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Class: XII
Subject: Math's
Topic: Area Under Curve
No. of Questions: 20
Duration: 60 Min
Maximum Marks: 60

1. The area of the region enclosed by the curves y= sin x and y = cos x and y – axis is

A. $\sqrt{2}-1$

B. $\sqrt{2}+1$

C. $\sqrt{2}$

D. 1

Ans. A

Solution:

$$\int_0^{\pi/4}\cos x\,dx-\int_0^{\pi/4}\sin x\,dx=\sqrt{2}-1$$

2. The area enclosed between the curves $y = x^3$ and $y = x$ is

A. 0

B. ½

C. ¼

D. 1

Ans. B

Solution:

When x=1,y=1

Area

$$2(1/2-\int_0^1 x^3dx)=1/2$$

3. The area enclosed between the parabola $y^2 = 4ax$ and the line y = 2x is

A. $\frac{1}{2}a^2$

B. $\frac{1}{3}a^2$

C. $\frac{1}{4}a^2$

D. $\frac{1}{6}a^2$

Ans. B

Solution:

$$\int_0^a \sqrt{4ax}\,dx - \int_0^a 2x\,dx = \frac{a^2}{3}$$

4. Let A be the area bounded by the curve $y = (\tan x)^n$, and the lines $x = 0, \frac{\pi}{4}$. Then for $n > 2$,

$A_n + A_{n-1} =$

A. $\frac{1}{n}$

B. $\frac{1}{n-2}$

C. $\frac{1}{n-1}$

D. N

Ans. C

Solution:

$$A_n = \int_0^{\pi/4} (\tan x)^n\,dx = \frac{1}{n-1} - A_{n-2}$$

$$A_n + A_{n-2} = \frac{1}{n-1}$$

5. The area of the region bounded by the parabola $y^2 = 4ax$ and its LR is

A. $\frac{4}{3}a^2$

B. $\frac{2}{3}a^2$

C. $\frac{8}{3}a^2$

D. $2a^2$

Ans. C

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Solution:

The area of the region bounded by the parabola y^2=4 ax and its LR is $\frac{8}{3}a^2$

6. The area of the region bounded by the curve y = log x, the x - axis and the line x = 2 is

A. $\log 2$

B. 2

C. $\frac{3}{2}$

D. $2\log 2 - 1$

Ans. D

Solution:

$$\int_1^2 \log x\,dx = [x\log x - x]_1^2 = 2\log 2 - 2 + 1 = 2\log 2 - 1$$

7. If the area enclosed between the parabola $y^2 = 24x$ and the line ay = x is 36 sq. units, then the value of a is

A.2
B. 1/2
C. 3
D. 1

Ans. B
Solution:

$$\int_0^{24a^2} \sqrt{24x}\,dx - \frac{1}{2}(24a^2)(24a) = 36$$

then

$a = 1/2$

8. AOB is the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where OA = a, OB = b.

Then the area between the arc AB and the chord AB of the ellipse is

A. $\frac{ab}{2}\left(\frac{\pi}{2} - 1\right)$

B. $\frac{ab}{2}(\pi - 1)$

C. $\frac{ab}{4}(\pi - 1)$

D. None of these

Ans. A
Solution:
Area $= \frac{\pi ab}{4} - ab/2$
$\frac{ab}{2}\left(\frac{\pi}{2} - 1\right)$

9. The area enclosed between the parabola $y = x^2 - x + 2$ & the line $y = x + 2$ in sq. units is equal to
   A. 4/3
   B. 2/3
   C. 1/3
   D. 8/3
   Ans.A
   Solution:
   required ...area
   $$\int_0^2 [(x+2) - (x^2 - x + 2)]dx$$
   $$= [\frac{x^2}{2} + 2x]_0^2 - [\frac{x^3}{3} - \frac{x^2}{2} + 2x]_0^2 = 4/3 \text{ sq..unit}$$

10. The area of the region bounded by the parabola $y^2 = -(x - 4)$ and y - axis is
   A. 8/3
   B. 16/3
   C. 32/3
   D. None of these
   Ans. B
   Solution:
   $$\int_{-2}^{0} x\,dy = \int_{-2}^{0} (y^2 - 4)dy = 16/3$$

11. The area of the region enclosed within the curve $|x| + |y| = 1$ is
   A. $\sqrt{2}$
   B. 2
   C. $\sqrt[2]{2}$
   D. 4
   Ans.B
   Solution:
   Area=4.(1/2)=2

12. The area of region lying in I quadrant enclosed by the x - axis, the line $x = \sqrt{3}y$ and the circle
   A. π/2
   B. π/3
   C. π/6
   D. π/4
   Ans. B

Solution:

