Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

askIITians
ENGINEERING | MEDICAL | FOUNDATION

Class: XII
Subject: Math's
Topic: Definite Integration
No. of Questions: 20
Duration: 60 Min
Maximum Marks: 60

1. $\int_0^{\pi/2}(\sin^{-1} x + \cos^{-1} x)\, dx =$
   a. $\frac{\pi}{2}$
   b. $\frac{\pi}{4}$
   c. Doesn't exist
   d. $\frac{\pi^2}{4}$

   Ans. D

   Sol:d $\int_0^{\pi/2} (\sin^{-1} x + \cos^{-1} x)dx = \int_0^{\pi/2} \frac{\pi}{2} dx$

   $= \frac{\pi^2}{4}$

2. $\int_0^{x/4} \tan^2 x\, dx =$
   a. $\frac{\pi}{4}$
   b. $1 - \frac{\pi}{4}$
   c. $\frac{\pi}{4} - 1$
   d. $1 + \frac{\pi}{4}$

   Ans. B

   Sol:b $\int_0^{\pi/2}$ X dx =

   $\int_0^{\pi/2}$ [$\sec^2 x$ -1] dx = 1 – π/4

3. $\int_0^1 \frac{x^3}{\sqrt{1-x^2}}$ dx equals
   a. 1
   b. $\frac{1}{3}$
   c. $\frac{2}{3}$
   d. $-\frac{1}{2}$

   Ans. C

   Sol:C Put $1-x^2 = v^2$

$$\int_0^1 \frac{x^3}{\sqrt{1-x^2}} dx$$

$$= \int_1^0 (-1+v^2)dv = \left[-v+\frac{v^3}{3}\right]_1^0 = -(-1+1/3) = 2/3$$

4. $\int_{-1}^{1} x \tan^{-1} x \, dx =$
   a. 0
   b. $\frac{\pi}{4}+\frac{1}{2}$
   c. $\frac{\pi}{2}-1$
   d. None of these

   Ans. C

   Solution:

   Given function is aneven function

$$\int_{-1}^{1} x \tan^{-1} x \, dx = \int_0^1 2x \tan^{-1} x$$

$$= 2\left[\frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\{x - \tan^{-1} x\}\right]_0^1 = \frac{\pi}{2} - 1$$

5. $\int_0^{x/4} \frac{\sqrt{\tan x}}{\sin x \cos x} dx =$

   a. 0
   b. 1
   c. -2
   d. 4

   Ans. C

   Solution:

$$\int_0^{x/4} \frac{\sqrt{\tan x}}{\sin x \cos x} dx = \int_0^1 \frac{\sqrt{v}}{v} dv$$
$$= 2\left[\sqrt{v}\right] = 2$$

6. $\underset{n \to \infty}{\text{Lt}} \left[\frac{n+1}{n^2+1^2} + \frac{n+2}{n^2+1^2} + \dots\dots\dots + \frac{1}{n} \right] =$

   a. $\frac{\pi}{4}$
   b. $\frac{\pi}{2}$
   c. $\frac{\pi}{4} + \log\sqrt{2}$
   d. None of these

   Ans. C

   Solution:

$$\underset{n \to \infty}{\text{Lt}} \left[\frac{n+1}{n^2+1^2} + \frac{n+2}{n^2+1^2} + \dots\dots\dots + \frac{1}{n} \right]$$
$$= \underset{n \to \infty}{lt} \ 1/n \left[\frac{1+1/n}{1+(1/n)^2} + \dots\dots + \frac{1+n/n}{1+(n/n)^2} \right] = \int_0^1 \frac{1+x}{1+x^2} dx = \left[\tan^{-1} x + \frac{1}{2}\log(1+x^2) \right]_0^1$$
$$= \pi/4 + \log\sqrt{2}$$

