then $y = 0$

Then $-\frac{dx}{dy} = -3$

16. The length of the sub tangent to the curve $\sqrt{x} + \sqrt{y} = 3$ at (4,1) is

A. 4

B. 2

C. ½

D. None of these

Ans. B

Solution:

$$\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-1/2$$

*length..of..the..sub*tan *gent*

$$\left|\frac{y}{dy/dx}\right|=2$$

17. A point on the curve $y = 6x - x^2$ at which the tangent to the curve is included at an angle of 45° to the line $x + y = 0$ is

A. (-3,9)

B. (-3,-27)

C. (3,9)

D. (0,0)

Ans. D

Solution:

Then the slope of the tangent be 90 degree

Equation of the tangent $x = 0$

The point is (0,0)

18. If the tangent and normal to thte curve $y = x^3 + x^2 - 11$ at P (2,1) meets the x-axis at A and B respectively, then AB=

A. 257/16

B. 16

C. 3/2

D. 255/6

Ans.A

Solution:

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Here dy/dx at the given point=16
Equation of the tangent is y-16x=-31
Normal 16y+x=18
Point A(31/16,0)
B(18,0)
Distance=257/16

19. If the curves $y = ax^3 + b$ and $y = bx^2 + 1$ touch each other at $x = -1$,

Then a =

A. -4
B. 4
C. 0
D. None of these

Ans. D

Solution:

Just equate both y and put x=-1, you will get

$-a + b = b + 1$

Then $a = -1$

20. For the function $x(1-x)^2, 0 \leq x \leq 2$, the minimum and maximum values are respectively

A. $0; \frac{9}{4}$
B. $0; \frac{4}{27}$
C. $0; \frac{9}{12}$
D. $1; \frac{4}{27}$

Ans. B

Solution:

Let $y = x(1 - x)^2$
Here putting dy/dx=0 we get x=1,1/3
Min value=0 [putting x=1]
Max value=4/27[putting x = 1/3]