$for..y = 1$

$x = \sqrt{3}$

$reqd..area = \frac{\sqrt{3}}{2} + \int_{\sqrt{3}}^{2} \sqrt{4 - x^2}\, dx = \frac{\pi}{3}$

13. The area bounded by one arch of the curve y = sin mx and x - axis is
    A. 2m
    B. 2/m
    C. 4m
    D. None of these

    Ans. A

    Solution:

    The area bounded by one arch of the curve y = sin mx and x-axis is 2/m

    $\int_{0}^{\pi} \sin mx\, dx = 2/m$

14. The area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and line $\frac{x}{a} + \frac{y}{b} = 1$ is
    A. $\pi ab - ab$
    B. $\frac{ab}{4}(\pi - 2)$
    C. $\frac{ab}{2}(\pi - 1)$
    D. None of these

    Ans. B

    Solution:

    $\frac{\pi ab}{4} - \frac{ab}{2}$

    $\frac{ab}{4}(\pi - 2)$

15. The area of the region bounded by $y = 1 + 8/x^2$, y = 0, x = 2 and x = 4 is
    A. 1
    B. 2
    C. 4
    D. 8

    Ans.C

    Solution:

    $\int_{2}^{4} (1 + 8/x^2)\, dx = 4$

16.
S_3 are respectively the areas of these parts numbered from top to bottom, then $S_1 : S_2 : S_3$ is

The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2,

A. 1:2:3
B. 1:2:1
C. 1:1:1
D. 2:1:2

Ans. C

Solution:

Area of $s_1=s_2=s_3$

Their ratio 1:1:1

17. The area enclosed between the parabola $y^2 = 6x$ and $x^2 = 4y$ is

A. 8
B. 16
C. 32
D. None of these

Ans. A

Solution:

The area bounded by $y^2=4ax$ and $x^2=4by$ is given by $(4a)(4b)/3$

Here area=24/3=8

18. The area bounded by the curve $y = x(3 - x)^2$, the x – axis and the ordinates of the maximum and minimum points of the curve is

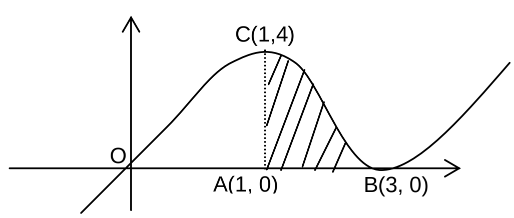
A. 2 sq. units
B. 6sq. units
C. 4 sq. units
D. 8 sq. units

Solution: $y = x(3 - x)^2$

After solving, we get x =1 and x = 3 are points of maximum and minimum respectively.

Now the shaded region is the required region

$$\therefore\ A = \int_1^3 x(3-x)^2\,dx = 4 \text{ sq. units}$$



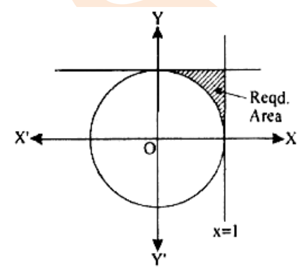
19. What is the area of a plane figure bounded by the points of the lines max (x, y) = 1 and $x^2 + y^2 = 1$?

(A) $\frac{p}{2}$ sq. units (B) $\frac{p}{3}$ sq. units

(C) $\frac{\pi}{4}$ sq. units (D) π sq. units

Solution: By definition the lines max, (x, y) = 1 means.

x = 1 and y ≤ 1 or y = 1 and x ≤ 1



Required area

$$= \int_0^1 \left[1 - \sqrt{1-x^2}\right] dx$$

$$= \left[x - \frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\sin^{-1}x\right]_0^1$$

$$= 1 = 0 = \frac{1}{2}\left(\frac{\pi}{2}\right) = 1 = \frac{\pi}{4} sq units$$

20. The area bounded by the curve y = (x –1) (x – 2) (x – 3) lying between the ordinates x = 0 and x = 3 is

(A) $\frac{7}{4}$ sq. units (B) 4 sq. units

(C) $\frac{11}{4}$ sq. units (D) 3 sq. units

Solution: Reqd. area = $\int_0^3 |y|\,dx = \int_0^1 |y|\,dx + \int_1^2 |y|\,dx + \int_2^3 |y|\,dx$

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$$= -\left(\frac{x^4}{4} - 2x^3 + \frac{11}{2}x^2 - 6x\right)_0^1 + \left(\frac{x^4}{4} - 2x^3 + \frac{11}{2}x^2 - 6x\right)_1^2 - \left(\frac{x^4}{4} - 2x^3 + \frac{11}{2}x^2 - 6x\right)_2^3$$

$$= \frac{9}{4} + \frac{1}{4} + \frac{1}{4} = \frac{11}{4} \text{ sq units.}$$