7. The value of the integral $I = \int_0^1 x(1-x)^n \, dx$ is

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

a. $\frac{1}{n+1}$

b. $\frac{1}{n+2}$

c. $\frac{1}{n+1} - \frac{1}{n+2}$

d. $\frac{1}{n+2} + \frac{1}{n+2}$

Ans. C

Solution:

$$I = \int_0^1 x\,(1-x)^n\,dx = \int_0^1 (1-x)x^n\,dx$$

$$= \frac{1}{n+1} - \frac{1}{n+2}$$

8. $\int_0^3 |2-x|dx =$

a. $\frac{3}{2}$

b. $-\frac{3}{2}$

c. $\frac{5}{2}$

d. 2

Ans. C

Solution:

$$\int_0^3 |2-x|\,dx = \int_0^2 (2-x)dx + \int_2^3 (x-2)dx$$

$$= 5/2$$

9. If [x] is the greatest integer function, then $\int_0^1 \left([x] + \frac{\log(1+x)}{1+x^2}\right) dx =$

a. $\frac{1}{2}+\frac{\pi}{4}\log 2$

b. $\frac{\pi}{8}\log 2$

c. $\frac{1}{2}+\frac{\pi}{8}\log 2$

d. $\frac{\pi}{4}\log 2$

Ans. B

Solution:

$$\int_0^1 [x]dx = \int_0^1 0.dx = 0 \text{ and } \int_0^1 \frac{\log(1+x)}{1+x^2}dx = \frac{\pi}{8}\log 2.(From\ memory)$$

10. $\int_0^{\pi/2} \sin^2 x\, dx + \int \cot^2 x\, dx =$

a. $\frac{\pi}{2}+\cot x - x$

b. $\frac{\pi}{4}+\cot x - x$

c. $\frac{\pi}{2}+\cot x + x$

d. $\frac{\pi}{4}-\cot x - x$

Ans. D

Solution:

$$\int_0^{\pi/2} \sin^2 x\, dx + \int \cot^2 x\, dx = \int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}dx + \int \cos ec^2 x dx - \int dx$$

$$= \frac{\pi}{4} - \cot x - x + c$$

11. If f(x) is differenti able and $\int_0^{t^2} x\, f(x)\, dx = \frac{2}{5} t^5$, find $f\left(\frac{4}{25}\right)$ equals

a. $\frac{2}{5}$

b. $-\frac{5}{2}$

c. $\frac{5}{2}$

d. 1

Ans. A

Solution:

$$\int_0^{t^2} x\, f(x)\, dx = \frac{2}{5} t^5, it..shows..that\ f(x) = \sqrt{x}, \text{then } f\left(\frac{4}{25}\right) = 2/5$$

12. If $a_n = \int_0^{\pi/2} \sin 2t\,(1-\cos t)^n\, dt$, then the value of $\lim_{n\to\infty} \sum_{n=0}^{\infty} a_n$ is

a. 1

b. 2

c. 10

d. 8

Ans. B

Solution

$$a_n = \int_0^{\pi/2} \sin 2t\,(1-\cos t)^n\, dt$$

$$a_n = 2\int_0^{\pi/2} \cos t \sin t\,(1-\cos t)^n\, dt \qquad \begin{bmatrix} 1-\cos t = u \\ \sin t dt = du \\ \cos t = 1-u \end{bmatrix}$$

$$a_n = 2\int_0^{1} (1-u)\,u^n\, dt$$

$$a_n = 2\left[\frac{u^{n+1}}{n+1} - \frac{u^{n+2}}{n+2}\right]_0^1$$

$$a_n = 2\left[\frac{1}{n+1} - \frac{1}{n+2}\right]$$

$$\lim_{n\to\infty} \sum_{n=0}^{\infty} a_n$$

$$= 2\left[1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} \ldots\right]$$

$$= 2$$

13. $\int_0^{\pi/4} \sqrt{\dfrac{1-\sin 2x}{1+\sin 2x}}\, dx =$

a. $\log 2$
b. 0
c. $\frac{1}{2}\log 2$
d. $2\log 2$

Ans. C

Solution:

$$\int_0^{\pi/4} \sqrt{\frac{1-\sin 2x}{1+\sin 2x}}\, dx$$

$$= \int_0^{\pi/4} \sqrt{\frac{(\cos x-\sin x)^2}{(\cos x+\sin x)^2}}\, dx = [\log|\cos x+\sin x|]_0^{\pi/4} + c = \frac{1}{2}\log 2$$

14. If $I_1 = \int_e^{e^2} \dfrac{dx}{\log x}$ and $I_2 = \int_1^2 \dfrac{e^x}{x}\, dx$, then

a. $2I_1 = I_2$
b. $I_1 = I_2$
c. $I_1 = 2I_2$
d. $I_1 + I_2 = 0$

Ans. B

Solution:

Put logx=t
Then (1/x)dx=dt

$$\text{If } I_1 = \int_e^{e^2} \frac{dx}{\log x} = \int_1^2 \frac{e^t}{t}dt \text{ and } I_2 = \int_1^2 \frac{e^x}{x}\, dx, \text{ then}$$

$$I_1 = I_2$$

15. $\int_{-\pi/2}^{\pi/2} \sin|x|\, dx =$

a. 0
b. π
c. 2
d. None of these

Ans. C

Solution:

$$\sin|x| = even..function$$

$$\int_{-\pi/2}^{\pi/2} \sin|x|\,dx = \int_0^{\pi/2} 2\sin x\,dx = 2[-\cos x]_0^{\pi/2} = 2$$

16. $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x}\,dx =$

a. $\frac{\pi}{4}$
b. 1
c. π
d. 0

Ans. D

Solution:

$$I = \int_0^{\pi/2} \frac{\cos(\pi/2 - x) - \sin(\pi/2 - x)}{1 + \cos(\pi/2 - x)\sin(\pi/2 - x)}dx$$

$$= \int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x}dx = -I$$

$$2I = 0$$

$$I = 0$$

17. Evaluate: $\int_0^{100} e^{x-[x]}dx$

a. $100[e-1]$
b. $100[e+1]$
c. 100
d. 50

Right Answer Explanation: A

$$\int_0^{100} e^{x-[x]}dx$$

$x-[x]$ is a perodic function of period 1

$$\int_0^{100} e^{x-[x]}dx$$

$$= 100\int_0^{1} e^{x}dx$$

$$= 100[e-1]$$

18. If $f(x) = \int_1^x \frac{dt}{t}$, then f(x) satisfies
    a. f (xy) = f (x) + f (y)
    b. $f\left(\frac{x}{y}\right) = f(x) + f(y)$
    c. f (x + y) = f (x) + f (y)
    d. none of these

    Ans. A

    Solution:

    Put v=logt

    Then dt/t=dv

    Then f (xy) = f (x) + f (y)

19. If $k\int_0^1 x\ f(3x)\ dx = \int_0^3 t\ f(t)\ dt$, then the value of k is equal to
    a. 3
    b. 1/3
    c. 9
    d. 1/9

    Ans. C

    Solution:

Put $t = 3x$ in the LHS integral

Then $dx = \frac{1}{3}dt$. As $x \in [0,1], t \in [0,3]$

$$\therefore k\int_0^1 x\ f(3x)dx = \int_0^3 t\ f(t)dt \Rightarrow k.\int_0^3 \frac{1}{3}f(t).\frac{1}{3}dt = \int_0^3 t\ f(t)dt \Rightarrow k = 9.$$

20. $\int_0^1 \frac{\sqrt{\tan^{-1} x}}{1+x^2}dx =$

a. $\frac{2}{3}\left(\frac{\pi}{2}\right)^{\frac{3}{2}}$

b. $\frac{2}{3}\left(\frac{\pi}{4}\right)^{\frac{3}{2}}$

c. $\frac{1}{2}\left(\frac{\pi}{4}\right)^{\frac{1}{2}}$

d. None of these

Ans. B

Solution:

$$\int_0^1 \frac{\sqrt{\tan^{-1} x}}{1+x^2}dx = \int_0^{\pi/4}\sqrt{v}dv = \frac{2}{3}\left(\frac{\pi}{4}\right)^{3/2}$$